

An Extended Fuzzy PROMETHEE based on Fuzzy Rule based System for Supplier Selection Problem

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Abstract

Background/Objectives: Supplier Selection Problem (SSP) has become critical issue for purchasing department because of its significant effect toward successful logistic and Supply Chain Management (SCM). Due to the fact that various conflicted criteria must be considered, SSP is inherently Multiple Criteria Decision Making (MCDM) problem. **Methods/Statistical Analysis:** The aim of this paper is to solve SSP under group decision making and fuzzy environment. To do so, as main contribution of this paper, a hybrid approach which employs both Fuzzy Rule Based System (FRBS) and Preference Ranking Organization Method for Enrichment Evaluations (PROMETHEE), is proposed to select suitable supplier. **Findings:** One advantage of MCDM techniques is providing Decision Maker's (DM) preferences and utilities. To do so, we have modeled DM's utilities using fuzzy logic and then implemented in PROMETHEE method. **Improvement/Application:** Ultimately, numerical illustration and sensitivity analyses are performed to demonstrate the applicability of the proposed method for a supplier selection problem.

Keywords: Fuzzy Rule based Systems, MADM, PROMETHEE, Supplier Selection Problem

1. Introduction

In today's extremely competitive corporate environment, the role of suppliers is a key one for the success of the supply chain and therefore supplier selection is a vital decision in Supply Chain Management (SCM)¹. An efficient purchasing with flexible participation with suppliers assures the robustness of competition in markets. Many researchers have been carried out in Supplier Selection Problem (SSP). Jiang, et al.² investigated an empirical study to examine whether different supplier selection criteria and integration mechanisms can improve customer satisfaction and business performance. In the literature, SSP investigated in terms Of Multi-Criteria Decision Making (MCDM) and various exact and efficient heuristic methodologies are proposed for solving SSPs³.

SSP is classified into two classes⁴: 1. Single source in which restrictions is not considered in supplier selection process and the final decision is the best supplier; 2.

Multiple sources in which some constraints of the supplier including capacity, quality and lead time are considered. In the second class, one supplier cannot satisfy whole requirements of demands; then, remain part should be provided in others suppliers. Thus, the problem follows to answer into two questions: the best supplier(s) as well as the purchasing quantities of supplier(s). In this paper, we deal with single source class in which suppliers are prioritized and the best one is chosen. Deboer, et al.¹ proposed a well-known framework for analyzing supplier selection problem consisting 1. Problem definition, 2. Formulation of Criteria, 3. Qualification and 4. Choice. The SSP is a typical multi attribute decision making problem. Several MCDM techniques proposed in SSPs literature⁵.

In SSPs, criteria formulation is one of the most important steps in supplier selection process. Many descriptive studies attempt to investigate the criteria of the Purchasers for selecting the suitable suppliers.

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Dickson⁶ presented a seminal study and introduced quality, delivery and performance history as three most important criteria. Weber, et al.⁷ analyzed 74 papers on supplier selection criteria and identified price as the most cited criterion, followed by delivery and quality. Kannan and Tan⁸ presented an empirical study of the importance of supplier selection and assessment criteria of American manufacturing companies for items to be used in products already in production. Out of 30 selection criteria, on-time delivery and quality were ranked as the most important. Kahraman, et al.⁹ presented four parts of SSP criteria that consist of supplier characteristics, product efficiency, service quality and cost. However, in another research by Frödell¹⁰ reported cost is the most important criterion in 12 zones of industries. Ku, et al.¹¹ reviewed literature of SSP and summarized supplier selection criteria as cost or price, quality, service, supplier's profile, risk, buyer-supplier partnership, cultural and communication barriers and trade restrictions.

One of the prominent features of SSP is uncertainty in making decisions because of inherent ambiguity in evaluating criteria. Most parameters in real world situations are vague, imprecise and incomplete. Fuzzy sets theory is most important and applicable approaches in facing with uncertainty in supplier selection process^{12,13}. Fuzzy theory analyzes inexact criteria to hybridize quantitative and qualitative measures in selection process. Fuzzy set theory, introduced by Zadeh¹⁴, presented to deal with this incomplete information and model them. In fuzzy logic applications, linguistic variables are applied to facilitate rules and facts¹⁵. Therefore, in SSP, many experts employ linguistic variables as fuzzy numbers in order to determine important features and performance of alternatives.

Many researches concentrate on SSP in fuzzy environment. Carrera and Mayorga¹⁶ proposed a modular Fuzzy Inference System approach in supplier selection for new product development. Wang, et al.¹⁷ proposes fuzzy hierarchical TOPSIS for solving a SSP. Boran, et al.¹⁸ proposed a TOPSIS method combined with intuitionistic fuzzy set to select appropriate supplier in group decision making environment. They utilized intuitionistic fuzzy weighted averaging operator to aggregate individual opinions of decision makers for rating the importance of criteria and alternatives. Keskin, et al.¹⁹ presented fuzzy an adaptive resonance theory to evaluate and select the suppliers. Hsu, et al.²⁰ used fuzzy preference relations with using fuzzy quality data to rank suppliers and select the

best one. Büyük Özkan and Cifci²¹ developed an approach based on fuzzy Analytic Network Process (ANP) within multi-person decision-making schema under incomplete preference relations in Sustainable supply chain. Shemshadi, et al.²² proposed fuzzy VIKOR for SSP based on entropy measure for objective weighting. Amindoust, et al.²³ proposed a ranking method based on fuzzy interference system for Sustainable supplier selection. Büyük Özkan and Cifci²⁴ hybridized three MCDM methods including fuzzy DEMATEL, fuzzy ANP and fuzzy TOPSIS to evaluate green suppliers. Lima Junior, et al.²⁵ presented a fuzzy inference approach for SSP under A fuzzy inference and categorization approach for supplier selection using compensatory and non-compensatory decision rules. Dursun and Karsak²⁶ developed a fuzzy multi-criteria group decision making approach that makes use of the Quality Function Deployment (QFD) concept for supplier selection process. Lima Junior, et al.²⁷ investigated a comparative analysis of fuzzy TOPSIS and fuzzy AHP methods in the context of supplier selection decision making. Ai, et al.²⁸ proposed an approach based on hesitant fuzzy set to solve SSP. Sultana, et al.²⁹ combined Fuzzy Delphi, Fuzzy AHP and Fuzzy TOPSIS for solving SSP. Khalilo, et al.³⁰ proposed a method on the basis of the computing with words for selection problem. Moayeri, et al.³¹ compared the fuzzy AHP and TOPSIS methods for selection problem.

One of the well-known and most recent MADM techniques is PROMETHEE method (Preference Ranking Organization Method for Enrichment Evaluation). Araz and Ozkarahan³² utilized PROMETHEE method to evaluate the performance of suppliers by simultaneously considering supplier capabilities and performance metrics and to provide a preference relation between suppliers. Chen, et al.³³ proposed fuzzy PROMETHEE, a mathematical model for that accounts for the presence of vagueness, uncertainty and imprecision of information in the SSP. Chai, et al.³⁴ developed a Superiority and Inferiority Ranking (SIR) group decision approach for SSP under intuitionistic fuzzy environments, in which the SIR method can be regarded as an extension of the conventional PROMETHEE method.

Some researchers used PROMETHEE method in various field of decision making. Dagdeviren³⁵ proposed an integrated approach which employs Analytic Hierarchy Process (AHP) and PROMETHEE together for the equipment selection problem. Taha and Rostam³⁶ proposed a hybrid approach based on fuzzy AHP and

fuzzy PROMETHEE for machine tool selection in flexible manufacturing cell. Behzadian, et al.³⁷ provided PROMETHEE group decision support system approach that integrates the design preferences of the Quality Function Deployment (QFD) team. Hashemian, et al.³⁸ proposed a hybrid fuzzy group decision-making approach for supplier evaluation based on fuzzy AHP and fuzzy PROMETHEE under group decision making. Chen³⁹ extended PROMETHEE method using a signed distance-based approach within the environment of interval type-2 fuzzy sets for multiple criteria decision analysis. Ghazinoory, et al.⁴⁰ used fuzzy PROMETHEE for wind turbine in road mapping process.

In conventional PROMETHEE methods, DM can only select predetermined preference function as his/her preferences while this preference function used in traditional methods cannot completely provide preferences of DM. Therefore, considering DM's preferences and opinions, to provide aspirations of DM, leads to more accurate decision making process. Often in real case problems preferences and knowledge of DMs are as linguistic terms in form of IF-THEN rules. Therefore, as main contribution of this paper, we follow to develop conventional PROMETHEE method using Fuzzy Rule Based System (FRBS) based on fuzzy logic for modeling utility of DM.

This paper presents a SSP model under group decision making. As main contribution of our study, a new PROMETHEE method based on fuzzy logic is developed to select best supplier. The rest of paper is organized as follow: Next section is devoted to some Preliminary concepts including PROMETHEE method and fuzzy logic. In Section 3, we describe our proposed extended PROMETHEE for selection process. Section 4 presents the application of the proposed method in solving a SSP. At end, the summary and conclusions of this paper are presented.

2. Preliminaries

In this section, we review main idea and executive steps of PROMETHEE method and also, some basic concepts and definitions related to Fuzzy Rule Based System, which will be needed in the our hybrid method of selection process.

2.1 The PROMETHEE Method

Brans, et al.^{41,42} considered a new family of outranking methods, called PROMETHEE for solving MADM

problems. A considerable number of successful applications has been treated by PROMETHEE methodology in various fields such as Banking, Industrial Location, Manpower planning, Water resources, Investments, Medicine, Chemistry, Health care, Tourism, Ethics in OR, Dynamic management. The success of the methodology is basically due to its mathematical properties and to its particular friendliness of use⁴³. A more detail review of PROMETHEE method has been provided in Behzadian, et al.⁴⁴. Moreover, it can be seen a review on fuzzy PROMETHEE method in Chen³⁹. We describe procedure of implementation PROMETHEE I and II as follow:

We consider a multi-attribute decision-making problem with a set of possible alternatives which is evaluated on a set of criteria. Without loss of generality, suppose that all the criteria have to be maximized. The problem can be represented as:

$$\max \{g_1(a_i), g_2(a_i), \dots, g_j(a_i), \dots, g_n(a_i) | a_i \in A \quad (1)$$

Where $A = \{a_i | i = 1, 2, \dots, m\}$ is a set of possible alternatives and $g = \{g_j | j = 1, 2, \dots, n\}$ is a set of considered criteria; $g_j(a_i)$ represents performance of alternative a_i with respect to the j th criterion.

A decision-maker expresses his preference of alternative a over alternative b considering the criterion g_j by computing a single-criterion preference degree $p_j(a, b)$ which is in function of $d_j(a, b) = g_j(a) - g_j(b)$. The value of this preference function $p_j(a, b)$ is included between 0 and 1. If a is better than b , then $p_j(a, b) > 0$; otherwise, $p_j(a, b) = 0$.

The selection of a specific preference function, Brans and Vincke⁴² proposed six basic types: 1. Usual criterion, 2. U-shape criterion, 3. V-shape criterion, 4. Level criterion, 5. V-shape with indifference criterion and 6. Gaussian criterion.

Then $\pi(a, b)$ as a preference index over all the criteria is calculated by $\pi(a, b) = \sum_{j=1}^n w_j p_j(a, b)$, where W_j is the weight of j th criteria.

Moreover, in order to evaluate the alternatives of a by using the outranking relation, following flows must be defined.

$$\text{The leaving flow: } \phi^+(a) = \frac{1}{m-1} \sum_{b \in A} \pi(a, b) \quad (2)$$

This score represents the global strength of action a in comparison to all other alternatives. Indeed, this score has to be maximized.

The entering flow:

$$\left(\rho \frac{\partial v}{\partial t} + \rho(v \cdot \nabla)v, \psi\right) + (\nabla \cdot \sigma, \psi) = (g_3, \psi)_0 + (f_3, \psi) \quad (3)$$

This score represents the global weakness of a in comparison to all other alternatives. Indeed, this score has to be minimized.

The net flow: $\phi(a) = \phi^+(a) - \phi^-(a)$ (4)

According to Brans, et al.^{41,42}, PROMETHEE 1 determines the partial preorder on the alternatives of A based on the leaving and entering outranking flows that satisfied the following principle:

a is preferred to b ($a P^{(1)}b$) iif;
 a is preferred to b ($a p^{(1)}b$) iif;

$$\begin{cases} \phi^+(a) > \phi^+(b) \text{ and } \phi^-(a) < \phi^-(b), \text{ or} \\ \phi^+(a) = \phi^+(b) \text{ and } \phi^-(a) < \phi^-(b), \text{ or} \\ \phi^+(a) > \phi^+(b) \text{ and } \phi^-(a) = \phi^-(b) \end{cases}$$

a is in different to b ($a I^{(1)}b$) iif; $\phi^+(a) = \phi^+(b)$ and $\phi^-(a) = \phi^-(b)$

a is incomparable to b ($a Rb$) iif;

$$\begin{cases} \phi^+(a) > \phi^+(b) \text{ and } \phi^-(a) > \phi^-(b) \\ \phi^+(a) < \phi^+(b) \text{ and } \phi^-(a) < \phi^-(b) \end{cases}$$

Furthermore, PROMETHEE II gives a complete preorder induced by the net flow and defined by:

a is preferred to b ($a P^{(II)}b$) iif; $\phi(a) > \phi(b)$

a is indifferent to b ($a I^{(II)}b$) iif; $\phi(a) = \phi(b)$

It seems easier for the DM to achieve the decision problem by using the complete preorder in PROMETHEE II instead of the partial one given by PROMETHEE I. However, the partial preorder provides more realistic information by considering only confirmed outranking with respect to the leaving and entering flows. On the other hand, the relation of incomparability can also be severely useful. In real-world applications, considering both PROMETHEE I and PROMETHEE II is recommended. The complete ranking is easy to use, but the analysis of the incomparability often helps to finalize a proper decision.

2.2 Basis of Fuzzy Systems

In many cases, decision making and optimization are in uncertain environment and a mathematical deterministic equation does not exist. In this situation, human knowledge and experiment have been utilized to decision making

and optimization. Generally, human knowledge can be categorized in two groups: one is unconscious knowledge and the other one, conscious knowledge. In unconscious knowledge, experts cannot describe their knowledge as linguistic terms but in conscious knowledge, experts can explain their knowledge as linguistic terms. Zadeh^{14,15}, first introduced the concept of fuzzy logic “computing with words”. Fuzzy logic can be used to formulate the human knowledge. Fuzzy systems are knowledge-based or rule-based systems. The heart of a fuzzy system is a knowledge base consisting of the so-called fuzzy IF-THEN rules. A fuzzy IF-THEN rule is an IF-THEN statement in which some words are characterized by continuous membership functions.

Mamdani fuzzy system was developed in 1975 by Mamdani⁴⁵ is most common type of fuzzy system that has applied in special fields because of its ability in knowledge representation. The starting point of constructing a fuzzy system is to obtain a collection of fuzzy rules from human experts and establish a fuzzy rule base. A fuzzy rule is the implication stated as If-

Then rule which divided into two main parts:

- “If” part which described premise section of fuzzy rule
- “Then” part which described conclusion section of fuzzy rule

A general form of fuzzy rule in this paper is as follows:

$$R: \text{ if } x_1 \text{ is } \mu_R^{(1)} \text{ and } \dots \text{ and } x_n \text{ is } \mu_R^{(n)} \text{ Then } y \text{ is } \mu_R \quad (5)$$

Where x_1^0 and x_2^0 are input variables and y is output variable. Moreover, $\mu_{x_1}(x_1^0)$ and $\mu_{x_2}(x_2^0)$ are linguistic variables, that is, for vague concepts like “about small” or “very height” which are represented by fuzzy sets. In addition to fuzzy rule base as main component of fuzzy system, a fuzzy system includes three other component consisting of fuzzy inference engine, fuzzification and defuzzification. Basic configuration of mamdani fuzzy systems is shown as Figure 1.

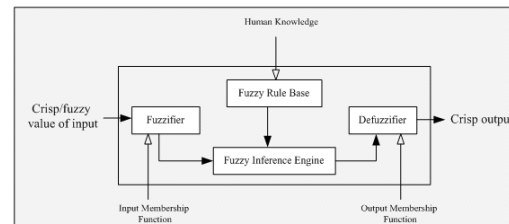


Figure 1. Basic configuration of mamdani fuzzy systems.

In order to convert the inputs into the output, fuzzy inference system based on the fuzzy logic is utilized by composite if-then rules of developed rule base. To implement fuzzy inference system, in this research, following assumptions have been considered⁴⁶:

- A singleton fuzzifier in the terms of fuzzification interface
- Mamdani implication engine in order to inference the fuzzy system
- A centroid defuzzifier in the terms of defuzzification interface

Let x_1 and x_2 are two input variables, also x_1^0 and x_2^0 are their input fuzzy singleton. Fuzzy inference procedure is illustrated as following steps:

Step 1: Involve inputs fuzzy singletons into their universe sets.

Step 2: Combine inputs fuzzy sets which are consisted of fuzzy singletons and obtain the active rules.

Step 3: Calculate $\mu_{x_1}(x_1^0)$ and $\mu_{x_2}(x_2^0)$ as membership functions.

Step 4: Determine matching degree for each rules as

$$\alpha_j = \min(\mu_{x_1}(x_1^0), \mu_{x_2}(x_2^0)) \tag{6}$$

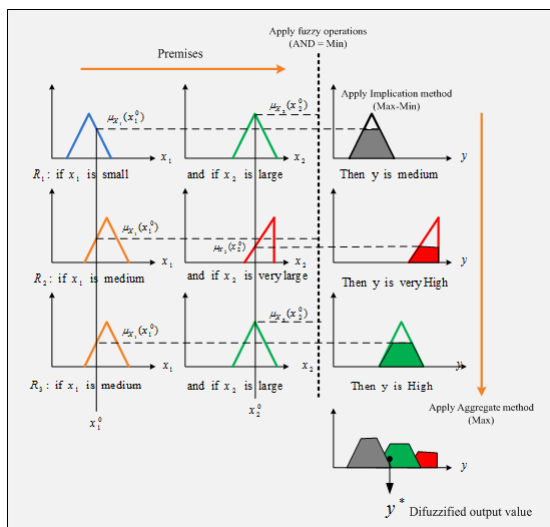


Figure 2. Illustration of the Mamdani fuzzy inference system scheme.

Step 5: Execute Max-Min operator to determine the outputs area.

Step 6: Steps 3-5 should be iterated for all active rules.

Step 7: Aggregate all outputs and obtain crisp value of output by applying centroid defuzzification in Equation 7. Let \bar{y}^j be the center of output for j th rule.

$$y^* = \frac{\sum_{j=1}^J \bar{y}^j \times \alpha_j}{\sum_{j=1}^J \alpha_j} \tag{7}$$

Figure 2 illustrates fuzzy inference procedure with considering three rules.

3. The Proposed Method

In this section we develop an extended PROMETHEE method based on fuzzy systems. As reviewed PROMETHEE method, except step of preference function selection, DM has not any role in decision making process. Moreover, it is possible which conventional preference function cannot provide preferences of DM, completely. Considering DM’s preference and opinions leads process of selection to provide aspirations of DM. Therefore, modeling the preferences of DM as a utility function is valuable in decision making. Often in real case problems, preferences of DM are as linguistic terms in form IF-THEN rules. Thus, construction utility function as a mathematical function is a challenge in decision making process. In this paper, fuzzy rule base system based on fuzzy logic is applied for deriving utility of DM within the PROMETHEE method. To do so, we present a step-by-step explanation of the proposed fuzzy PROMETHEE using the fuzzy rule based system and linguistic variables under group decision making. A graphical scheme of proposed method is depicted in Figure 3.

It should be mentioned that we had a GDM problem with m alternatives ($a_i, i=1, 2, \dots, m$), n criteria ($g_j, j=1, 2, \dots, n$) and K experts ($E_k, k=1, 2, \dots, K$) to determine the performance rating using linguistic variables. Linguistic variables were transformed into trapezoidal fuzzy numbers.

$$\tilde{E}_k = \begin{bmatrix} \tilde{x}_{11k} & \tilde{x}_{12k} & \dots & \tilde{x}_{1nk} \\ \tilde{x}_{21k} & \tilde{x}_{22k} & \dots & \tilde{x}_{2nk} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{x}_{m1k} & \tilde{x}_{m2k} & \dots & \tilde{x}_{mnk} \end{bmatrix}, \tilde{x}_{ijk} = (x_{ijk}^l, x_{ijk}^m, x_{ijk}^n, x_{ijk}^u), k=1,2,\dots,K$$

Where \tilde{x}_{ijk} was the trapezoidal fuzzy numbers indicating the performance rating of i th alternative with regards to the j th criteria for the k th expert. The steps of proposed fuzzy algorithm are as below.

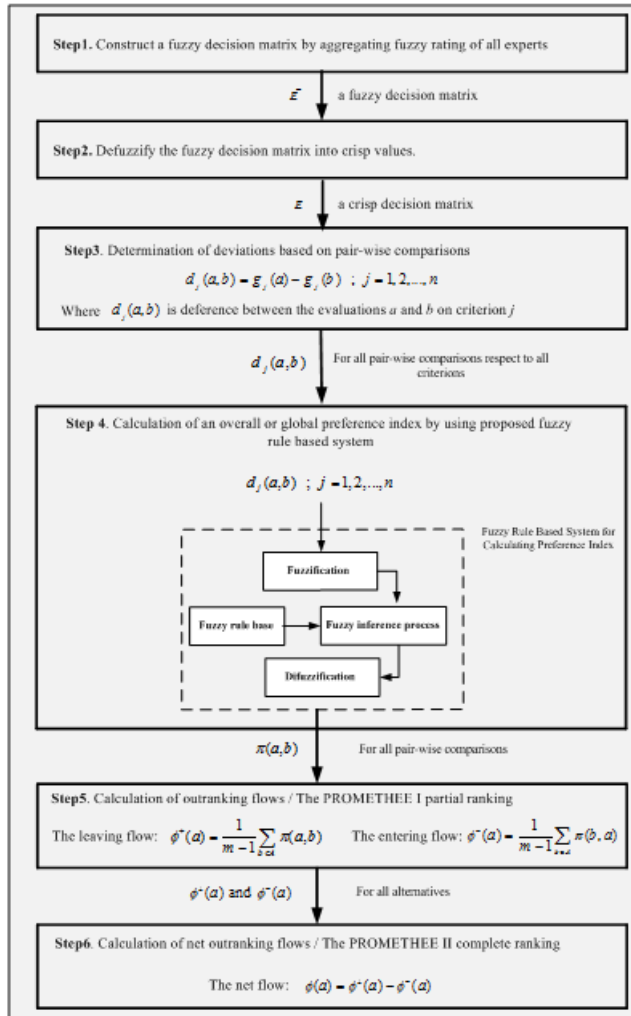


Figure 3. The scheme of proposed PROMETHEE method based on fuzzy system.

Step 1. Construct a fuzzy decision matrix (\tilde{E}) by aggregating fuzzy rating of all experts.

Let $\tilde{x}_{ij} = (x_{ij}^l, x_{ij}^m, x_{ij}^n, x_{ij}^u)$ is the aggregated fuzzy rating which can be calculated as:

$$x_{ij}^l = \min_k \{x_{ijk}^l\}, \quad x_{ij}^m = \frac{1}{K} \sum_{k=1}^K x_{ijk}^m, \quad x_{ij}^n = \frac{1}{K} \sum_{k=1}^K x_{ijk}^n, \quad x_{ij}^u = \max_k \{x_{ijk}^u\} \tag{8}$$

Therefore final fuzzy decision matrix can be constructed as:

$$\tilde{E} = \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \dots & \tilde{x}_{1n} \\ \tilde{x}_{21} & \tilde{x}_{22} & \dots & \tilde{x}_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{x}_{m1} & \tilde{x}_{m2} & \dots & \tilde{x}_{mn} \end{bmatrix}$$

Step 2. Defuzzify the fuzzy decision matrix into crisp values (E).

In order to derive the crisp values of arrays of decision matrix we may use the following equation:

$$defuzz(x_{ij}) = \frac{\int \mu(x).xdx}{\int \mu(x).dx} = \frac{\int_{x_{ij}^l}^{x_{ij}^m} (\frac{x-x_{ij}^l}{x_{ij}^m-x_{ij}^l}).xdx + \int_{x_{ij}^m}^{x_{ij}^n} xdx + \int_{x_{ij}^n}^{x_{ij}^u} (\frac{x_{ij}^u-x}{x_{ij}^u-x_{ij}^n}).xdx}{\int_{x_{ij}^l}^{x_{ij}^m} (\frac{x-x_{ij}^l}{x_{ij}^m-x_{ij}^l}).dx + \int_{x_{ij}^m}^{x_{ij}^n} dx + \int_{x_{ij}^n}^{x_{ij}^u} (\frac{x_{ij}^u-x}{x_{ij}^u-x_{ij}^n}).dx} = \frac{-x_{ij}^l x_{ij}^m + x_{ij}^m x_{ij}^n + \frac{1}{3}(x_{ij}^u-x_{ij}^n)^2 - \frac{1}{3}(x_{ij}^m-x_{ij}^l)^2}{-x_{ij}^l-x_{ij}^m+x_{ij}^n+x_{ij}^u} \tag{9}$$

Step 3. Determination differences based on pair-wise comparisons as follows:

$$d_j(a_1, a_2) = g_j(a_1) - g_j(a_2); j = 1, 2, \dots, n \tag{10}$$

Where $d_j(a_1, a_2)$ is deference between the rating a_1 and a_2 on criterion j . in order to simplify further computations, we can establish pair-wise comparisons tables. Table 1 shows pair-wise comparisons of a_1 related to other alternatives on all criteria and it must be constructed for all alternatives.

Table 1. Pair-wise comparisons table of a_1 related to other alternatives on all criteria

alternatives	g1	g2	...	g _n	$\pi(a_1, b)$
a2	$d_1(a_1, a_2)$	$d_2(a_1, a_2)$		$d_n(a_1, a_2)$	$\pi(a_1, a_2)$
a3	$d_1(a_1, a_3)$	$d_2(a_1, a_3)$		$d_n(a_1, a_3)$	$\pi(a_1, a_3)$
⋮	⋮	⋮		⋮	⋮
am	$d_1(a_1, a_m)$	$d_2(a_1, a_m)$		$d_n(a_1, a_m)$	$\pi(a_1, a_m)$

Step 4. Calculation of an overall or global preference index.

Unlike conventional PROMETHEE method which DM only select a preference function, this paper attempts to increase DM's role in decision making process. In order to obtain global preference index, we propose a fuzzy rule based system based on preferences of DM in form of IF-THEN rules. To do so, we have used fuzzy rule based system method mentioned in Section 2.2 to calculate global preference index.

Input variables of our proposed FRBS are $d_j(a, b)$ ($j=1, 2, \dots, n$). For each antecedent variable, three linguistic terms are proposed, named "Low", "Medium" and "High". Output variable of FRBS is $\pi(a, b)$. For consequent

variable, five linguistic terms are defined, called “Very Low”, “Low”, “Medium”, “High” and “Very High”. Triangular fuzzy numbers are defined for antecedent and consequent variables. It is necessary to mention that main component of FRBS is fuzzy rule based of DMs preferences and knowledge. The base of rules consists of *if-then* rules as follows:

$$\text{if } \underbrace{d_1(a,b) \text{ is very high and } \dots \text{ and } d_n(a,b) \text{ is medium}}_{\text{set of antecedent conditions}} \text{ then } \underbrace{\pi(a,b) \text{ is high}}_{\text{Consequent}} \quad (11)$$

The inference process to obtain $\pi(a_i,b)$ from $d_j(a,b)$ is defined as in the Mamdani method indicated in Figure 2.

After establishing pair-wise comparisons tables, calculated $d_j(a,b)$ are involved into designed FRBS and then fuzzy inference procedure as described in Section 2.2 is applied to obtain final output of FRBS as global preference index ($\pi(a_i,b)$). For example global preference indexes of alternative a_1 over other alternatives can be exhibited in last column of Table 1. In order to simplify future computations, we complete Table 2 at the end of this step.

Table 2. Global preference index for all pair-wise comparisons alternatives

	a1	a2	a3	...	am
a1	–	$\pi(a_1,a_2)$	$\pi(a_1,a_3)$...	$\pi(a_1,a_m)$
a2	$\pi(a_2,a_1)$	–	$\pi(a_2,a_3)$...	$\pi(a_2,a_m)$
⋮	⋮	⋮	⋮	⋮	⋮
a _m	$\pi(a_m,a_1)$	$\pi(a_m,a_2)$	$\pi(a_m,a_3)$...	–

Step 5. Calculation of outranking flows / The PROMETHEE I partial ranking.

In this section outranking flows including leaving flow and entering flow are calculated as follows:

The leaving flow:
$$\phi^+(a) = \frac{1}{m-1} \sum_{b \in A} \pi(a,b) \quad (12)$$

The entering flow:
$$\phi^-(a) = \frac{1}{m-1} \sum_{b \in A} \pi(b,a) \quad (13)$$

At the end of this section the PROMETHEE I partial ranking of alternatives are provided.

Step 6. Calculation of net outranking flows / The PROMETHEE II complete ranking.

In order to provide a complete ranking of alternatives, PROMETHEE II is applied by calculating net flow as follows:

The net flow:
$$\phi(a) = \phi^+(a) - \phi^-(a) \quad (14)$$

4. Illustrative Example

In this section, we evaluate performance of our proposed method for a SSP. To do so, a numerical example has been applied. Suppose that a company wants to select a suitable supplier for a main material which strongly effects on production process. To do so, a committee of 10 experts is established. After initial screening, four candidate suppliers (a_1, a_2, a_3 and a_4) remain for further evaluation. All 10 experts were asked to provide a list of criteria that could be used to evaluate suppliers and ultimately four criteria consisting of C1: *quality*, C2: *Price*, C3: *lead time delivery*, C4: *Flexibility* is selected. Judgments of experts are considered as linguistic variables. We consider five linguistic variables for the performance rating of the alternative. These linguistic variables and its fuzzy numbers are shown in Table 3 and Figure 4.

Table 3. The linguistic variables for the performance rating and their associated fuzzy numbers

Linguistic Variable	Fuzzy number
Very Poor (VP)	(0,0,0.1,0.2)
Poor (P)	(0.1,0.2,0.2,0.3)
Moderately Poor (MP)	(0.2,0.3,0.4,0.5)
Faire (F)	(0.4,0.5,0.5,0.6)
Moderately good (MG)	(0.5,0.6,0.7,0.8)
Good (G)	(0.7,0.8,0.8,0.9)
Very good (VG)	(0.8,0.9,1,1)

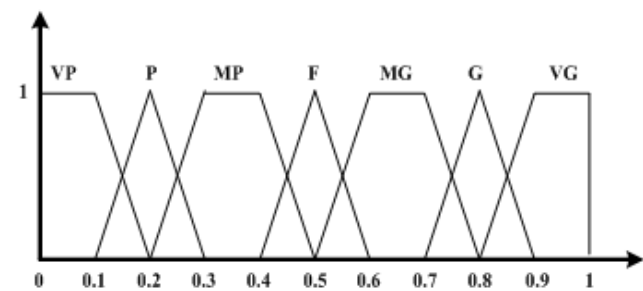


Figure 4. Linguistic variables for the fuzzy rates of alternatives.

The performance rating of the alternative with respect to the four criteria provided by each individual expert are presented in Table 4.

We have applied our proposed method indicated in Figure 3 to select best supplier by following steps.

Table 4. The performance rating of the suppliers with respect to criteria provided by DMs

Experts	Suppliers	Criteria				Experts	Suppliers	Criteria			
		C1	C2	C3	C4			C1	C2	C3	C4
e1	a1	MG	MG	G	VG	e6	a1	VG	G	MG	G
	a2	MG	G	G	MG		a2	G	G	G	VG
	a3	G	F	MG	VG		a3	G	MG	MG	G
	a4	VG	G	MG	G		a4	MG	MG	G	VG
e2	a1	G	G	G	VG	e7	a1	MG	G	G	MG
	a2	MG	MG	MG	MG		a2	VG	G	MG	G
	a3	VG	F	MG	VG		a3	MG	MG	MG	MG
	a4	G	VG	G	G		a4	MG	MG	G	VG
e3	a1	MG	G	MG	MG	e8	a1	VG	F	MG	VG
	a2	MG	G	G	G		a2	MG	G	MG	MG
	a3	G	MG	MG	G		a3	G	MG	MG	G
	a4	G	VG	MG	MG		a4	G	VG	MG	MG
e4	a1	MG	MG	G	VG	e9	a1	G	MG	MG	G
	a2	MG	MG	MG	MG		a2	MG	MG	G	VG
	a3	G	MG	MG	G		a3	VG	F	MG	VG
	a4	VG	G	MG	G		a4	G	VG	G	G
e5	a1	MG	G	G	MG	e10	a1	G	MG	MG	G
	a2	VG	F	MG	VG		a2	VG	F	MG	VG
	a3	G	VG	MG	MG		a3	VG	G	MG	G
	a4	G	G	G	VG		a4	G	VG	G	VG

Table 5. Aggregated fuzzy values of supplier rates

Alternatives	Criteria			
	C1	C2	C3	C4
a ₁	(0.50, 0.67, 0.73,0.90)	(0.50, 0.73, 0.77,0.90)	(0.50, 0.73, 0.77,0.90)	(0.50, 0.80, 0.90, 1.00)
a ₂	(0.50, 0.60, 0.70, 0.80)	(0.50, 0.73, 0.77,0.90)	(0.50, 0.73, 0.77,0.90)	(0.50, 0.67, 0.73, 0.90)
a ₃	(0.70, 0.83, 0.87,1.00)	(0.40, 0.53, 0.57,0.80)	(0.50, 0.60, 0.70, 0.80)	(0.70, 0.87, 0.93, 1.00)
a ₄	(0.70, 0.83, 0.87,1.00)	(0.70, 0.87, 0.93,1.00)	(0.50, 0.67, 0.73,0.90)	(0.50, 0.73, 0.77, 0.90)

Table 6. Defuzzified aggregated fuzzy values of supplier rates

Alternatives	Criteria			
	C1	C2	C3	C4
a ₁	0.70	0.72	0.72	0.79
a ₂	0.65	0.72	0.72	0.70
a ₃	0.85	0.58	0.65	0.87
a ₄	0.85	0.87	0.70	0.72

Step 1. The aggregated fuzzy numbers according to the Equation 8 are calculated as a fuzzy decision matrix and results are shown in Table 5.

Step 2. The aggregated fuzzy values of supplier rates are defuzzified using Equation 9 which results are shown in Table 6.

Table 7. Pair-wise comparisons of alternative a1

Alternatives	Criteria				$\pi(a_1, b)$
	C1	C2	C3	C4	
a ₂	0.05	0	0	0.09	0.0909
a ₃	0	0.14	0.07	0	0.1798
a ₄	0	0	0.02	0.07	0.0800

Step 3. Determination differences based on pair-wise comparisons using Equation 10. Results are shown in Tables 7-10.

Table 8. Pair-wise comparisons of alternative a2

Alternatives	Criteria				$\pi(a_2, b)$
	C1	C2	C3	C4	
a_1	0	0	0	0	0.0000
a_3	0	0.14	0.07	0	0.1798
a_4	0	0	0.02	0	0.0800

Table 9. Pair-wise comparisons of alternative a3

Alternatives	Criteria				$\pi(a_3, b)$
	C1	C2	C3	C4	
a_1	0.15	0	0	0.08	0.1030
a_2	0.20	0	0	0.17	0.2500
a_4	0	0	0	0.15	0.0800

Table 10. Pair-wise comparisons of alternative a4

Alternatives	Criteria				$\pi(a_4, b)$
	C1	C2	C3	C4	
a_1	0.15	0.15	0	0	0.0800
a_2	0.20	0.15	0	0.02	0.1482
a_3	0	0.29	0.05	0	0.0800

Step 4. Global preference index are calculated using proposed FRBS in Section 4.

In order to design FRBS, we have constructed a fuzzy rule base from DMs opinion on form of *if-then* terms. After a joint meeting, DMs introduced 35 rules as shown in Table 11. Pair-wise differences obtained in Tables 7-10 are used as inputs of proposed FRBS and Global preference index ($\pi(a_i, a_k)$; $i, k = 1, 2, 3, 4$; $i \neq k$) are calculated and shown in Table 12.

Table 11. The fuzzy rule base of proposed FRBS

Rule Number	If part				Then part $\pi(a, b)$
	$d_1(a, b)$	$d_2(a, b)$	$d_3(a, b)$	$d_4(a, b)$	
1	High	High	High	Low	Very High
2	Low	High	Medium	High	High
3	High	Medium	Medium	Medium	High
4	Medium	Low	Medium	High	Medium
⋮	⋮	⋮	⋮	⋮	⋮
35	Medium	Low	Low	High	Low

Step 5. Outranking flows including leaving and entering flows are calculated using Equation 12 and 13. Results are indicated in Table 13. In this step PROMETHEE

II partial ranking of suppliers are provided as $a_1 P^{(T)} a_2, a_1 a_1 R a_3, a_1 P^{(I)} a_4, a_3 P^{(I)} a_4, a_4 P^{(I)} a_2,$ and $a_3 R a_4$. Figure 5 shows a value outranking graph of the constructed partial ranking. As can be seen, a_3 is incomparable to a_1 and a_4 . Therefore, it is not suitable to make a decision based on partial ranking. Hence in the next step complete ranking is presented.

Table 12. Global preference index for Pair-wais Comparisons of alternatives

Alternatives	$\pi(a_i, a_k), i, k=1, 2, 3, 4$ and $i \neq k$			
	a_i			
	a_1	a_2	a_3	a_4
a_k a_1	–	0.0000	0.1030	0.0800
a_2	0.0909	–	0.2500	0.1482
a_3	0.1798	0.1798	–	0.0800
a_4	0.0800	0.0800	0.0800	–

Step 6. Net outranking flow for each supplier is calculated using Equation 14.

Table 13. The results of leaving flows, entering flows, and net flows

Alternatives	$\phi^+(a)$	$\phi^-(a)$	$\phi(a)$	Rank
a_1	0.117	0.061	0.056	1
a_2	0.087	0.163	-0.076	4
a_3	0.144	0.147	-0.002	3
a_4	0.103	0.080	0.023	2

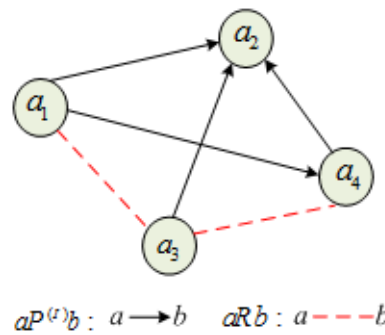


Figure 5. The partial ranking in considered SSP,

The values of net outranking flow are calculated as $\phi(a_1) = 0.056, \phi(a_2) = 0.076, \phi(a_3) = 0.02$ and $\phi(a_4) = 0.023$. In this step, the PROMETHEE II complete ranking of suppliers is provided. According to calculated values of net outranking flow the ranking of suppliers by the PROMETHEE II complete ranking are $a_1 P^{(II)} a_4, a_4 P^{(II)} a_3,$

$a_3 \succ^{(III)} a_2$, and . Therefore, complete ranking of suppliers can be shown as $a_1 \succ^{(II)} a_4 \succ^{(II)} a_3 \succ^{(II)} a_2$. Figure 6 shows the value outranking graph of the constructed complete preorder. The summary of leaving, entering, and net flows is presented in Table 13. As stated, ranking results indicate that supplier a_1 is the best among all alternatives.

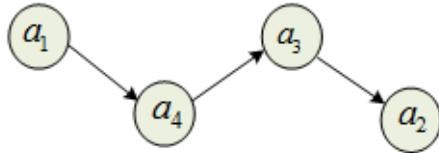


Figure 6. The complete ranking in considered SSP.

5. Conclusion

A successful supply chain for companies is more dependent on selecting suitable suppliers. However, the nature of supplier selection is a complex multi-criteria problem including both quantitative and qualitative factors which may be in conflict and uncertain. We studied a SSP under group decision making situation and fuzzy linguistic variables for rating of alternative.

In selection process, as main contribution, a fuzzy PROMETHEE based on a FRBS for SSP was proposed. In conventional PROMETHEE methods, DM only selects predetermined preference function as his/her preferences. However, due to more consideration of DM's preferences and opinions in decision making process, the conventional preference functions in PROMETHEE method could be inadequate and inexact to capture the right preferences of DM's. That is why; a FRBS was integrated with the conventional PROMETHEE to overcome this problem.

In our proposed FRBS, deviations based on pairwise comparisons were input variables and the value of global preference index for each couple of alternatives was output variable. In our proposed methodology, preferences and opinions of DMs were collected in form of if-then rules. In real decision making problem, DMs are often unwilling to determine weight of criteria exactly in numerical values. Therefore, we have caught the weights of criteria as indirectly way from DM within if-then rules.

6. References

- De Boer L, Labro E, Morlacchi P. A review of methods supporting supplier selection. *European Journal of Purchasing and Supply Management*. 2001 Jun; 7(2):75–89.
- Jiang ZF, Zhuang TY, Lin SX. Empirical analysis of the effects of supplier selection and integration on customer satisfaction and business performance. *IEEE International Conference on management of innovation and technology*; Singapore, China. 2006. p. 931–5.
- Ho W, Xu X, Dey PK. Multi-criteria decision making approaches for supplier evaluation and selection: A literature review. *European Journal of Operational Research*. 2010 Apr; 202(1):16–24.
- Xia W, Wu Z. Supplier selection with multiple criteria in volume discount environments. *Omega the international Journal of Management Science*. 2007 Oct; 35(5):494–504.
- Wu C, Barnes D. A literature review of decision-making models and approaches for partner selection in agile supply chains. *Journal of Purchasing and Supply Management*. 2011 Dec; 17(4):256–74.
- Dickson GW. An analysis of vendor selection systems and decisions. *Journal of Purchasing*. 1966; 2(1):5–17.
- Weber CA, Current JR, Benton WC. Vendor selection criteria and methods. *European Journal of Operational Research*. 1991 Jan; 50(1):2–18.
- Kannan VR, Tan KC. Supplier selection and assessment: Their impact on business performance. *Journal of Supply Chain Management*. 2002 Sep; 38(3):11–21.
- Kahraman C, Cebeci U, Ulukan Z. Multi-criteria supplier selection using fuzzy AHP. *Logistics Information Management*. 2003; 16(6):382–94.
- Frödell M. Criteria for achieving efficient contractor–supplier relations. *Engineering, Construction and Architectural Management*. 2011; 18(4):381–93.
- Ku C, Chang C, Ho H. Global supplier selection using fuzzy analytic hierarchy process and fuzzy goal programming. *Quality and Quantity*. 2010 Jun; 44(4):623–40.
- Chai J, Liu JNK, Ngai EWT. Application of decision-making techniques in supplier selection: A systematic review of literature. *Expert Systems with Applications*. 2013 Aug; 40(10):3872–85.
- Wu WY, Lin CT, Kung JY. Supplier selection in supply chain management by using fuzzy multiple-attribute decision-making method. *Journal of Intelligent and Fuzzy Systems*. 2013; 24(1):175–83.
- Zadeh LA. Fuzzy sets. *Information and Control*. 1965; 8:338–53.
- Zadeh LA. The concept of a linguistic variable and its application to approximate reasoning. *Information Sciences*. 1975; 8(3):199–249.
- Carrera DA, Mayorga RV. Supply chain management: a modular Fuzzy Inference System approach in supplier selection for new product development. *Journal of Intelligent Manufacturing*. 2008 Feb; 19(1):1–12.
- Wang J, Cheng C, Kun-Cheng H. Fuzzy hierarchical TOPSIS for supplier selection. *Applied Soft Computing*. 2009 Jan; 9(1): 377–86.
- Boran FE, Genc S, Kurt M, Akay D. A multi-criteria intuitionistic fuzzy group decision making for supplier selection with TOPSIS method. *Expert Systems with Applications*. 2009 Oct; 36(8):11363–68.
- Keskin GA, Ilhan S, Ozkan C. The fuzzy ART algorithm: A

- categorization method for supplier evaluation and selection. *Expert System with Applications*. 2010 Mar; 37(2):1235–40.
20. Hsu B, Chiang C, Shu M. Supplier selection using fuzzy quality data and their applications to touch screen. *Expert Systems with Applications*. 2010 Sep; 37(9): 6192–200.
 21. Büyükoçkan G, Cifci G. A novel fuzzy multi-criteria decision framework for sustainable supplier selection with incomplete information. *Computers in Industry*. 2011 Feb; 62(2):16474.
 22. Shemshadi A, Shirazi H, Toreihi M, Tarokh MJ. A fuzzy VIKOR method for supplier selection based on entropy measure for objective weighting. *Expert Systems with Applications*. 2011 Sep; 38(10):12160–7.
 23. Amindoust A, Ahmed S, Saghafinia A, Bahreininejad A. Sustainable supplier selection: a ranking model based on fuzzy inference system. *Applied Soft Computing*. 2012 Jun; 12(6):1668–77.
 24. Buyüo Özkan G, Cifci G. A novel hybrid MCDM approach based on fuzzy DEMATEL, fuzzy ANP and fuzzy TOPSIS to evaluate green suppliers. *Expert Systems with Applications*. 2012 Feb; 39(3):3000–11.
 25. Lima Junior FR, Osiro L, Carpinetti LCR. A fuzzy inference and categorization approach for supplier selection using compensatory and non-compensatory decision rules. *Applied Soft Computing*. 2013 Oct; 13(10):4133–47.
 26. Dursun M, Karsak EE. A QFD-based fuzzy MCDM approach for supplier selection. *Applied Mathematical Modelling*. 2013 Apr; 37(8):5864–75.
 27. Lima Junior FR, Osiro L, Carpinetti LCR. A comparison between Fuzzy AHP and fuzzy TOPSIS methods to supplier selection. *Applied Soft Computing*. 2014 Aug; 21:194–209.
 28. Ai FY, Yang J, Zhang PD. An approach to multiple attribute decision making problems based on hesitant fuzzy set. *Journal of Intelligent and Fuzzy Systems*. 2014; 27(6):2749–55.
 29. Sultana I, Ahmed I, Azeem A. An integrated approach for multiple criteria supplier selection combining fuzzy Delphi, fuzzy AHP and fuzzy TOPSIS. *Journal of Intelligent and Fuzzy Systems*. 2015 Oct; 29(4):1273–87. DOI:10.3233/IFS-141216.
 30. Khalilo A, Sadi-Nezhad S, Najafi SE, Hosseinzadeh Lotfi F. Using computing with words for selecting projects in field of fuel consumption reduction. *Indian Journal of Science and Technology*. 2015 Jul; 8(15). DOI: 10.17485/ijst/2015/v8i15/70582.
 31. Moayeri M, Shahvarani A, Behzadi MH, Hosseinzadeh-Lotfi F. Comparison of fuzzy AHP and fuzzy TOPSIS methods for math teachers selection. *Indian Journal of Science and Technology*. 2015 Jul; 8(13). DOI: 10.17485/ijst/2015/v8i13/54100.
 32. Araz C, Ozkarahan I. Supplier evaluation and management system for strategic sourcing based on a new multi-criteria sorting procedure. *International Journal of Production Economics*. 2007 Apr; 106(2):585–606.
 33. Chen Y, Wang T, Wu C. Strategic decisions using the fuzzy PROMETHEE for IS outsourcing. *Expert Systems with Applications*. 2011 Sep; 38(10):13216–22.
 34. Chai J, Liu JNK, Xu Z. A new rule-based sir approach to supplier selection under intuitionistic fuzzy environments. *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems*. 2012 Jun; 20(3):451–71.
 35. Dagdeviren M. Decision making in equipment selection: an integrated approach with AHP and PROMETHEE. *Journal of Intelligent Manufacturing*. 2008 Aug; 19(4):397406.
 36. Taha Z, Rostam S. A hybrid fuzzy AHP-PROMETHEE decision support system for machine tool selection in flexible manufacturing cell. *Journal of Intelligent Manufacturing*. 2012 Dec; 23(6):2137–49.
 37. Behzadian M, Motlagh SMH, Ignatius J, Goh M, Sepehri MM. PROMETHEE Group Decision Support System and the House of Quality. *Group Decision and Negotiations*. 2013 Mar; 22(2):189–205.
 38. Hashemian SM, Behzadian M, Samizadeh R, Ignatius J. A fuzzy hybrid group decision support system approach for the supplier evaluation process. *International Journal of Advance Manufacturing Technology*. 2014 Jul; 73(5):05–1117–1105.
 39. Chen TY. A PROMETHEE-based outranking method for multiple criteria decision analysis with interval type-2 fuzzy sets. *Soft Computing*. 2014 May; 18(5):923–40.
 40. Ghazinoory S, Daneshmand-Mehr M, Arasti MR. Developing a model for integrating decisions in technology road mapping by fuzzy PROMETHEE. *Journal of Intelligent and Fuzzy Systems*. 2014 Mar; 26(2): 625–45
 41. Brans JP, Mareschal B, Vincke Ph. PROMETHEE: A new family of outranking methods in multi-criteria analysis. *Operational Research*. North-Holland: Elsevier Science Publishers B.V.; 1984 Jan. p. 408–21.
 42. Brans JP, Vincke Ph. A preference ranking organization method (The PROMETHEE method for MCDM). *Management Science*. 1985 Jun; 31(6): 647–56.
 43. Brans JP, Mareschal B. PROMETHEE methods. In: Figueira J, Greco S, Ehrgott M, editors. *Multiple criteria decision analysis: State of the art surveys*. Springer Science + Business Media, Inc., Boston; 2005. 78. p. 163–86.
 44. Behzadian M, Kazemzadeh RB, Albadvi A, Aghdasi M. PROMETHEE: A comprehensive literature review on methodologies and applications. *European Journal of Operational Research*. 2010 Jan; 200(1):198–215.
 45. Mamdani EH, Assilian S. An experiment in linguistic synthesis with a fuzzy logic controller. *International Journal of Man Machine Studies*. 1975 Jan; 7(1):1–13.
 46. Zimmermann HJ, *Fuzzy set theory and its applications*. Boston: Kluwer Academic Publishers; 1996.