

A New Approach for Small Satellite Gyroscope and Star Tracker Fusion

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Abstract

This paper discusses the use of hybridization of measurements from different sensors (namely the gyroscope and star tracker) in order to estimate the attitude of a satellite. In this study, an adaptive fuzzy logic algorithm is developed in order to add robustness to the existing extended Kalman-filtering method of sensor fusion. Models of the sensor are developed and a simulation is programmed in Matlab in order to examine the effectiveness of the new algorithm. Results from the simulations show that the fuzzy logic algorithm allows for better pointing accuracy during the period of Star tracker unavailability. This method used the EKF filter effectively and optimally when the system parameters and noises are known. However in practice some of these parameters are uncertain, leading to the ineffectiveness when applied to system with high accuracy requirements or complex system. In order to overcome this constraint, fuzzy algorithm can be used to evaluate the reliability of the each component then estimate the appropriate tuning parameters. This approach will facilitate small satellite attitude fault-tolerance estimator.

Keywords: Fusion, Fuzzy Logic Estimator, Satellite Gyroscope, Star Tracker

1. Introduction

Sensor fusion generally refers to the process of combining measurements from several different sensors in order to obtain a better result than considering each sensor separately. Sensor fusion is especially useful for such system equipped with different types of sensors which are differently characterized by output types, sampling frequencies and accuracies. Common attitude sensors on board an Earth-observation small satellite are grouped into two types:

- Inertial sensors: such as accelerometer and velocity sensors.
- Reference sensors: such as Sun sensor, Earth sensor, magnetometer and star tracker (or stellar sensor).

Each type of the above sensors has its own advantages and disadvantages. Particularly, inertial sensors are of high accuracy (typically to the one hundredth of a degree) but suffering from drift over time and may contain moving parts; reference sensors do not provide

highly accurate output (accuracy of around one to several degrees) and depend on the reference frame, but are low cost, robust and containing no moving parts. Moreover, an in-orbit satellite is considered as a non-linear system due to influences by unwanted factors including:

- External Forces: Perturbation forces, solar pressure, gravity gradient.
- Internal Forces: Vibration effects by solar panel mechanism, momentum wheels, disturbance due to the actuation of propulsion valves or pyrotechnique
- Sensors' inaccuracies.

The diagram in Figure 1 details the components of an AOCS subsystem on-board a small satellite.

This paper focus on the attitude estimator by fusing gyroscope and stellar sensor. Extended Kalman filter (EKF) is one of the most common algorithms because of its simplicity and effectiveness for low-Earth orbit satellites with light requirements in attitude accuracy. However, this method is impacted by uncertain parameters such as the system noise, measurement noise and especially in

the case of failure or unavailability of one sensor. This fact causes the estimator delivering unreliable measurements, i.e. when the star tracker is blinded by the Moon or the Sun, or when the gyroscope suffer from over-drift or scale factor. As a result, it is necessary to apply adaptive algorithms, fuzzy algorithm in this paper, to develop a fault-tolerance estimator.

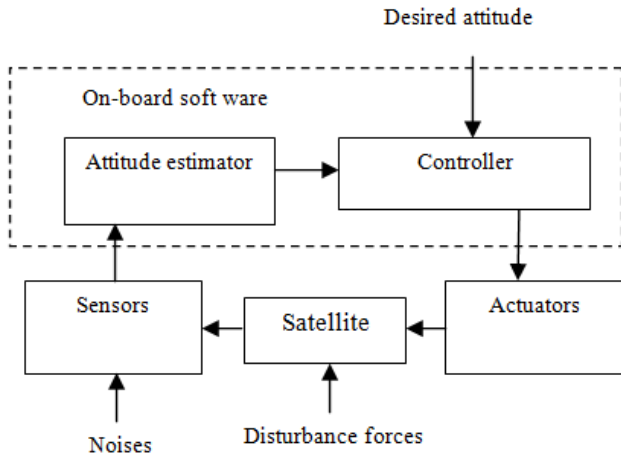


Figure 1. Small satellite attitude and orbit control subsystem.

2. Quaternion Representation for Satellite Attitude

Satellite attitude can be represented by: Euler angles, directional cosine matrices, and quaternions. Among them, quaternions are preferable thanks to its efficiency and stability in computation, and are widely used in simulation programs and attitude estimation software.

A quaternion is a 4-dimension vector, as follows:

$$q = [q_1 \ q_2 \ q_3 \ q_4]^T \equiv [\hat{q} \ q_4]^T \tag{1}$$

Where:

$$\hat{q} = [q_1 \ q_2 \ q_3]^T = \hat{e} \sin\left(\frac{\theta}{2}\right) \tag{2}$$

$$q_4 = \cos\left(\frac{\theta}{2}\right)$$

\hat{e} is the Euler axis unit matrix and θ is the angle of rotation. As this is a 4-dimension vector and used to represent a 3-dimension rotation, the components of quaternion cannot be independent and have to satisfy the following condition (normalization condition):

$$q^T q = q_1^2 + q_2^2 + q_3^2 + q_4^2 = 1 \tag{3}$$

The corresponding directional cosine matrix can be computed as:

$$A(q) = (q_4^2 - \|\hat{q}\|^2) I_{3 \times 3} + 2\hat{q}\hat{q}^T - 2q_4\hat{q}^{\wedge x} \Psi(q) \tag{4}$$

Where:

$I_{3 \times 3}$ is the unit matrix

$$\Xi(q) = \begin{bmatrix} q_4 I_{3 \times 3} + \hat{q}^{\wedge x} \\ -\hat{q}^T \end{bmatrix}$$

$$\Psi(q) = \begin{bmatrix} q_4 I_{3 \times 3} + \hat{q}^{\wedge x} \\ -\hat{q}^T \end{bmatrix} \tag{5}$$

$$\hat{q}^{\wedge x} = \begin{bmatrix} 0 & -q_3 & q_2 \\ q_3 & 0 & -q_1 \\ -q_2 & q_1 & 0 \end{bmatrix}$$

The dynamic equation shall take the form:

$$\dot{q} = \frac{1}{2} \Xi(q) \omega = \frac{1}{2} \Omega(\omega) q \tag{6}$$

Where ω is the 3-dimensional velocity vector and:

$$\Omega(\omega) = \begin{bmatrix} -\omega^x & \omega \\ -\omega^T & 0 \end{bmatrix} \tag{7}$$

Therefore, the key advantage of using quaternions is that the kinetic equations are linear and non-singular. Additionally, successive rotations can be obtained by quaternion multiplication. For example:

$$A(q)A(q) = A(q \otimes q) \tag{8}$$

The product of two quaternions is bilinear:

$$q \otimes q = [\Psi(q) \ q]q = [\Xi(q) \ q] \dot{q} \tag{9}$$

The inverse of a quaternion matrix is given as:

$$q^{-1} = \begin{bmatrix} -\hat{q} \\ q_4 \end{bmatrix} \tag{10}$$

Where $A(q^{-1}) = A^T(q)$

It is noticeable that:

$q \otimes q^{-1} = [0001]^T$ is the unit quaternion.

3. Modelling of Attitude Sensors

A. Modelling of Gyroscope

Gyroscope is considered as an orientation measuring device with relatively good accuracy. Mechanical sensors operating on the principle of conservation of angular momentum, usually consists of a rotating wheel or disk

around its shaft. One of the main disadvantages of these sensors is its moving parts, leading to the declining in their usage. They are being replaced by optical fiber or laser gyroscope. Both of them operate on the Sagnac effect. They are of high accuracy; containing no moving parts and high reliability. However they are suffering from drifts and noises. In particular, the commonly used optic gyroscopes' drifts are highly susceptible to the Earth's magnetic field. This needs to be seriously considered when using gyroscopes for attitude determination.

For a 3-axis attitude stabilized satellite, each axis will be equipped with a gyroscope for its attitude prediction. As such, there will be error resulted from the mechanical misalignment of the mounting of the gyroscopes on the satellite platform.

Mathematical model of a gyroscope can be represented in Equation (11).

$$\begin{aligned} \dot{\omega} &= (I_{3 \times 3} + S)\omega + \beta + \eta_v \\ \dot{\beta} &= \eta_u \\ \dot{s} &= \eta_s \\ \dot{k}_u &= \eta_u \\ \dot{k}_L &= \eta_L \end{aligned} \tag{11}$$

$$S \equiv \begin{pmatrix} s_1 & k_{U1} & k_{U2} \\ k_{L1} & s_2 & k_{U3} \\ k_{L2} & k_{L3} & s_3 \end{pmatrix}$$

Where: $\tilde{\omega}$ as the measured velocity, β as the drift, S as the scale factor matrix

s, k_U and k_L as the misalignment error;

η_u, η_v, η_u and η_L as the Gaussian white noises with the zero mean value and:

$$\begin{aligned} E\{\eta_v(t)\eta_v^T(t)\} &= \sigma_v^2 \delta(t-\tau) I_{3 \times 3} \\ E\{\eta_u(t)\eta_u^T(t)\} &= \sigma_u^2 \delta(t-\tau) I_{3 \times 3} \\ E\{\eta_v(t)\eta_v^T(t)\} &= \sigma_v^2 \delta(t-\tau) I_{3 \times 3} \\ E\{\eta_L(t)\eta_L^T(t)\} &= \sigma_L^2 \delta(t-\tau) I_{3 \times 3} \end{aligned} \tag{12}$$

Where $E\{\}$ denotes the expectation and $\delta(t-\tau)$ denotes the Dirac pulse function.

B. Modelling of Star Tracker

A star tracker is high accuracy sensor. The sensor image the stars and compares the captured pictures with a star map on board, the differences then be used to calculate the satellite orientation. The sensor's accuracy depends on several factors such as the number of stars it can track,

its star map and the quality of its optical components. Its accuracy is usually in the range of several arc seconds.

Star tracker is modeled as:

$$q_s = \delta q \otimes q \tag{13}$$

Where:

q_s is the quaternion measured attitude,

q is the quaternion real attitude,

δq is the sensor noise.

4. Gyroscope - Star Tracker Fusion using Extended Kalman Filter (EKF)

The general structure of an EKF filter is given in¹. However, multiple sensors of different types are used on board; there must be some modifications to the measurement matrix H_k and the error vector computation ε for each sensor. The state vector of the estimator is given as:

$$x = [q \ \beta_x \ \beta_y \ \beta_z]^T \tag{14}$$

Where: $q = [q_1 \ q_2 \ q_3 \ q_4]$ as the satellite attitude quaternion. This vector is defined by (1), (2) and (3):

The error quaternion is given as:

$$\delta q = q \otimes \hat{q}^{-1} \tag{15}$$

Where $\delta q = [\delta q_1 \ \delta q_2 \ \delta q_3 \ \delta q_4]^T$ and the inverse quaternion $q^{-1} = [-q_4 \ q_1 \ q_2 \ q_3]^T$. Taking the difference of (15) leads to:

$$\delta \dot{q} = \dot{q} \otimes \hat{q}^{-1} + q \otimes \hat{q}^{-1} \tag{16}$$

Eventually, the kinematic equation is calculated as:

$$\dot{\hat{q}} = \frac{1}{2} \Xi(\hat{q}) \hat{\omega} = \frac{1}{2} \Omega(\hat{\omega}) \hat{q} \tag{17}$$

In order to find the quaternion propagation equation, the above kinematic equation will be expanded using a series of exponential function as:

$$\begin{aligned} e^{\frac{\Omega(\hat{\omega})t}{2}} &= I_{4 \times 4} \sum_{k=0}^{\infty} \left\{ \frac{\left[\frac{1}{2} \Omega(\hat{\omega})t \right]^{2k}}{(2k)!} + \left\| \hat{\omega} \right\|^{-1} \Omega(\hat{\omega}) \sum_{k=0}^{\infty} \frac{\left[\frac{1}{2} \Omega(\hat{\omega})t \right]^{2k+1}}{(2k+1)!} \right\} \\ &= I_{4 \times 4} \cos\left(\frac{1}{2} \left\| \hat{\omega} \right\| t\right) + \Omega(\hat{\omega}) \frac{\sin\left(\frac{1}{2} \left\| \hat{\omega} \right\| t\right)}{\left\| \hat{\omega} \right\|} \end{aligned} \tag{18}$$

The propagation equation is given simply as:

$$\hat{q}_{k+1}^- = \bar{\Omega}(\hat{\omega}_k^+) \hat{q}_k^+ \quad (19)$$

With $\hat{\omega}_k^+$ and \hat{q}_k^+ as prediction vectors after update and $\bar{\Omega}(\hat{\omega}_k^+)$ is defined as:

$$\bar{\Omega}(\hat{\omega}_k^+) = \begin{pmatrix} I_{3 \times 3} \cos\left(\frac{1}{2}\|\hat{\omega}_k^+\|\Delta t\right) & \hat{\Psi}_k^+ \\ -\hat{\Psi}_k^+ & \cos\left(\frac{1}{2}\|\hat{\omega}_k^+\|\Delta t\right) \end{pmatrix} \frac{\cos\left(\frac{1}{2}\|\hat{\omega}_k^+\|\Delta t\right) \hat{\omega}_k^+}{\|\hat{\omega}_k^+\|} \quad (20)$$

$$\hat{\omega}_k^+ = \hat{\omega}_k - \hat{\beta}_k^+ \hat{\beta}_k^- \hat{\beta}_k^+ \quad (21)$$

Δt is the sampling period.

The covariance matrix is calculated as follows:

$$\begin{aligned} P_{k+1}^- &= \Phi_k P_k^- P_k^{T^-} + G_k Q_k G_k^T \\ \Phi_k &= \begin{pmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{pmatrix} \\ \Phi_{11} &= I_{3 \times 3} - [\hat{\omega}_k^{+X}] \frac{\sin(\|\hat{\omega}_k^+\|\Delta t)}{\|\hat{\omega}_k^+\|} + [\hat{\omega}_k^{+X}]^2 \frac{[1 - \cos(\|\hat{\omega}_k^+\|\Delta t)]}{\|\hat{\omega}_k^+\|^3} \\ \Phi_{21} &= 0_{3 \times 3} \\ \Phi_{22} &= I_{3 \times 3} \\ G_k &= \begin{pmatrix} -I_{3 \times 3} & 0_{3 \times 3} \\ 0_{3 \times 3} & I_{3 \times 3} \end{pmatrix} \\ Q_k &= \begin{pmatrix} (\sigma_v^2 \Delta t + \frac{1}{3} \sigma_u^2 \Delta t^3) I_{3 \times 3} & -(\frac{1}{2} \sigma_u^2 \Delta t^2) I_{3 \times 3} \\ -(\frac{1}{2} \sigma_u^2 \Delta t^2) I_{3 \times 3} & (\sigma_v^2 \Delta t) I_{3 \times 3} \end{pmatrix} \end{aligned} \quad (22)$$

With σ_u^2 and σ_v^2 as the standard deviations.

In order to determine the update function, the state prediction error is given as:

$$\Delta \hat{X}_k^+ \equiv [\Delta \hat{\theta}_k^{+T} \quad \Delta \hat{\beta}_k^{+T}] \quad (23)$$

For small angles it is possible to approximate $\delta \hat{q} \approx \frac{\delta \theta}{2}$ and $q_4 \approx 1$. As such, the 4-dimension vector can be replaced by the 3-dimension Euler angle vector.

The next step is to determine the measurement matrix H_k . The measurement model is given as:

$$y_k \equiv h_k(\hat{X}_k) + V_k \quad (24)$$

With y_k as the measured vector, \hat{X}_k as the state, h_k as the measurement matrix and V_k as the noise.

For the star tracker:

$$q_k = \delta q_k \otimes q_k^- = [\Xi(q_k^-) q_k^-] \delta q_k \quad (25)$$

Its measurement matrix is given as:

$$H_k(\hat{X}_k^-) \equiv \frac{\delta h_k(\hat{X}_k^-)}{\hat{X}_k^-} \Big|_{\hat{X}_k^-} = \left[\frac{1}{2} \Xi(q_k^-) 0_{3 \times 3} \right] \quad (26)$$

Lastly, the update function for the state error vector is given as:

$$\Delta \hat{x}_k^+ = K_k [y_k - h_k(\hat{x}_k^-)] \quad (27)$$

With K_k as the filter gain, $h_k(\hat{X}_k^-)$ as the measurement estimation, for star tracker this estimation is the quaternion q_k^- .

Update function for the gyroscope:

$$\hat{\beta}_k^+ = \hat{\beta}_k^- + \Delta \hat{\beta}_k^+ \quad (28)$$

Covariance matrix and filter gains are calculated as followed:

$$\begin{aligned} P_k^+ &= [I - K_k H_k(\hat{x}_k^-)] P_k^- \\ K_k &= P_k^- H_k^T(\hat{x}_k^-) [H_k(\hat{x}_k^-) P_k^- H_k^T(\hat{x}_k^-) + R_x]^{-1} \end{aligned} \quad (29)$$

Summing up, the gyro-stellar fusion using EKF is given in Table 1.

Table 1. EKF Filter for Gyro-Stellar Fusion

Initialize	$\hat{q}(k_0) = \hat{q}_0$ $\beta(k_0) = \beta_0$ $P(k_0) = P_0$
Propagation	$\hat{\omega}_k^+ = \hat{\omega}_k - \hat{\beta}_k^+$ $\hat{\beta}_k^- = \hat{\beta}_k^+$ $\hat{q}_{k+1}^- = \bar{\Omega}(\hat{\omega}_k^+) \hat{q}_k^+$ $\hat{P}_{k+1}^- = \Phi_k P_k^- \Phi_k^T + G_k Q_k G_k^T$
Gain computation	$K_k = P_k^- H_k^T(\hat{x}_k^-) [H_k(\hat{x}_k^-) P_k^- H_k^T(\hat{x}_k^-) + R_x]^{-1}$ $H_k(\hat{x}_k^-) = \left[\frac{1}{2} \Xi(q_k^-) 0_{3 \times 3} \right]$
Update	$\Delta \hat{x}_k^+ = K_k [y_k - h_k(\hat{x}_k^-)]$ $\hat{\beta}_k^+ = \hat{\beta}_k^- + \Delta \hat{\beta}_k^+$ $\hat{q}_k^+ = \hat{\beta}_k^- + \frac{\Xi(q_k^-)}{2} \delta \hat{\theta}_k^+$ $P_k^+ = [I - K_k H_k(\hat{x}_k^-)] P_k^-$

The simulation results of this estimator will be illustrated in the session VI.

5. Proposed Fuzzy Algorithm for Tuning the EKF Gyro-Stellar Estimator

During the operation on a Sun-synchronous orbit, the attitude determination subsystem has to handle the following constraints:

- Star tracker: Despite of its high accuracy, it is not always possible to deliver measurements with the same accuracy. In particular if there are too few stars in the field of view of the sensor the accuracy will be degraded. In addition, when there are bright objects (like the Sun or the Moon) in its field of view, its measurements will be unreliable due to its error.
- Gyroscope: the drift over time has a large impact on the accuracy of the angular measurements. And, its scale factor error and calibration error must be carefully considered.

On the other hand, Kalman filter only works well when the system noise model w_k (with covariance matrix Q_k) and the measurement noise v_k (with covariance matrix R_k) are white noise processes with zero mean. However, in practice all the parameters of the system cannot be known exactly and these noise processes are not white noises. This fact leads to the surplus error of the EKF ($y_k - h_k(\hat{x}_k)$) not being white noise any more. If this error gets too large due to uncertain elements the EKF filter will not be able to converge. These are only two factors that affects the convergence of the EKF filter, there may be many more other such as the drift or scale factor of the gyroscope, the misalignment of the mechanical mounting process of the sensors.

Based on the above analysis, in order to maintain the accuracy, stability and fault-tolerance of the attitude estimator there has to be a mechanism to monitor the evolution of this surplus error, and a mechanism for compensating the gains of the EKF filter. Fuzzy logic algorithm is proposed to monitor and compensate the parameters Q_k and R_k of the filter. This is proposed as follows:

$$\begin{aligned} Q_{k_new} &= Q_0 \alpha^{-2(k-1)} \\ R_{k_new} &= R_0 \alpha^{-2(k-1)} \end{aligned} \tag{30}$$

With $\alpha \geq 1$ as the tuning coefficient, if $\alpha = 1$ then this is the normal EKF filter, Q_0, R_0 are constant matrices.

The fuzzy algorithm works on the following principle: keep monitoring the mean and standard deviation of the

surplus error of the EKF filter, then assess and calculate the calibration value α according to this fluctuation. In practice this process aims at assessing the sensors' reliability, if a sensor's reliability is high then the filter will increase the confidence weight and prioritize the use its measurement and vice versa. This is also the background to develop evaluation and ranking algorithm, creating the foundation for attitude determination system with fault-resistance capability.

The fuzzy algorithm takes the mean and standard deviation of the surplus error as inputs and the output is the tuning coefficient α . The block diagram of the algorithm is given in Figure 2.

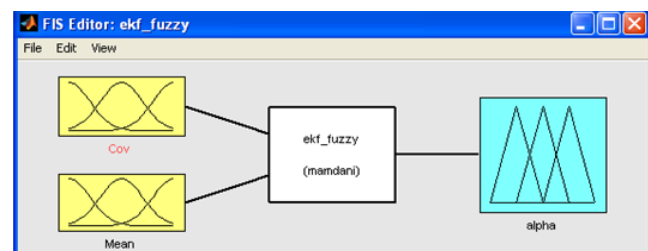


Figure 2. Fuzzy based tuning block.

Next step, the Membership function (MF) is developed. The MF function of the inputs is divided into 3 levels: zero, small and large corresponding to the value of the inputs. The MF function for the mean value input is given in Figure 3.

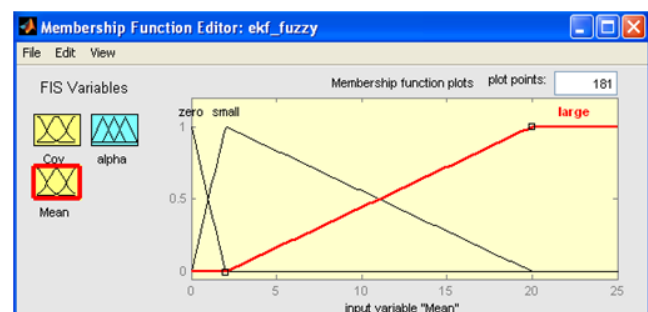


Figure 3. MF function for input mean value.

For the output α , 4 levels are the range of $[1; 1.2]$ as: zero, small, average and large. Its MF function is given in Figure 4.

The fuzzy referring is proposed in Table 2.

Table 2. Fuzzy referring for Gyro-Stella estimator

	Value of α	Input 1: value of the mean		
		Small	Large	
Input 2:	Zero	Small	Small	Large
standard	Small	Zero	Large	Average
deviation	Large	Large	Average	Zero

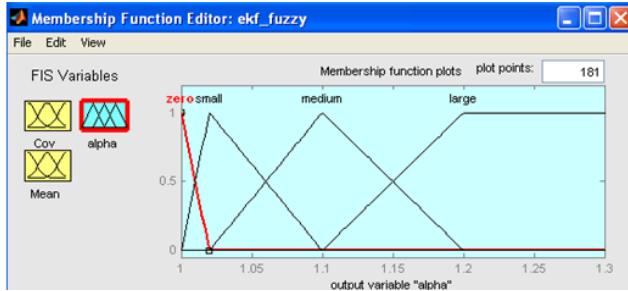


Figure 4. MF function for output.

6. Simulation

The simulation is implemented in MATLAB with the following parameters:

Satellite specifications:

- Inertial matrix:

$$J = \begin{pmatrix} 13.5 & 0 & 0 \\ 0 & 21 & 0 \\ 0 & 0 & 18 \end{pmatrix}_{kg.m^2}$$

- Sampling frequency: 20 Hz
- Simulation time: 20 s
- Star tracker: 4 radian (3σ) error
- Gyroscope: $6^\circ/h$ drift, angular rate walk $ARW=0.15^\circ/\sqrt{h}$.
- Perturbation torque:

$$\tau_{ext} = [0.005 \ -0.008 \ 0.005]^T$$

- Initial condition:

$$x = [1 \ 0 \ 0 \ 0 \ 0]^T$$

The result of the simulation is depicted by Figure 5.

Suppose that the star tracker suffers a loss of some measurements (due to noise) during the operation then the result of the simulation is illustrated in Figure 6.

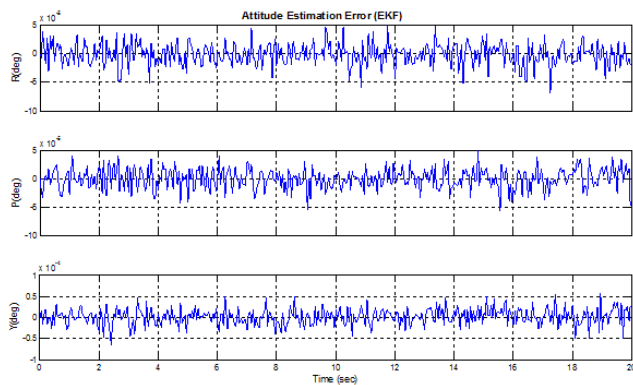


Figure 5. Simulation results by normal EKF filter.

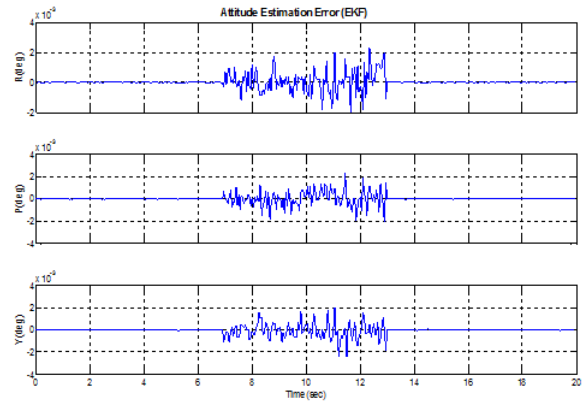


Figure 6. Simulation results by normal EKF filter (impacted by noise).

It can be seen that when the measurement data of a star tracker loses reliability then the EKF can only use data from the gyroscopes to estimate the attitude. However due to the drifts of the gyroscopes the longer the duration of sensor loss the more the error of the filter will accumulate, decreasing the reliability of the filter, and even leads to divergence. As a compensation method, fuzzy algorithm is applied. This algorithm calculates the reliability of the sensors using the surplus error, then estimate the calibration coefficient. The result after applying fuzzy algorithm is given in Figure 7.

Comparing the results, the fuzzy algorithm has improved greatly the effectiveness of the filter, especially when measurement anomaly appears in the star tracker.

In the scope of this article, fuzzy algorithm is applied only to EKF filter with star tracker and gyroscopes as two inputs. However, this technique can also be applied to other sensors fusion with more than two types of sensors in order to improve their accuracy and stability. This is also the foundation to develop adaptive sensor fusion with fault-tolerance capability.

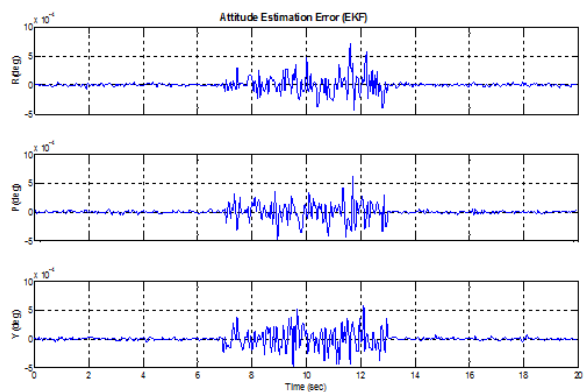


Figure 7. Simulation results by fuzzy tuned EKF filter (impacted by noise).

7. Conclusions

This article describes the method of sensor-fusion using the extended Kalman filter to estimate the attitude of a small satellite. This method used the EKF filter effectively and optimally when the system parameters and noises are known. However in practice some of these parameters are uncertain, leading to the ineffectiveness when applied to system with high accuracy requirements or complex system. In order to overcome this constraint, fuzzy algorithm can be used to evaluate the reliability of the each component then estimate the appropriate tuning parameter. This approach will facilitate small satellite attitude fault-tolerance estimator.

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