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A Comprehensive Analysis of Optimal Performance Parameters of Stand-Alone Generator

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Abstract

Background/Objectives: This paper aims to evaluate performance characteristics of standalone Self-Excited Induction Generator (SEIG) under varying load condition. **Methods/Statistical Analysis:** Prime mover speed, excitation capacitance and load significantly influence the performance of SEIG. Due to nonlinear magnetization, estimation of nonlinear magnetic characteristics involves clumsy mathematical computation. It gives an opportunity to model nonlinear magnetization characteristics suitable to obtain minimum impedance and optimal capacitance for improved performance. **Findings:** The effect of regression on nonlinear magnetization curve has been discussed while estimation of magnetization reactance by employing regression functions. It provides optimal value of magnetization reactance to compute performance variables of SEIG. Solution techniques are applied to calculate frequency and excitation capacitance and compared their results. Regression functions with solution techniques have been applied on comprehensive data of induction machines to exhibit validity and accuracy of proposed scheme. **Applications/Improvements:** The proposed techniques reveal lower capacitance requirement as compared to existing piece- wise linearization model.

Keywords: Comprehensive Analysis, Nonlinear Constraint Optimization, Optimal Performance Parameters, Stand-Alone Generator

1. Introduction

With advancement in new edge machine technology, SEIG has gained attention of researchers in the field of renewable energy exploration and exploitation. SEIG has increasingly been used in generating electrical energy from conventional and non-conventional energy sources[1,2]. It has relative advantages over conventional synchronous generators like low cost, brushless and rugged construction, reduced size, better transient performance, lower maintenance and inherent protection against short-circuit[3-7]. Performance analysis of SEIG includes study of variation in performance variables viz. estimation of required excitation capacitance, speed variation, voltage regulation and frequency regulation under different operating constraints[8,9].

An induction machine operates as SEIG, if rotor is driven by an external prime mover and an appropriate capacitor bank is connected across its stator terminals to meet the excitation requirement[10]. Excitation capacitor affects air gap voltage, frequency, amount of current flowing through stator winding and efficiency of SEIG. Therefore optimal calculation of excitation capacitance is required for optimal steady state performance analysis of SEIG. Literature includes symmetrical components based on equivalent circuit model[11], eigen values and eigen sensitivity based iterative method[12,13], per-phase equivalent circuit approach[14] and generalized machine theory based d-q axis model[15,16] for determination of excitation capacitance. Among these methods, symmetrical component model fails to analyze dynamic/transient performance directly[11]. It doesn't provide minimum and maximum values of excitation capacitance of SEIG. Eigen value and eigen sensitivity based iterative method determines minimum and maximum value of excitation capacitance[12,13]. A direct approach has been proposed to determine the value

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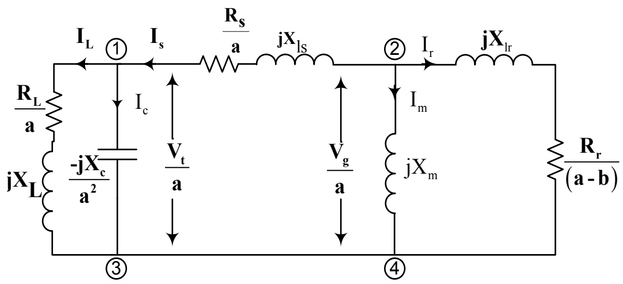

Figure 1. Per-phase equivalent circuit of SEIG.

of excitation capacitance by solving two non-linear equations using Newton-Raphson method with resistive and reactive load[17,18]. Per-phase equivalent circuit approach includes load impedance and neglects core losses. It can be analyzed either by loop-impedance method[4,14] or nodal admittance method[19] to compute the minimum capacitance required for self-excitation under loaded condition using trial-and-error technique[20]. Loop impedance and nodal admittance methods are lengthy, prone to errors and time consuming. It requires initial guess, selection of search space and needs to segregate admittance/impedance to real and imaginary parts[17].

Required excitation capacitance for SEIG can be estimated by determining magnetizing reactance from magnetization curve. Nonlinear nature of magnetization curve has been analyzed by piecewise linearized model[18-22] and polynomial approximation[23-25] for the ease of computation. These models provide approximate value of magnetizing reactance due to poor fitness of magnetization curve. This approximate value of magnetizing reactance gives fairly accurate values of performance variables including excitation capacitance. Optimum value of performance variables can be determined using precise value of magnetization reactance by proper fitness of magnetization curve. It can be accomplished by employing different regression functions in place of piecewise linearized approximation.

This paper proposes simplified method to calculate performance variables of SEIG. It reduces effort required to solve nonlinear equations involved in nodal admittance method of per-phase equivalent circuit for calculation of optimal excitation capacitance. Levenberg-Marquardt algorithm has been employed to observe the goodness of fit of magnetization curve for eight regression functions including exponential, higher degree polynomial, Gaussian, power and sinusoidal functions. It determines optimum value of magnetizing reactance. Performance variables of SEIG have been calculated by 'eigen value computation' solution technique. Further, these results are compared with trust-region dogleg, trust-region reflective, Levenberg-Marquardt and MATLAB 'fzero' solution techniques. All the regression methods and solution techniques have been employed on ten induction machines of different ratings to prove effectiveness of proposed technique.

2. Steady State Analysis of SEIG

Air gap voltage (V_g) and per unit frequency (a) of SEIG varies with per unit rotor speed (b), excitation capacitance (C) and load impedance (Z_L). Magnetizing reactance (X_m) is considered as a variable quantity which depends on saturation level of magnetic circuit. Nonlinear magnetization curve is the process of voltage buildup with respect to magnetizing reactance (X_m) at a particular value of capacitance as shown in Figure 1[20]. For ease of calculation, core losses and mechanical losses can be neglected due to magnetic saturation without affecting accuracy of performance analysis substantially. An appropriate circuit representation and exact mathematical modelling is required to evaluate steady state performance analysis of SEIG for different operating conditions. Per-phase equivalent circuit of a three-phase SEIG with excitation capacitance and resistive-inductive load for steady state analysis has been shown in Figure 1. The six variable parameters viz. air gap voltage, excitation capacitor reactance, per unit frequency, per unit rotor speed, load impedance and magnetizing reactance should be known for performance analysis of SEIG.

Loop impedance and nodal admittance methods have been used to evaluate variable parameters for steady state analysis of SEIG[27]. The values of variable parameters has been evaluated by equations (iii-vi) for loop impedance method and equation (xiii-xiv) for nodal admittance method as given in Appendix – I.

3. Performance Analysis of SEIG

System stability, dynamic behavior and machine design analysis involves simulation of nonlinear magnetization curve. It is defined by piecewise linearized model, in which a nonlinear curve is sub-divided into parts and each part is defined by separate straight line equation. A single function with minimum error over whole range needs to be evaluated in order to obtain proper curve fitting. The function is supposed to be easy in application and mathematically simple. Approximation of magnetization curve by Fourier series, power series, hyperbola and transcendental function has been found valid only over specific ranges of curve[28]. Determination of such approximate expressions for non-saturated ranges is easier as compared to knee range of magnetization curve.

Regression technique is used to obtain best fit mathematical function for given set of data. Linear least squares fitting is applied iteratively to a linearized form of function until convergence is achieved for nonlinear least squares fitting. Least square comprises of minimizing the sum of squares of errors between a set of measured data points and a parameterized function. For non-linear curve fitting, Levenberg-Marquardt algorithm[29] has been used to optimize the parameters of the function $f(X_i, \alpha)$ so that sum of squares of deviations $S(\alpha)$ becomes minimum.

$$S(\alpha) = \sum_{i=1}^{N} w_i \left[Y_i - f(X_i, \alpha) \right]^2 \tag{1}$$

Where X_i is row vector for i^{th} observation, α is parameter to be computed, w_i is weight of i^{th} observation and N is the number of observations. The iteration process is initialized by providing initial guess to parameter α. In case of multiple minima, the algorithm converges to global minimum only if the initial guess is close to final solution. Levenberg-Marquardt algorithm is used to solve nonlinear least squares problems. It is a combination of two minimization methods: gradient descent algorithm when the parameters are far from their optimal value and Gauss-Newton algorithm when the parameters are close to their optimal value. In gradient descent method, sum of squared errors is reduced by updating the parameters in the direction of greatest reduction of least squares objective. In Gauss-Newton method, sum of squared errors is reduced by assuming least squares function as locally quadratic, and finding the minimum of quadratic.

Non-negative damping factor is adjusted at each iteration. If reduction of $S(\alpha)$ is rapid, a smaller value is used to bring the algorithm closer to Gauss–Newton algorithm; whereas if an iteration gives insufficient reduction in the residual, damping factor can be increased to bring algorithm closer to gradient descent algorithm.

3.1 Regression Functions

A function has to be determined to fit the nonlinear magnetization curve over the entire useful range. Some representations of functions used in this paper are given in Table 1.

3.2 Solution Techniques

Variable parameters of SEIG can be determined by solving regression function using proper solution technique. Trust-region dogleg, trust-region reflective, Levenberg-Marquardt approach and optimization tool 'fzero' of MATLAB has been employed.

3.2.1 Eigen Value Computation Approach[30]

Eigen value problem is of substantial theoretical importance and wide-ranging application viz. solving systems of differential equations, analyzing population growth models, and calculating powers of matrices. The characteristic polynomial of degree n is represented by a n x n matrix A, has exactly n complex roots (with zero or nonzero imaginary part). The polynomial can be factored into the product of n linear terms;

$$\det(\lambda I - A) = (\lambda - \lambda_1)(\lambda - \lambda_2)(\lambda - \lambda_3)\ldots(\lambda - \lambda_n) \tag{2}$$

$$= c_1\lambda^n + c_2\lambda^{n-1} + \ldots + c_n\lambda + c_{n+1} \tag{3}$$

Where each λ_i is a complex number. The numbers λ_1, $\lambda_2, \lambda_3, \ldots \lambda_n$ (which may not be all distinct) are roots of the polynomial, and are represented as eigen values of A. $c_1, c_2, \ldots, c_n, c_{n+1}$ are the coefficients of polynomial A.

The eigen values may have non-zero imaginary parts even for all real values of A. Also eigen values may be irrational numbers for all rational or integer values of A. The non-real roots of a real polynomial with real coefficients can be grouped into pairs of complex conjugate values, namely with the two members of each pair having same real part and imaginary parts that differ only in sign. If the degree is odd, then by the intermediate value theorem at least one of the roots is real. Therefore, any real matrix with odd order has at least one real eigen value - whereas a real matrix with even order may have no real eigen values.

In this paper eigen value computation approach is used by 'poly' and 'roots' commands of MATLAB. poly(A) generates the characteristic polynomial of A, and roots (poly(A)) finds the roots of that polynomial, which are the eigen values of A. 'poly (A) returns the coefficients of polynomial ordered in descending powers in a row vector whereas 'roots' returns the roots of polynomial A in a column vector. For vectors, 'roots' and 'poly' are inverse functions of each other up to ordering, scaling, and round-off error.

Table 1. Regression Functions

Fitting Type	Regression Function	Equation	Description
I	One-Term Exponential Function	$V_g = pe^{qX_m}$	Rate of change of a quantity is proportional to initial amount of quantity. Sign of coefficient q and/or s represents exponential decay or growth of function.
II	Two-Term Exponential Function	$V_g = pe^{qX_m} + re^{sX_m}$	
III	Gaussian Function	$V_g = pe^{\left[-\left(\frac{X_m - q}{r}\right)^2\right]}$	p , q and r represents amplitude, centroid (location) and peak width respectively.
IV	Polynomial First Order Function	$V_g = pX_m + q$	Taylor series expansion of the unknown nonlinear function using least squares method.
V	Polynomial Second Order Function	$V_g = pX_m^2 + qX_m + r$	
VI	Polynomial Third Order Function	$V_g = pX_m^3 + qX_m^2 + rX_m + s$	
VII	Power Function	$V_g = pX_m^q + r$	Parameter p, q and r are the intercept on y-axis, scaling factor and exponent/power.
VIII	Sinusoidal Function	$V_g = p\sin\left(qX_m + r\right)$	p , q and r represents amplitude, frequency and phase constant.

3.2.2 Trust-Region Dogleg Approach[31]

Solution of a system of n nonlinear equations $F(x)$ with n unknowns is obtaining each equation of the system equal to zero for any value of x. Trust-region algorithm is based on region search rather than classical methods of optimization, line search. In this algorithm, start with a guess value for solution of optimization problem and a neighborhood region has been constructed near the guess point. The neighborhood region is known as trust-region if the current point provides minimum function value. This algorithm obtains a search direction d_i in each iteration such that

$$J\left(x_i\right)d_i = -F\left(x_i\right) \quad (4)$$

$$x_{i+1} = x_i + d_i, \quad (5)$$

Where $J(x_i)$ is n by n Jacobian matrix

$$J\left(x_i\right) = \begin{bmatrix} \Delta F_1\left(x_i\right)^T \\ \Delta F_2\left(x_i\right)^T \\ \cdot \\ \cdot \\ \cdot \\ \Delta F_n\left(x_i\right)^T \end{bmatrix} \quad (6)$$

Trust-region dogleg algorithm is more robust and effective than Gauss-Newton method as it requires only one linear solves per iteration. The dogleg algorithm attempts to follow a similar path by first finding the minimum along the gradient and then finding the minimum along a trajectory from the current point to the bottom of the quadratic model. The minimum along the second path is either the trust region boundary or the quadratic solution. In this paper, this approach is applied through MATLAB 'fsolve' command.

3.2.3 Trust-Region Reflective Approach[29]

The trust-region reflective approach is used for optimization of any function. The function $f(x)$ initialized at a and move to get lower value function $g(x)$ by estimating $f(x)$ at a new position for a step s, which fairly exhibits the behavior of function in a neighborhood trust-region N around the initial value a. The current point is updated to be $(a+s)$ if $f(a+s) < f(x)$; otherwise, the current point remains unchanged and trust-region N is shrunk and the trial step computation is repeated.

Trust-region reflective is a large-scale algorithm as it uses linear algebra and not operate on full matrices. This algorithm can be used on small problem as internal algorithms either preserve sparsity or do not generate matrices. Small-scale algorithm requires significant amount of memory and long time to execute due to use of full matrices and dense linear algebra.

Hessian matrix H is symmetric and positive definite only in the neighborhood trust region. Preconditioned Conjugate Gradients (PCG) method is employed to solve large symmetric positive definite systems of linear Equations $Hp = -g$, where p is PCG output direction and g is gradient. Symmetric positive definite metrics $M(=C^2)$ is a pre-conditioner of H, where $C^{-1}HC^{-1}$ is a matrix with clustered eigen values. MATLAB 'fsolve' is used to employ this approach.

3.2.4 Levenberg-Merquardt Approach[31]

The Levenberg-Marquardt (LM) algorithm is an iterative technique used to solve nonlinear least square problems. It is employed to find the minimum of function $F(x)$;

$$F(x) = \frac{1}{2}\sum_{i=1}^{n}\left(f_i(x)\right)^2 \quad (7)$$

The Levenberg-Marquardt algorithm examines in the direction of solution p to the equation;

$$\left(J_k^T J_k + \lambda_k I\right) p_k = -J_k^T f_k \quad (8)$$

It outperforms simple gradient descent and other conjugate gradient methods in a wide variety of problems. It is a pseudo-second order method which means that it works with only function evaluations and gradient information but it estimates the Hessian matrix using the sum of outer products of the gradients. Least squares problems arise when fitting a parameterized function to a set of measured data points by minimizing the sum of the squares of the errors between data points and function. In this paper MATLAB 'fsolve' command has been used for this approach.

A comprehensive comparative analysis of ten induction machines is presented in this paper. Ratings and parameters of all the machines[17,23,25,32-38] are required to plot magnetization curve and for estimation of excitation capacitance. Table 2 shows the machine ratings and parameters like rated voltage, current, type of connection, frequency, rated speed and per phase equivalent circuit parameters (R_s, R_r, X_{ls} and X_{lr}) of all the machines.

Nonlinear nature of magnetization curve of induction machines has been analyzed by proposed regression functions. A single function provides magnetizing reactance over whole range according to its fitness. Solution techniques are applied to calculate performance parameters of SEIG using magnetizing reactance having best goodness of fit.

4. Results and Discussions

Piecewise linearized model has been adopted to analyze nonlinear behavior of magnetization curve, which provides approximate value of excitation capacitance. In this paper, an effort has been made to solve the nonlinearity of magnetization curve by using different regression functions for the whole range. Eight regression functions have been employed for ten machines and compared their fitness to reflect the effect of fitness on performance of SEIG.

The details of each fitting expression like values of coefficients, Sum of Square Error (SSE), Degree of Freedom (DFE) in error, Root Mean Square Error (RMSE) and R-square for all machines has been shown in Table 3. Goodness of fit can be determined by SSE, R-square, DFE and RMSE (Appendix – II).

The comparison of different regression functions using graphical methods to evaluate the goodness of fit has been depicted in Table 3. The value of SSE and RMSE closer to zero indicates a fit that is more appropriate for prediction. The value of R-square closer to 1 (one) indicates greater proportion of variance is accounted for the model and fit is more appropriate for prediction. It has been observed from Table 3 that goodness of fit for all the machines is different with all the regression functions to give different values of X_m for specific value of V_g. Most appropriate fitting of magnetization curve is polynomial of third order (fitting – VI) for five machine (out of ten) and two-term exponential (fitting – II) for remaining five machines. These fitting – II and VI has the maximum value of R-square, minimum SSE and RMSE among the eight given expression under consideration, which clearly indicates the goodness of fit. Due to better fitting, polynomial fitting gives optimal value of X_m for fixed value of V_g, in comparison to piecewise linearized model.

Air gap voltage, magnetization reactance, excitation capacitance, per unit frequency, per unit speed and load impedance is six variables required to study the performance analysis of SEIG. Variation of two variables with reference to other two variables can be analyzed by keeping remaining two variable as constant. Table 4 shows the values of X_m, a and c for a particular value of V_g (=1.0

Table 2. Rating and Equivalent-Circuit Parameters (Per-Phase) of Induction Machines Operating as SEIG

M/C No.	Rating		No. of Poles	Rated Voltage (volts)	Line Current (Amp.)	Phase Current (Amp.)	Connection Type	Machine Parameters				Base Impedance (ohms)	Base Frequency (Hz)	Rated Speed (rpm)
	HP	kW						R_s (p.u.)	X_s (p.u.)	R_r (p.u.)	X_r (p.u.)			
1[32]	1	0.75	4	380	1.9	1.9	Star	0.08232	0.0766	0.06967	0.0766	115.4	50	1500
2[17]	2.72	2	4	380	5.4	5.4	Star	0.0982	0.112	0.0621	0.0952	40.63	50	1500
3[33]	3	2.2	4	230	8.6	4.965212	Delta	0.07232	0.10471	0.038	0.10471	46.32	50	1500
4[34]	10.2	7.5	4	415	14.6	14.6	Star	0.06094	0.08391	0.04692	0.08391	16.41	50	1500
5[35]	3	2.2	4	220	9.4	5.427093	Delta	0.0844	0.112	0.0621	0.0981	40.54	50	1500
6[36]	5	3.75	4	415	7.5	4.33	Delta	0.0601	0.09777	0.04372	0.09777	95.84	50	1500
7[37]	0.9	0.67	4	220	3.81	3.81	1-Φ	0.10769	0.16154	0.04539	0.08077	13.00	50	1500
8[23]	30	22	6	400	40	23.09401	Delta	0.03233	0.08661	0.04157	0.08661	17.32	50	1000
9[38]	9.52	7	4	400	14.7	14.7	Star	0.06668	0.16574	0.0823	0.16574	15.748	50	1500
10[25]	1	0.75	4	220	2.31	2.31	1-Φ	0.111	0.157	0.132	0.157	95.238	60	1800

p.u.) at unity p.u. speed and purely resistive unity load ($R_L = 1.0$ *p.u.*, $X_L = 0.0$ *p.u.*) for all machines with proposed eight regression expressions using eigen value computation approach.

It is observed from Table 4 that values of magnetizing reactance, p.u. frequency and excitation capacitance are varying with regression function for all machines. It gives different values then piecewise linear model and polynomial model. Estimation of excitation capacitance is prime important for performance analysis of SEIG as voltage build up is depending on the value of excitation capacitance. Precise value of X_m for a particular value of V_g provides exact values of p.u. frequency and excitation capacitance. Table 4 also shows that the impact of regression function is significant on the values of p.u. frequency and excitation capacitance. Optimum value of variables has been determined using the function having best fit among all the eight regression functions under study. The deviation in the values of X_m, a and C with fitting expression validates the proposed methodology. The most precise value can be considered by taking best fit expression from Table 3 for optimum calculation of excitation capacitance and performance analysis of SEIG for all machines.

Literature reveals that nonlinearity of magnetization curve has been solved using piecewise linearized model for seven machines (out of ten) and polynomial model for remaining three machines. Table 4 shows that polynomial of third order and two-term exponential functions are the best fitting expression. Eigen value approach has been applied to determine the values of all variables by employing best fitting expression on magnetization curve. Existing performance analysis results of all machines under consideration has been compared with the results obtained by best fit expression. Table 5 shows the comparison of existing and proposed methodologies of computing performance parameters i.e. magnetizing reactance, excitation capacitance and per unit frequency at unity power factor (purely resistive) load and rated speed (1.0 p.u.) for all machines. It has been observed from Table 5, that the magnitude of required excitation capacitance is decreased by using proposed technique.

Per unit speed and load impedance at output terminals are constant for calculating the performance parameters of Table 5 and 6. Load impedance significantly influences the performance parameters of SEIG of all machines. Excitation capacitance will decreases as load impedance increases. The decrease is gradual at higher values (above

Table 3. Comparison of Regression Functions

Function	Parameter		Machine 1	Machine 2	Machine 3	Machine 4	Machine 5	Machine 6	Machine 7	Machine 8	Machine 9	Machine 10
(Fitting-I) One-Term Exponential $V_g = a.e^{bx_m}$	Coefficient	a=	7.351	1.348	3.226	2.517	3.247	1.705	2.958	2.886	1.531	1.759
		b=	-1.654	-0.3475	-0.7279	-0.4139	-0.6981	-0.2898	-0.6616	-0.4592	-0.1704	-0.3639
	SSE		1.026	3.381	0.469	3.861	1.749	0.6687	0.6294	1.5	1.857	0.5854
	R-Square		0.9423	0.8019	0.9425	0.7527	0.8007	0.9157	0.9111	0.9084	0.9423	0.8631
	DFE		533	234	379	279	245	685	711	349	634	349
	RMSE		0.04386	0.1202	0.03518	0.1176	0.0845	0.03124	0.02975	0.06555	0.05412	0.04096
(Fitting-II) Two-Term Exponential $V_g = ae^{bx_m} + ce^{dx_m}$	Coefficient	a=	$-2.6x10^{-7}$	-0.00069	-99.89	$-2.12x10^{-7}$	$-9x10^{-16}$	$-9x10^{-12}$	$-5.8x10^{-9}$	3.992	1.797	-0.00045
		b=	8.72	2.622	0.561	4.653	13.67	8.004	8.356	0.2241	-0.2185	2.88
		c=	3.315	1.207	100.9	1.619	2.202	1.478	1.707	-2.868	0.001381	1.096
		d=	-0.9978	-0.1406	0.5574	-0.1841	-0.4755	-0.2171	-0.328	0.2986	0.4913	-0.0316
	SSE		0.02562	0.02419	0.00532	0.04934	0.2534	0.00454	0.01283	0.0063	0.6563	0.00107
	R-Square		0.9986	0.9986	0.9993	0.9968	0.9711	0.9994	0.9982	0.9996	0.9796	0.9998
	DFE		531	232	377	277	243	683	709	347	632	347
	RMSE		0.006945	0.01021	0.00376	0.01335	0.03229	0.00258	0.00425	0.00426	0.03223	0.00175
(Fitting-III) Gaussian $V_g = ae^{-(\frac{x_m-b}{c})^2}$	Coefficient	a=	1.04	1.179	0.9949	1.182	1.015	1.021	1.007	0.9951	1.362	1.02
		b=	1.086	0.4826	1.474	1.85	1.601	1.488	1.558	2.135	-1.47	1.452
		c=	0.558	2.106	1.067	1.483	0.9986	2.297	0.8539	1.83	7.398	1.382
	SSE		0.2148	1.02	0.0088	0.1.456	0.9031	0.2926	0.1011	0.7951	0.2548	0.06025
	R-Square		0.9979	0.9402	0.9989	0.9068	0.8971	0.9631	0.9857	0.9951	0.9765	0.9859
	DFE		532	233	378	278	244	684	710	348	632	348
	RMSE		0.02009	0.06616	0.00483	0.07236	0.06084	0.02068	0.01194	0.01512	0.02533	0.01316
(Fitting-IV) Polynomial First Order $V_g = ax_m + b$	Coefficient	a=	-1.346	-0.3276	-0.6077	-0.431	-0.6096	-0.262	-0.6084	-0.3629	-0.0892	-0.3434
		b=	2.624	1.332	1.976	1.984	2.037	1.484	1.999	1.842	1.131	1.538
	SSE		0.4094	2.086	0.2193	2.86	1.327	0.5074	0.4646	0.5607	4.702	0.4516
	R-Square		0.977	0.8778	0.9731	0.8169	0.8488	0.936	0.9344	0.9658	0.854	0.8944
	DFE		533	234	379	279	245	685	711	349	634	349
	RMSE		0.02771	0.09442	0.02405	0.1012	0.0736	0.02722	0.02556	0.04008	0.08612	0.03597
(Fitting-V) Polynomial Second Order $V_g = ax_m^2 + bx_m + c$	Coefficient	a=	-1.514	-0.1638	-0.4722	-0.3882	-0.6806	-0.141	-1.053	-0.1306	0.01661	-0.4194
		b=	2.791	0.1231	1.207	1.414	2.101	0.3826	3.209	0.4287	-0.2948	1.187
		c=	-0.1753	1.14	0.2591	-0.114	-0.6068	0.7692	-1.434	0.6867	1.677	0.1794
	SSE		0.09478	0.451	0.00322	0.8372	0.7263	0.237	0.067	$9.9x10^{-29}$	0.5542	0.02991
	R-Square		0.9947	0.9736	0.9996	0.9464	0.9173	0.9701	0.9905	1.0000	0.9828	0.993
	DFE		532	233	378	278	244	684	710	348	633	348
	RMSE		0.01335	0.04399	0.00292	0.05488	0.05456	0.01861	0.00971	$5.4x10^{-16}$	0.02959	0.00927
(Fitting-VI) Polynomial Third Order $V_g = ax_m^3 + bx_m^2 + cx_m + d$	Coefficient	a=	-6.262	-0.1209	0.09789	-0.4972	-1.44	-0.3148	-2.553	$2.4x10^{-16}$	0.000597	-0.4184
		b=	24.16	0.3223	-1.037	3.157	7.918	2.018	12.82	-0.1306	0.005521	1.871
		c=	-32.09	-0.3875	2.281	-6.792	-14.81	-4.464	-21.83	0.4287	-0.232	-2.923
		d=	15.53	1.228	-0.4163	6.04	10.34	4.326	13.55	0.6866	1.572	2.595
	SSE		0.02039	0.07259	0.00282	0.2119	0.5594	0.07337	0.02184	$4.6x10^{-29}$	0.5318	$3.5x10^{-28}$
	R-Square		0.9988	0.9957	0.9997	0.9864	0.9363	0.9907	0.9969	1.0000	0.9835	1.0000
	DFE		531	232	377	277	243	683	709	347	632	347
	RMSE		0.006197	0.01769	0.00273	0.02766	0.04798	0.01036	0.00555	$3.6x10^{-16}$	0.02901	$1x10^{-15}$

(Fitting-VII) Power $V_g = ax_m^b + c$	Coefficient	a=	-0.107	-0.0364	-0.02118	-1.74x10^{-4}	-0.00138	-4.9x10^{-3}	-0.00106	-0.00945	2.28	-0.00236
		b=	4.337	3.262	3.983	7.274	6.806	4.048	8.019	3.211	-0.5127	6.025
		c=	1.228	1.114	1.12	1.161	1.038	1.05	1.049	1.118	-0.3784	1.039
	SSE		0.07603	0.1958	0.0057	0.3206	0.6159	0.1979	0.03423	0.00264	1.427	0.0028
	R-Square		0.9957	0.9874	0.9993	0.9795	0.9298	0.975	0.9952	0.9998	0.9557	0.9993
	DFE		532	226	378	278	244	684	710	348	633	348
	RMSE		0.01195	0.02943	0.00388	0.03396	0.05024	0.01701	0.00694	0.00276	0.04748	0.00284
(Fitting-VIII) Sinusoidal $V_g = a\sin(bx_m + c)$	Coefficient	a=	1.083	1.168	1.016	1.176	1.015	1.026	1.01	1.018	37.51	1.02
		b=	1.91	0.5709	1.071	0.8568	1.235	0.5521	1.511	0.5815	0.002376	0.9436
		c=	5.969	1.332	6.402	-2.11x10^{-5}	5.923	7.08	5.534	0.4959	3.111	6.506
	SSE		0.1162	0.574	0.00278	0.9954	0.771	0.2522	0.07612	0.0038	4.703	0.03778
	R-Square		0.9935	0.9664	0.9997	0.9363	0.9122	0.9682	0.9892	0.9998	0.854	0.9912
	DFE		532	233	378	278	244	684	710	348	633	348
	RMSE		0.01478	0.04963	0.00271	0.05984	0.05621	0.0192	0.01035	0.00331	0.08619	0.01042

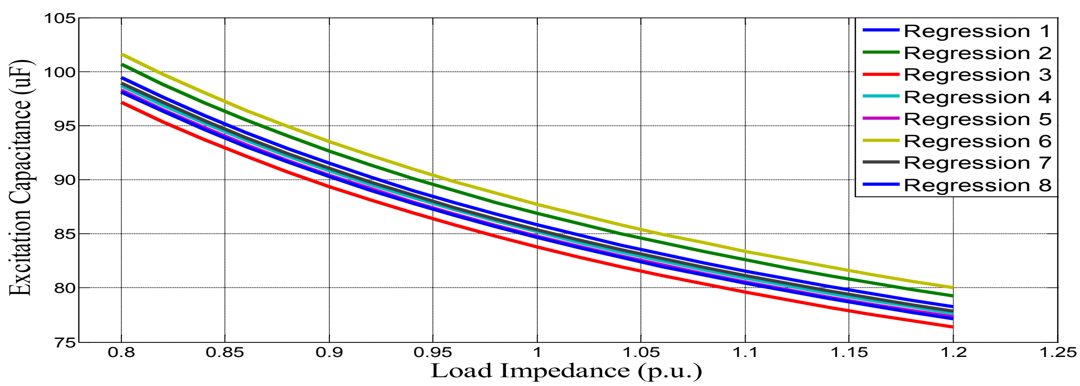

Figure 2. Variation of excitation capacitance with load impedance for machine – 5.

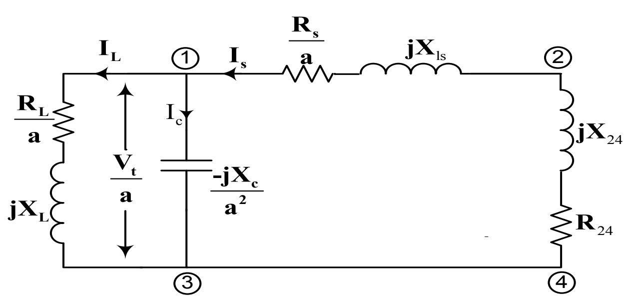

Figure 3. Simplified per-phase equivalent circuit of SEIG.

1.0 p.u.) of load impedance, while rapid at lower values (below 1.0 p.u.) of load impedance. Figure 2 shows the variation of excitation capacitance with load impedance for machine – 5 using all the eight regression expression. This figure has been plotted for under-loaded and over-loaded values of load impedance (*0.8 p. u.- 1.2 p. u.*) at constant speed (*=1.0 p. u.*), magnetizing reactance and corresponding air gap voltage. The excitation capacitance decreases more rapidly for underload to unity load conditions as compared to unity load to overload conditions of load impedance.

Performance variables V_g, X_m, C, a, b and Z_L are required to study the performance analysis of SEIG that can be determined by using mesh impedance method or node admittance method (appendix I). Eigen value approach has already been discussed and applied to calculate different variables in Table 4 and 5. Trust-region dogleg, Levenberg-Marquardt, trust-region reflective and MATLAB 'fzero' are four other approaches, has been applied to calculate performance variable. A comparison between all the four approaches for all machines with their best fittings among eight expressions has been shown in Table 6. The values of all the four approaches are then compared with the results of eigen value approach.

Table 6 shows that the values of p.u. frequency and excitation capacitance, calculated by trust-region dogleg, Levenberg-Marquardt, trust-region reflective and MATLAB 'fzero' are almost equal to the values by eigen value approach at higher function tolerance of more than 10^{-15}. The results deviate significantly for trust-region dogleg and trust-region reflective approaches at lower function tolerance of less than 10^{-08}. The results of Table 6

Table 4. Comparison of magnetizing Reactance, Frequency and Capacitance at Constant Air Gap Voltage

Fitting	Parameter	Machine									
		1	2	3	4	5	6	7	8	9	10
I	X_m (p.u.)	1.206068	0.859344	1.609071	2.23017	1.687053	1.84115	1.639228	2.308083	2.49954	1.551926
	a (p.u.)	0.924134	0.911014	0.957146	0.992610	0.966352	0.991075	0.972991	1.030317	1.000740	1.000041
	C (μF)	35.53018	154.04797	69.27467	87.32084	55.39841	18.06342	178.35720	83.07004	75.60267	19.47279
II	X_m (p.u.)	1.19258	1.219429	1.761051	2.49298	1.660062	1.799507	1.617248	2.221152	2.706417	1.580134
	a (p.u.)	0.923904	0.925244	0.957928	1.024353	0.966077	0.995596	0.973685	0.992643	1.000043	0.999634
	C (μF)	35.89453	109.45093	64.60651	74.96367	56.29140	17.89582	178.50676	82.60058	70.32641	19.18560
III	X_m (p.u.)	1.196508	1.3371966	1.472845	2.4564145	1.47915192	1.81913898	1.62931785	2.135	2.6420784	1.25752246
	a (p.u.)	0.923972	0.927646	0.956272	1.005651	0.990946	0.981374	0.987087	0.991127	0.999365	0.999241
	C (μF)	35.78743	100.81818	74.28772	74.86339	51.49800	19.95506	151.18859	86.63500	72.10721	23.54290
IV	X_m (p.u.)	1.206538	1.01343	1.606055	2.283063	1.701115	1.84733	1.642012	2.320198	1.46861	1.566686
	a (p.u.)	0.924142	0.918990	0.957129	0.996948	0.979613	1.010936	1.045806	1.008211	0.995309	0.999085
	C (μF)	35.51761	130.20159	69.37589	83.13761	48.97624	16.59318	140.30934	75.33126	125.57262	19.37027
V	X_m (p.u.)	1.192469	1.37371	1.53183	2.489964	1.69014	1.808244	1.62522	2.184266	2.71038	1.62948
	a (p.u.)	0.923902	0.928277	0.956674	1.004267	1.005909	0.976865	1.048288	1.008568	1.000044	0.999630
	C (μF)	35.89759	98.47340	72.00637	74.15993	42.49283	21.09457	143.18712	79.77157	70.22937	18.66011
VI	X_m (p.u.)	1.192873	1.320053	1.51315	2.561393	1.631602	1.77459	1.563642	2.18356	2.687304	1.55975
	a (p.u.)	0.923909	0.927332	0.956551	1.023097	1.005645	1.014555	0.972412	1.033994	1.000335	1.001124
	C (μF)	35.88654	101.96970	72.70922	72.88151	43.95026	17.21120	186.32483	89.20924	70.72700	19.31473
VII	X_m (p.u.)	1.190571	1.419394	1.545672	2.55756	1.627314	1.77457	1.61294	2.195165	2.66865	1.59243
	a (p.u.)	0.923869	0.929006	0.956763	1.007843	1.007216	0.995423	0.973092	1.032721	0.999067	1.002084
	C (μF)	35.94959	95.72388	71.49637	71.71805	43.83124	18.15134	180.31671	87.98347	71.50932	18.89694
VIII	X_m (p.u.)	1.193211	1.369403	1.52165	2.480138	1.702926	1.81052	1.6286	2.17236	1.65401	1.63876
	a (p.u.)	0.923915	0.928205	0.956608	1.004321	1.007130	1.014578	1.017247	1.009048	1.002116	0.998496
	C (μF)	35.87726	98.74280	72.38654	74.43155	42.02910	16.89931	129.38306	80.19173	110.18129	18.63700

Table 5. Comparison of Proposed and Existing Methodology

Machine	Fitting Type	V_g (p.u.)	Proposed Methodology			Existing Methodology			% Error	
			X_m (p.u.)	*a* (p.u.)	*C* (µF)	X_m (p.u.)	*a* (p.u.)	*C* (µF)	*a*	*C*
1	VI	1.05674	1.148279	0.923096	37.15574	1.142981	0.922994	37.31346	-0.011	0.4227
2	II	1.04	0.9953751	0.918252	132.51495	0.99368	0.918181	132.73694	-0.0077	0.1672
3	VI	0.961939	1.65555	0.957404	67.75510	1.651554	0.957383	67.88178	-0.0022	0.1866
4	II	0.902283	2.7445739	0.953125	121.45913	2.705667	0.953067	122.71292	-0.0061	1.0217
5	II	1.034004	1.5897475	0.931142	85.15044	1.532	0.930510	87.76567	-0.0679	2.9798
6	II	1.032495	1.652264	0.952016	31.67033	1.65	0.952004	31.70424	-0.0013	0.1070
7	II	1.015734	1.573789	0.943867	286.93675	1.57	0.943818	287.50410	-0.0052	0.1973
8	VI	1.012894	2.08525	0.956224	134.65914	2.084299	0.956222	134.70454	-0.0002	0.0337
9	VI	1.0201	2.58178	0.906244	193.44829	2.3749	0.904664	204.11328	-0.1747	5.2250
10	VI	0.99693	1.581348	0.842374	43.81832	1.581	0.842329	43.83791	-0.0053	0.0447

Table 6. Comparison of Variable Performance Parameters using Different Solution Techniques

M/C No.	Fitting Type	X_m (p.u.)	Eigen value Approach		Trust-Region Dogleg Approach		Trust-Region Reflective Approach		Levenberg-Marquardt Approach		Fzero Approach	
			a (p.u.)	*C* (µF)	*a* (p.u.)	*C* (µF)	*a* (p.u.)	*C* (µF)	*a* (p.u.)	*C* (µF)	*a* (p.u.)	*C* (µF)
1	VI	1.192873	0.969663	39.18943	0.97	39.17294	0.97	39.17294	0.97	39.17294	0.97	39.17294
2	II	1.219429	0.834007	223.22391	0.834	223.23769	0.834	223.23769	0.834	223.23769	0.834	223.23769
3	VI	1.51315	0.869410	166.42621	0.869	167.29358	0.869	167.29358	0.869	167.29358	0.869	167.29358
4	II	2.49298	0.880159	294.53966	0.88	295.01038	0.88	295.01038	0.88	295.01038	0.88	295.01038
5	II	1.660062	0.931132	100.75940	0.931	100.79840	0.931	100.79840	0.931	100.79840	0.931	100.79840
6	II	1.799507	0.912563	45.03377	0.913	44.90917	0.913	44.90917	0.913	44.90917	0.913	44.90917
7	II	1.617248	0.863691	579.23654	0.864	577.08807	0.864	577.08807	0.864	577.08807	0.864	577.08807
8	VI	2.18356	0.971176	217.30935	0.971	217.17268	0.971	217.17268	0.971	217.17268	0.971	217.17268
9	VI	2.687304	0.856125	314.80150	0.856	315.11137	0.856	315.11137	0.856	315.11137	0.856	315.11137
10	VI	1.55975	0.802710	74.20801	0.803	74.08529	0.803	74.08529	0.803	74.08529	0.803	74.08529

show that performance variables can be computed accurately by different approaches with negligible deviation in values.

5. Conclusion

Performance analysis of SEIG includes estimation of per-phase equivalent circuit parameters and operating parameters. Magnetizing characteristics has been determined using regression functions for nonlinear magnetization curve. Third order polynomial function shows best goodness of fit among regression functions. Magnetizing reactance, per unit frequency and excitation capacitance, at constant air gap voltage, varies with adopted regression functions. Similarly, air gap voltage varies at constant excitation capacitance. Excitation capacitance decreases with increase in load impedance. Selection of regression function significantly affects performance analysis of SEIG. Different regression functions are applied on ten induction machines to obtain best fitting. Thus optimum magnetizing reactance has been obtained. Optimal value of magnetizing reactance provides optimal excitation capacitance and steady state performance of SEIG. Solution technique is required to calculate variable performance parameters. Different solution techniques have been applied on all machines at optimal magnetizing characteristics to compute the performance parameters of SEIG. Significant improvement of up to 5% has been observed by proposed technique as compared to existing techniques. Optimal performance of SEIG can be computed by using combination of proper fitting of magnetization curve and appropriate solution technique all together.

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APPENDIX – I

Per-phase equivalent circuit for steady state analysis of SEIG in which all impedances and voltages referred to stator at base frequency is referred to Figure 1. The load and excitation capacitance branches are decoupled in the equivalent circuit of SEIG. Machine must operate at threshold of saturation to get self-excited for minimum value of excitation capacitance. Therefore, machine core losses can be ignored.

Loop impedance and nodal admittance methods has been used for steady state analysis of SEIG.

(a) Loop Impedance Method

Load impedance, capacitive reactance, stator impedance, magnetizing reactance and rotor impedance (referred to stator) of per phase equivalent circuit has been considered for loop impedance method.

Applying Kirchoff's law in Figure 1;

$$Z_T I_s = \left(Z_{13} + Z_{12} + Z_{24}\right) I_s = 0 \qquad \text{(i)}$$

$$Z_T = \frac{Z_c Z_L}{Z_c + Z_L} + Z_s + \frac{Z_r Z_m}{Z_r + Z_m} \qquad \text{(ii)}$$

Where

$$Z_L = \frac{R_L}{a} + jX_L\,; \qquad Z_c = -\frac{jX_c}{a^2}\,; \qquad Z_s = \frac{R_s}{a} + jX_{ls}\,;$$

$$Z_r = \frac{R_r}{(a - b)} + jX_{lr}\,; \qquad Z_m = jX_m$$

Z_T will be zero as stator current cannot be zero under steady state self-excitation. Two nonlinear equations by taking real and imaginary parts of Z_T equal to zero separately denoted as a function of X_c and a.

$$F_1\left(X_c, a\right) = P_1 a^3 + P_2 a^2 + \left(P_3 X_c + P_4\right) a + P_5 X_c = 0 \qquad \text{(iii)}$$

$$F_2\left(X_c, a\right) = \left(Q_1 X_c + Q_2\right) a^2 + \left(Q_3 X_c + Q_4\right) a + Q_5 X_c = 0 \qquad \text{(iv)}$$

Where

$$P_1 = -\left(2X_m + X_1\right) X_1 R_L\,; \qquad P_2 = -P_1 b\,;$$

$$P_3 = \left(R_L + R_s + R_r\right)\left(X_m + X_1\right); \quad P_4 = R_s R_L R_r\,;$$

$$P_5 = -\left(R_s + R_L\right)\left(X_m + X_1\right) b\,; \quad Q_1 = \left(2X_m + X_1\right) X_1\,;$$

$$Q_2 = R_L\left(R_s + R_r\right)\left(X_m + X_1\right); \quad Q_3 = -Q_1 b\,;$$

$$Q_4 = -R_s R_L\left(X_m + X_1\right) b$$

$$Q_5 = -R_r\left(R_s + R_L\right)$$

Two nonlinear equations by taking real and imaginary parts of Z_s equal to zero separately denoted as a function of X_m and a also.

$$F_3\left(X_m, a\right) = \left(R_1 X_m + R_2\right) a^2 + \left(R_3 X_m + R_4\right) a + R_5 = 0 \qquad \text{(v)}$$

$$F_4\left(X_m, a\right) = \left(S_1 X_m + S_2\right) a^3 + \left(S_3 X_m + S_4\right) a^2 + \left(S_5 X_m + S_6\right) a + \left(S_7 X_m + S_8\right) = 0 \qquad \text{(vi)}$$

Where

$$R_1 = R_L\left(R_s + R_r\right) + 2X_c X_1\,;$$

$$R_2 = R_L\left(R_s + R_r\right) X_1 + X_c X_1^2\,;$$

$$R_3 = -R_s R_L b - 2X_c X_1 b\,;$$

$$R_4 = -R_s R_L b X_1 - X_c X_1^2 b\,;$$

$$R_5 = -X_c R_r\left(R_s + R_L\right); \qquad S_1 = -2R_L X_1\,;$$

$$S_2 = -X_1^2 R_L\,; \qquad S_3 = -S_1 b\,; \qquad S_4 = -S_2 b\,;$$

$$S_5 = \left(R_L + R_s + R_r\right) X_c\,;$$

$$S_6 = \left(R_L + R_s + R_r\right) X_c X_1 + R_L R_r\left(X_c + R_s\right);$$

$$S_7 = -X_c\left(R_s + R_L\right); \qquad S_8 = S_7 X_1$$

Values of excitation capacitance and frequency at fixed air gap voltage, has been evaluated from equation (iii) and (iv). The Load characteristics has been determined by solving equation (v) and (vi) at fixed capacitance. If magnetizing reactance and generated frequency are known, analysis of performance characteristics of SEIG can easily be evaluated at given terminal conditions.

(b) Nodal admittance method

The calculation of per unit frequency 'a' is independent of X_c, as load and excitation capacitance branches can easily be decoupled. For this purpose, Figure 1 is redrawn as Figure I.1.

Where

$$R_{24} = \frac{(a-b) R_r X_m^2}{R_r^2 + (a-b)^2 (X_m + X_r)^2};$$

$$X_{24} = \frac{R_r^2 X_m + (a-b)^2 X_m X_{lr} (X_m + X_{lr})}{R_r^2 + (a-b)^2 (X_m + X_{lr})^2}$$

The total impedance Z_{14} of branch '124' is given by

$$Z_{14} = R_{14} + jX_{14} \quad \text{(vii)}$$

Where

$$R_{14} = \frac{R_s}{a} + R_{24}; \qquad X_{14} = X_{ls} + X_{24}$$

The relation for different admittances are;

$$Y_L = \frac{aR_L}{R_L^2 + a^2 X_L^2} - j\frac{a^2 X_L}{R_L^2 + a^2 X_L^2} \quad \text{(viii)}$$

$$Y_{14} = \frac{R_{14}}{R_{14}^2 + X_{14}^2} - j\frac{X_{14}}{R_{14}^2 + X_{14}^2} \quad \text{(ix)}$$

$$Y_c = j\frac{a^2}{X_C} \quad \text{(x)}$$

Apply nodal admittance method in Figure I.1

$$Y_s V_s = \frac{V_1}{a}(Y_L + Y_{14} + Y_c) = 0 \quad \text{(xi)}$$

Since the stator voltage will not be zero for successful voltage build-up, $V_1 \neq 0$, hence $Y_s = 0$.

$$(Y_L + Y_{14} + Y_c) = 0 \quad \text{(xii)}$$

By separating real and imaginary terms of the above equation (xii) to zero respectively;

$$\frac{aR_L}{R_L^2 + a^2 X_L^2} + \frac{R_{14}}{R_{14}^2 + X_{14}^2} = 0 \quad \text{(xiii)}$$

$$\frac{a^2}{X_c} - \frac{a^2 X_L}{R_L^2 + a^2 X_L^2} - \frac{X_{14}}{R_{14}^2 + X_{14}^2} = 0 \quad \text{(xiv)}$$

If the speed of machine is fixed and X_m is kept at minimum value, then per unit frequency 'a' and capacitive reactance 'X_c' are the only variables in Figure 1. The value of per unit frequency a will be determined from equation (xiii), as per unit frequency is the only variable in equation (xiii) and is independent of X_c. The value of X_c can be calculate from equation (xiv) by putting the value of 'a' from equation (xiii).

Equation (xiv) can be expressed as a 6th degree polynomial as follows after a series of algebraic manipulations;

$$h_6 a^6 + h_5 a^5 + h_4 a^4 + h_3 a^3 + h_2 a^2 + h_1 a + h_0 = 0 \quad \text{(xv)}$$

All the real and complex roots can be determined by solving equation (xv). Only real roots are to be considered for calculating the value of excitation capacitance C. At the time of solving equation (xiii) or (xv), the value of X_m is assumed to be known. The value of X_m can be determined by magnetization curve between V_g-X_m.

APPENDIX – II

The experimental value and computed value of dependent variable is x and y respectively. The sum of square error (SSE) for the measured and fitted magnetizing curves is calculated as

$$SSE = \sum (y - \hat{y})^2 \quad \text{(xvi)}$$

$$S_{xx} = \sum (x - \bar{x})^2 \quad \text{(xvii)}$$

$$S_{xy} = \sum (x - \bar{x})(y - \bar{y}) \quad \text{(xviii)}$$

$$S_{yy} = \sum (y - \bar{y})^2 \quad \text{(xix)}$$

Regression equation:

$$\hat{y} = b_0 + b_1 x \quad \text{(xx)}$$

Where

$$b_1 = \frac{S_{xy}}{S_{xx}}; \qquad b_0 = \frac{1}{n}\left(\sum y - b_1 \sum x\right) = \bar{y} - b_1 \bar{x}$$

Linear Correlation Coefficient R-Square = $\frac{S_{xy}}{\sqrt{S_{xx} S_{yy}}}$

Root Mean Square Error = $\sqrt{\frac{\sum (x-y)^2}{n}}$