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The Concept of Topological Entropy from the Viewpoint of an Intuitionistic Observer

Zahra Eslami Giski[1*] and Mohamad Ebrahimi[2]

[1]Department of Mathematics, Kerman Branch, Islamic Azad University, Kerman, Iran;
Eslamig_zahra@yahoo.com
[2]The Faculty of Mathematics and Computer, Shahid Bahonar University of Kerman, Kerman, Iran

Abstract

An accurate and vigilant observer is required to evaluate the changes in the nature. In mathematics, introducing a mathematical modeling to describe a natural dynamical system and a method to compare and evaluate different observers' perspectives is of utmost importance. Therefore, at the beginning of this paper the concept of topology from the viewpoint of an intuitionistic observer, which is defined as an intuitionistic fuzzy set, is introduced. Afterwards, the notion of entropy as a main tool determining the complexity and/or uncertainty of the system related to an intuitionistic observer is studied and some properties of these notions are proved. In addition, it is proved that the entropy related to an intuitionistic observer is an invariant object under conjugate relation.

Keywords: Dynamical System, Relative Intuitionistic Dynamical System, Relative Intuitionistic Topological Entropy, Topological Entropy

1. Introduction

Entropy is an important tool in science, especially in physics and mathematics. It classifies the dynamical systems according to conjugate relation and measures the complex behavior of the orbits in a dynamical system as well. In addition, it measures uncertainty in information theory and disorders in statistical mechanics. The concept of topological entropy was presented in 1965 as an invariant of topological conjugate relation for continues maps[1]. Later, Bowen[2] and Dinaburg[3] gave an equivalent definition for topological entropy and this notion was further extended by the other researchers[3-6]. Molaei, considering the view that every phenomenon is finally evaluated by the observers, introduced the relative semi dynamical systems and relative topological entropy on the relative semi dynamical systems[7,8]. Relative entropy is defined as the entropy from the viewpoint of an observer. Mathematical model for an observer is defined as a fuzzy set. Following the concept of fuzzy sets, Atanassov[9,10] presented and studied the idea of IF sets. Using the idea of IF sets, some researchers have tried to generalize the mathematical models[11-14]. The important goal of this paper is to extend the notions of relative topological entropy, relative semi dynamical system to relative intuitionistic entropy and relative intuitionistic dynamical system, respectively. Consequently, in section 2, the relative intuitionistic topological space is introduced and some properties of this notion are investigated. In section 3, after defining the Relative Intuitionistic Topological (RIT in short) entropy, it is proved that finer partitions have bigger RIT entropy and also if "λ" is a cover that is obtained by joining of the two covers "θ" and "σ" then RIT entropy " λ" is less than the summation of RIT entropies " θ" and "σ". RIT entropy on relative intuitionistic dynamical system is introduced in section 4 and it is proved that two $\langle \zeta_\Gamma, \upsilon_\Gamma \rangle$ - conjugate functions f and g have the same RIT entropies.

* *Author for correspondence*

2. Relative Intuitionistic Topological Space

This section has started with some basic definitions of intuitionistic fuzzy sets and then it has continued with defining RI topology space while proving its properties.

Definition 2.1

Suppose that X is a set which is nonempty. An Intuitionistic Fuzzy Set (IFS in short) Γ in $X$is a set with the form, $\Gamma=\{\langle x,\zeta_\Gamma(x),\upsilon_\Gamma(x)\rangle \mid x\in X\}$ such that $\Gamma=\{\langle x,\zeta_\Gamma(x),\upsilon_\Gamma(x)\rangle \mid x\in X\}$ and $\upsilon_A : X\to[0,1]$ are two functions while ζ_Γ denote the degree of membership and $u_A$indicative the degree of non-membership of every member $x\in X$ to the set Γ and for every $x\in X$, the condition $0\le\zeta_\Gamma(x)+\upsilon_\Gamma(x)\le 1$ must be true[10].

We show that, the set of all IF sets in X with IFS (X).

Definition 2.2

Consider IF sets $\Gamma=\{\langle x,\zeta_\Gamma(x),\upsilon_\Gamma(x)\rangle \mid x\in X\}$ and

$\Lambda=\{\langle x,\zeta_\Lambda(x),\upsilon_\Lambda(x)\rangle \mid x\in X\}$. [10] Then:

- $\Gamma\subseteq\Lambda\Leftrightarrow$ for every $x\in X,\ \zeta_\Gamma(x)\le\zeta_\Lambda(x)$ and $\upsilon_\Gamma(x)\ge\nu_\Lambda(x)$,
- $\Gamma=\Lambda\Leftrightarrow\Gamma\subseteq\Lambda$ and $\Lambda\subseteq\Gamma$,,
- $\Gamma^C=\{\langle x,\upsilon_\Gamma(x),\zeta_\Gamma(x)\rangle \mid x\in X\}$,
- $\Gamma\cap\Lambda=\{\langle x,\inf\{\zeta_\Gamma(x),\zeta_\Lambda(x)\},\sup\{\upsilon_\Gamma(x),\upsilon_\Lambda(x)\rangle \mid x\in X\}$,
- $\Gamma\cup\Lambda=\{\langle x,\sup\{\zeta_\Gamma(x),\zeta_\Lambda(x)\},\inf\{\upsilon_\Gamma(x),\upsilon_\Lambda(x)\rangle \mid x\in X\}$.

The IF sets $\chi_\varnothing = 0_\sim = \{\langle x, 0, 1\rangle | x\in X\}$ and $1_\sim = \{\langle x, 1, 0\rangle | x\in X\}$ in order, are called the empty set and, the set X.

In this essay, we will apply the symbol $\Gamma=\langle\zeta_\Gamma,\upsilon_\Gamma\rangle$ instead of $\Gamma=\{\langle x,\zeta_\Gamma(x),\upsilon_\Gamma(x)\rangle \mid x\in X\}$.

Definition 2.3

We define an IF set, $\Gamma=\langle\zeta_\Gamma,\upsilon_\Gamma\rangle$ as a Relative Intuitionistic Observer (RIO in short) of set X.

Definition 2.4

Let $\Gamma=\langle\zeta_\Gamma,\upsilon_\Gamma\rangle$ be a RIO of X. Collection $\tau_{(\zeta_\Lambda,\upsilon_\Lambda)}$ of subsets of $\Gamma=\langle\zeta_\Gamma,\upsilon_\Gamma\rangle$ is named Relative Intuitionistic Topology (RIT in short) on X if the collection satisfy in the following axioms:

1. $\chi_\varnothing,\langle\zeta_\Gamma,\upsilon_\Gamma\rangle\in\tau_{(\zeta_\Gamma,\upsilon_\Gamma)}$,
2. For every $\Lambda_1,\Lambda_2\in\tau_{(\zeta_\Gamma,\upsilon_\Gamma)}$, we have $\Lambda_1\cap\Lambda_2\in\tau_{(\zeta_\Gamma,\upsilon_\Gamma)}$,
3. For any family $\{\Lambda_\alpha \mid \alpha\in I\}\subseteq\tau_{(\zeta_\Gamma,\upsilon_\Gamma)}$ we have $\bigcup_{\alpha\in I}\Lambda_\alpha\in\tau_{(\zeta_\Gamma,\upsilon_\Gamma)}$.

The pair $(X,\tau_{(\zeta_\Gamma,\upsilon_\Gamma)})$ is named a Relative Intuitionistic Topology Space (RITS in short) and elements of $\tau_{(\zeta_\Gamma,\upsilon_\Gamma)}$ are called $\langle\zeta_\Gamma,\upsilon_\Gamma\rangle$ - open observer and also $\Lambda=\langle\zeta_\Lambda,\upsilon_\Lambda\rangle$ is a $\langle\zeta_\Gamma,\upsilon_\Gamma\rangle$ - closed if $\Lambda^C=\langle\upsilon_\Lambda,\zeta_\Lambda\rangle\in\tau_{(\zeta_\Gamma,\upsilon_\Gamma)}$.

Example 2.5

Suppose that, F is the set of all mappings ψ from $\mathbb{N}$ to {0, 1}. Also let $\zeta : X\to[0,1]$, $\upsilon : X\to[0,1]$ be defined by:

$$\zeta(\psi)=\begin{cases}0 & \text{if } \psi(i)=0, \text{for every } i\in\mathbb{N}\\ \frac{1}{2} & \text{o.w}\end{cases},$$

$$\upsilon(\psi)=\begin{cases}1 & \text{if } \psi(i)=0, \text{for every } i\in\mathbb{N}\\ \frac{1}{3} & \text{o.w}\end{cases}$$

Consider $\tau_{(\zeta,\upsilon)}=\{\chi_\varnothing,\langle\zeta,\upsilon\rangle,\langle\zeta_j,\upsilon_j\rangle : j\in\mathbb{N}\}$ where

$$\zeta_j(\psi)=\begin{cases}\frac{1}{2^{j+1}} & \text{if } \psi(j)=1\\ 0 & \text{o.w}\end{cases},$$

$$\upsilon_j(\psi)=\begin{cases}\sum_{n=0}^{j}\frac{1}{3^{2n+1}} & \text{if } \psi(j)=1\\ 1 & \text{o.w}\end{cases}$$

We check the properties of the definition 2.4 for $\tau_{(\zeta,\upsilon)}$.

- $\langle\zeta_j,\upsilon_j\rangle\subseteq\langle\zeta,\upsilon\rangle$ For every $j\in\mathbb{N}$,
- $\langle\zeta_j,\upsilon_j\rangle\cap\langle\zeta_k,\upsilon_k\rangle=\langle\inf\{\zeta_j,\zeta_k\},\sup\{\upsilon_j,\upsilon_k\}\rangle$,

$$\sup\{\upsilon_j(\psi),\upsilon_k(\psi)\}=\begin{cases}\sum_{n=0}^{k}\frac{1}{3^{2n+1}} & \text{if } \psi(j)=\psi(k)=1,\ k\ge j\\ \sum_{n=0}^{j}\frac{1}{3^{2n+1}} & \text{if } \psi(j)=\psi(k)=1,\ k\le j\\ 1 & \text{o.w}\end{cases},$$

$$\inf\{\zeta_j(\psi),\zeta_k(\psi)\}=\begin{cases}\frac{1}{2^{k+1}} & \text{if } \psi(j)=\psi(k)=1,\ k\geq j\\ \frac{1}{2^{j+1}} & \text{if } \psi(j)=\psi(k)=1,\ k\leq j\\ 0 & \text{o.w}\end{cases}$$

- Let $\{\langle\zeta_\alpha,\upsilon_\alpha\rangle:\ \alpha\in I\}\subseteq\tau_{\langle\zeta,\upsilon\rangle}$, $\beta=\min\{\alpha:\alpha\in I\}$ then $\bigcup_{\alpha\in I}\langle\zeta_\alpha,\upsilon_\alpha\rangle=\langle\sup_{\alpha\in I}\zeta_\alpha,\inf_{\alpha\in I}\upsilon_\alpha\rangle$. If for every, $s\in I,\psi(s)=0$ then $\langle\sup_{\alpha\in I}\zeta_\alpha,\inf_{\alpha\in I}\upsilon_\alpha\rangle(\psi)=\chi_\varnothing$. If there exists, $s\in I$ such that $\psi(s)\neq 0$ then:

$$\sup_{\alpha\in I}\zeta_\alpha(\psi)=\begin{cases}\zeta_\beta(\psi) & \text{if } \psi(\beta)=1\\ \zeta_r(\psi) & \text{if } \psi(\beta)=0\end{cases},$$

$$\inf_{\alpha\in I}\upsilon_\alpha(\psi)=\begin{cases}\upsilon_\beta(\psi) & \text{if } \psi(\beta)=1\\ \upsilon_r(\psi) & \text{if } \psi(\beta)=0\end{cases}$$

Where, r is the first number of I that is greater than β, andand $\psi(r)=1$.

Definition 2.6

Suppose that $\Lambda=\langle\zeta_\Lambda, u_\Lambda\rangle$ is an IFS. Then the interior and closure of Λ are defined by:

$$\mathrm{Int}(\Lambda)=\Lambda^\circ=\cup\{\mathrm{P}:\mathrm{P}\subseteq\Lambda,\ \text{and P is an}\langle\zeta_\Lambda,\upsilon_\Lambda\rangle-\text{open observer}\},$$

$$Cl(\Lambda)=\overline{\Lambda}=\cap\{\mathrm{P}:\mathrm{P}\ \textit{is an}\langle\zeta_\Lambda,\upsilon_\Lambda\rangle-\textit{closed and}\ \Lambda\subseteq\mathrm{P}\}.$$

Theorem 2.7

Let $(X,\tau_{(\zeta_\Gamma,\upsilon_\Gamma)})$ be a RITS and $\Lambda=\langle\zeta_\Lambda,\upsilon_\Lambda\rangle$ be an IF observer. Then Λ° *is* $\langle\zeta_\Lambda,\upsilon_\Lambda\rangle$ - open observer and $\overline{\Lambda}$ *is* $\langle\zeta_\Lambda,\upsilon_\Lambda\rangle$ - closed.

Proof

Consider,

$J=\{\alpha:\mathrm{P}_\alpha\ \text{is}\langle\zeta_\Lambda,\upsilon_\Lambda\rangle-\text{open observer and } \mathrm{P}_\alpha\subseteq\Lambda\}$.

$\Lambda^\circ=\bigcup_{\alpha\in J}\mathrm{P}_\alpha$ *and* $\mathrm{P}_\alpha\in\tau_{(\zeta_\Lambda,\upsilon_\Lambda)}$ so $\Lambda^\circ\in\tau_{(\zeta_\Lambda,\upsilon_\Lambda)}$ and is $\langle\zeta_\Gamma,\upsilon_\Gamma\rangle$ -open observer.

Let $M=\{\mathrm{P}:\mathrm{P}\ \text{is}\ \langle\zeta_\Lambda,\upsilon_\Lambda\rangle-\text{closed and}\ \Lambda\subseteq\mathrm{P}\}$,

$$\bigcap_{\mathrm{P}\in M}\mathrm{P}=\langle\inf_{\mathrm{P}\in M}\zeta_\mathrm{P},\sup_{\mathrm{P}\in M}\upsilon_\mathrm{P}\rangle=\langle\sup_{\mathrm{P}\in M}\upsilon_\mathrm{P},\inf_{\mathrm{P}\in M}\zeta_\mathrm{P}\rangle^C=(\bigcup_{\mathrm{P}\in M}M^C)^C.$$

Since $\bigcup_{\mathrm{P}\in M}M^C\in\tau_{(\zeta_\Gamma,\upsilon_\Gamma)}$ *so* $\bigcap_{\mathrm{P}\in M}\mathrm{P}$ is $\langle\zeta_\Lambda,\upsilon_\Lambda\rangle$ - closed.

In general, one should pay attention to this point that in the relative intuitionistic observer approach $\Lambda=\langle\zeta_\Lambda,\upsilon_\Lambda\rangle$ is not necessarily a subset of $\overline{\Lambda}$. The next example proves our claim.

Example 2.8

Let $X=\{a,b\}$ and $\zeta_\Gamma(a)=0.7$, $\nu_\Gamma(a)=0.2$, $\zeta_\Gamma(b)=0.8$, $\nu_\Gamma(b)=0.1$. We show $\Gamma=\{\langle 0.7, 0.2\rangle_a, \langle 0.8, 0.1\rangle_b\}$.

Consider

$\tau_{(\zeta_\Gamma,\upsilon_\Gamma)}=\{\chi_\varnothing,\{\langle 0.7,0.2\rangle_a,\langle 0.8,0.1\rangle_b\},\{\langle 0.5,0.2\rangle_a,\langle 0.3,0.5\rangle_b\},\{\langle 0.6,0.2\rangle_a,\langle 0.8,0.2\rangle_b\}\ \{\langle 0.5, 0.3\rangle_a, \langle 0.3, 0.5\rangle_b\}$ and $B=\{\{\langle 0.8, 0.1\rangle_a, \langle 0.4, 0.4\rangle_b\}\}$.

If $B\subseteq G$ and $G^c\in\tau_{(\zeta\Gamma,u\Gamma)}$ then $\upsilon_{G^C}(a)$ must be 0.8 or 0.9 or 1, that implies $\overline{\Lambda}=\chi_\varnothing$ so Λ it's not a subset of $\overline{\Lambda}$.

Remark If $u_\Gamma=0$ then $\tau_{(\zeta_\Gamma,\upsilon_\Gamma)}$ is a relative topological space. If $\upsilon_\Gamma=0$, $\zeta_\Gamma=\chi_X$ and all the elements of $\tau_{(\zeta_\Gamma,\upsilon_\Gamma)}$ are characteristic function of some subsets of X then $(X,\tau_{(\zeta_\Gamma,0)})$ is a topological space and finally $(X,\tau_{(\chi_X,0)})$ is fuzzy topological space so the notion of relative intuitionistic topological space is a generalization of the notions of the relative topological space, fuzzy topological space and topological space.

Definition 2.9

A RITS $(X,\tau_{(\zeta_\Gamma,\upsilon_\Gamma)})$ is named compact, if every collection $\{\langle\zeta^\alpha,\upsilon^\alpha\rangle:\alpha\in I\}$ of elements $\tau_{(\zeta\Gamma,\upsilon\Gamma)}$ with $\bigcup_{\alpha\in I}\langle\zeta^\alpha,\upsilon^\alpha\rangle=\langle\zeta_\Gamma,\upsilon_\Gamma\rangle$ has a finite sub collection $\{\langle\zeta^\beta,\upsilon^\beta\rangle:\beta\in P,\ P\ \text{is finite}\}$ such that $\bigcup_{\beta\in P}\langle\zeta^\beta,\upsilon^\beta\rangle=\langle\zeta_\Gamma,\upsilon_\Gamma\rangle$. Such collection is called a RI open cover for $\langle\zeta_\Lambda,\upsilon_\Lambda\rangle$

Definition 2.10

Suppose that φ is a function from a RITS $(X,\tau_{(\zeta_\Gamma,\upsilon_\Gamma)})$ into a RITS $(Y,\tau_{(\zeta_\Lambda,\upsilon_\Lambda)})$. Then φ is said to be RI continuous map (RIC map in short) if

$$\varphi^{-1}(\Lambda)=\{x,\langle\varphi^{-1}(\zeta_\Lambda)(x),\varphi^{-1}(\upsilon_\Lambda)(x)\rangle:x\in X\}\cap\{x,\langle\zeta_\Gamma(x),\upsilon_\Gamma(x)\rangle:x\in X\}\in\tau_{(\zeta_\Gamma,\upsilon_\Gamma)}$$

$$\varphi^{-1}(\Lambda)=\{x,\langle\varphi^{-1}(\zeta_\Lambda)(x),\varphi^{-1}(\upsilon_\Lambda)(x)\rangle:x\in X\}\cap\{x,\langle\zeta_\Gamma(x),\upsilon_\Gamma(x)\rangle:x\in X\}\in\tau_{(\zeta_\Gamma,\upsilon_\Gamma)}$$

For every $\Lambda=\{y,\langle\zeta_\Lambda(y),\upsilon_\Lambda(y)\rangle:\ y\in Y\}\in\tau_{(\zeta_\Lambda,\upsilon_\Lambda)}$ and φ is RI homeomorphism if φ is bijection and both φ and φ^{-1} are RIC maps.

Since ζ_Γ, u_Γ, ζ_Λ and u_Λ are fuzzy sets, we explain that

$$\varphi^{-1}(\zeta_\Lambda)(x)=\zeta_\Lambda(\varphi(x)),\ \varphi^{-1}(\upsilon_\Lambda)(x)=\upsilon_\Lambda(\varphi(x))$$

and

$$\varphi(\upsilon_\Gamma)(y) = \begin{cases} \sup\limits_{x\in\varphi^{-1}(y)} \upsilon_\Gamma(x) & \text{if } \varphi^{-1}(y) \neq \varnothing \\ 0 & \text{o.w} \end{cases}$$

$$\varphi(\upsilon_\Gamma)(y) = \begin{cases} \sup\limits_{x\in\varphi^{-1}(y)} \upsilon_\Gamma(x) & \text{if } \varphi^{-1}(y) \neq \varnothing \\ 0 & \text{o.w} \end{cases}$$

Proposition 2.11

Let $\Gamma, \Gamma_i\,(i \in I)$ be IFSs in X and $\Lambda, \Lambda_j\,(j \in J)$ be IFSs in Y and $\varphi : X \longrightarrow Y$ be a map then[15]:

- $\varphi^{-1}(\bigcup_j \Lambda_j) = \bigcup_j \varphi^{-1}(\Lambda_j)$,
- $\varphi^{-1}(1_\sim) = 1_\sim, \varphi^{-1}(0_\sim) = 0_\sim$,
- $\varphi(\bigcup_i \Gamma_i) = \bigcup_i \varphi(\Gamma_i)$,
- $\varphi(\bigcup_i \Gamma_i) = \bigcup_i \varphi(\Gamma_i)$,
- $\varphi^{-1}(\bar{\Lambda}) = \overline{\varphi^{-1}(\Lambda)}$.

Theorem 2.12

Let $(X, \tau_{(\zeta_\Gamma, \upsilon_\Gamma)})$ and $(Y, \tau_{(\zeta_\Lambda, \upsilon_\Lambda)})$ be RIT spaces and $\varphi : X \longrightarrow Y$ be a RIC map. If $P = \langle \zeta_P, \upsilon_P \rangle$ is $\langle \zeta_\Gamma, \upsilon_\Gamma \rangle$-closed then $\varphi^{-1}(P) \cup \langle \zeta_\Gamma, \upsilon_\Gamma \rangle$ is $\langle \zeta_\Gamma, \upsilon_\Gamma \rangle$-closed.

Proof: $P = \langle \zeta_P, \upsilon_P \rangle$ is $\langle \zeta_\Lambda, \upsilon_\Lambda \rangle$ closed so $P^C = \langle \upsilon_P, \zeta_P \rangle \in \tau_{(\zeta_\Lambda, \upsilon_\Lambda)}$. Since φ is RIC map, therefore

$$\varphi^{-1}(P^C) = \langle x, \varphi^{-1}(\upsilon_P)(x), \varphi^{-1}(\zeta_P)(x) \rangle \cap \langle x, \langle \zeta_\Gamma(x), \upsilon_\Gamma(x) \rangle$$

$$= (\varphi^{-1}(P) \cup \langle \upsilon_\Gamma(x), \zeta_\Gamma(x) \rangle)^C \in \tau_{(\zeta_\Gamma, \upsilon_\Gamma)}$$

thus $\varphi^{-1}(P) \cup \langle \zeta_\Gamma, \upsilon_\Gamma \rangle$ *is* $\langle \zeta_\Gamma, \upsilon_\Gamma \rangle$ -closed.

Theorem 2.13

Suppose that RIT space $(X, \tau_{(\zeta_\Gamma, \upsilon_\Gamma)})$ is compact, $(Y, \tau_{(\zeta_\Lambda, \upsilon_\Lambda)})$ be a RIT space and $\varphi : X \to Y$ be an onto RIC map such that $\langle \zeta_\Lambda \circ \varphi, \upsilon_\Lambda \circ \varphi \rangle = \langle \upsilon_\Gamma, \zeta_\Gamma \rangle$ then $(Y, \tau_{(\zeta_\Lambda, \upsilon_\Lambda)})$ is a compact RIT space.

Proof Let $\{\langle \zeta^\alpha, \upsilon^\alpha \rangle : \alpha \in I\}$ of elements $\tau_{(\zeta_\Lambda, \upsilon_\Lambda)}$ be a RI open cover for $\langle \zeta_\Lambda, \upsilon_\Lambda \rangle$.

$$\bigcup_{\alpha\in I} \langle \zeta^\alpha \circ \varphi, \upsilon^\alpha \circ \varphi \rangle \cap \langle \zeta_\Gamma, \upsilon_\Gamma \rangle = \langle \zeta_\Lambda \circ \varphi, \upsilon_\Lambda \circ \varphi \rangle \cap \langle \zeta_\Gamma, \upsilon_\Gamma \rangle = \langle \zeta_\Gamma, \upsilon_\Gamma \rangle$$

therefore $\{\langle \zeta^\alpha \circ \varphi, \upsilon^\alpha \circ \varphi \rangle \cap \langle \zeta_\Gamma, \upsilon_\Gamma \rangle : \alpha \in I\}$ is a RIO cover for $\langle \zeta_\Lambda, \upsilon_\Lambda \rangle$. Since $(X, \tau_{(\zeta_\Gamma, \upsilon_\Gamma)})$ is compact so there is a finite sub collection $\{\langle \zeta^\beta \circ \varphi, \upsilon^\beta \circ \varphi \rangle \cap \langle \zeta_\Gamma, \upsilon_\Gamma \rangle : \beta \in Q, Q \text{ is finite}\}$ such that

$$\bigcup_{\beta\in Q} \langle \zeta^\beta \circ \varphi, \upsilon^\beta \circ \varphi \rangle \cap \langle \zeta_\Gamma, \upsilon_\Gamma \rangle = \langle \zeta_\Gamma, \upsilon_\Gamma \rangle.$$

Thus

$$\bigcup_{\beta\in Q} \langle \zeta^\beta \circ \varphi, \upsilon^\beta \circ \varphi \rangle \cap \langle \zeta_\Lambda \circ \varphi, \upsilon_\Lambda \circ \varphi \rangle = \langle \zeta_\Lambda \circ \varphi, \upsilon_\Lambda \circ \varphi \rangle.$$

This implies $\bigcup_{\beta\in Q} \langle \zeta^\beta \circ \varphi, \upsilon^\beta \circ \varphi \rangle = \langle \zeta_\Lambda \circ \varphi, \upsilon_\Lambda \circ \varphi \rangle$.

Since φ is onto, we conclude that $\bigcup_{\beta\in Q} \langle \zeta^\beta, \upsilon^\beta \rangle = \langle \zeta_\Lambda, \upsilon_\Lambda \rangle$.

Theorem 2.14

Let $(X, \tau_{(\zeta_\Gamma, \upsilon_\Gamma)})$ be RIT. $\varphi : X \to Y$ is a map and $\tau_{(\zeta_\Gamma \circ \varphi, \upsilon_\Gamma \circ \varphi)} = \{\langle \zeta_i \circ \varphi,, \upsilon_i \circ \varphi \rangle : \quad \langle \zeta_i, \upsilon_i \rangle \in \tau_{(\zeta_\Gamma, \upsilon_\Gamma)}\}$, then $(Y, \tau_{(\zeta_\Gamma \circ \varphi, \upsilon_\Gamma \circ \varphi)})$ is a RIT and φ is a RIC map.

Proof

- $\langle \zeta_i \circ \varphi,, \upsilon_i \circ \varphi \rangle \in \tau_{(\zeta_\Gamma \circ \varphi, \upsilon_\Gamma \circ \varphi)}$ and $\chi_\varnothing \circ \varphi = \chi_\varnothing \in \tau_{(\zeta_\Gamma \circ \varphi, \upsilon_\Gamma \circ \varphi)}$
- If $\langle \zeta_1 \circ \varphi, \upsilon_1 \circ \varphi \rangle$ and $\langle \zeta_2 \circ \varphi, \upsilon_2 \circ \varphi \rangle$ belong to $\tau_{(\zeta_\Gamma \circ \varphi, \upsilon_\Gamma \circ \varphi)}$, then for any $y \in Y$,

$$\langle y, \zeta_1 \circ \varphi(y), \upsilon_1 \circ \varphi(y) \rangle \cap \langle y, \zeta_2 \circ \varphi(y), \upsilon_2 \circ \varphi(y) \rangle = \langle y, \inf\{\zeta_1 \circ \varphi(y), \zeta_2 \circ \varphi(y)\}, \sup\{\upsilon_1 \circ \varphi(y), \upsilon_2 \circ \varphi(y)\} \rangle =$$

$$\langle y, \zeta_1 \circ \varphi(y), \upsilon_1 \circ \varphi(y) \rangle \cap \langle y, \zeta_2 \circ \varphi(y), \upsilon_2 \circ \varphi(y) \rangle = \langle y, \inf\{\zeta_1 \circ \varphi(y), \zeta_2 \circ \varphi(y)\}, \sup\{\upsilon_1 \circ \varphi(y), \upsilon_2 \circ \varphi(y)\} \rangle =$$

$\langle y, (\inf\{\zeta_1, \zeta_2\} \circ \varphi)(y), (\sup\{\upsilon_1, \upsilon_2\} \circ \varphi)(y) \rangle$ thus $\langle \zeta_1 \circ \varphi, \upsilon_1 \circ \varphi \rangle \cap \langle \zeta_2 \circ \varphi, \upsilon_2 \circ \varphi \rangle \in \tau_{(\zeta_\Gamma \circ \varphi, \upsilon_\Gamma \circ \varphi)}$

$\langle y, (\inf\{\zeta_1, \zeta_2\} \circ \varphi)(y), (\sup\{\upsilon_1, \upsilon_2\} \circ \varphi)(y) \rangle$ thus $\langle \zeta_1 \circ \varphi, \upsilon_1 \circ \varphi \rangle \cap \langle \zeta_2 \circ \varphi, \upsilon_2 \circ \varphi \rangle \in \tau_{(\zeta_\Gamma \circ \varphi, \upsilon_\Gamma \circ \varphi)}$.

3. Relative Intuitionistic Topological Entropy for Compact RIT Spaces

In this section, the meaning of RIT entropy is stated and some interesting theorems about RIT entropies obtained from refining a cover or joining two covers are proved.

Definition 3.1

Suppose that $(X, \tau_{(\zeta_\Gamma, \upsilon_\Gamma)})$ is a compact RIT space. RIO

cover σ is denominated a subcover of RIO cover

$\theta = \{\langle \zeta^{\alpha}, \upsilon^{\alpha} \rangle : \alpha \in I, \langle \zeta^{\alpha}, \upsilon^{\alpha} \rangle \in \tau_{(\zeta_\Gamma, \upsilon_\Gamma)}\}$ *if* $\sigma \subseteq \theta$.

Definition 3.2

Suppose that α is a RIO cover for $\langle \zeta_\Gamma, \upsilon_\Gamma \rangle$, $(X, \tau_{(\mu_A, \nu_A)})$ is a compact RITS and $\Delta(\alpha)$denotes the number of elements of set that is a subcover of α with the smallest cardinality. We define RIT entropy of the cover α by:

$$H(\alpha) = \log(\Delta(\alpha)) \cdot$$

Proposition 3.3

- $H(\alpha) \geq 0$,
- $H(\alpha) = 0$ iff $\Delta(\alpha) = 1$ iff $\langle \zeta_\Gamma, \upsilon_\Gamma \rangle \in \alpha$.

Definition 3.4

A RIO cover β is said to be a refinement of RIO cover α, denoted by $\alpha \prec \beta$, if every member of β is a subset of a member of β.

Theorem 3.5

If $\alpha \prec \beta$ then $H(\alpha) \leq H(\beta)$.

Proof Let $\{B_i = \langle \zeta^i, \upsilon^i \rangle : i = 1, ..., m(\beta)\}$ be a subcover of RIO cover β with the smallest cardinality. For every $i \in \{1, ..., m(\beta)\}$ there exists $A_i \in \alpha$ such that $B_i \subseteq A_i$ so $\{A_1, A_2, ..., A_{m(\beta)}\}$ covers $\langle \zeta_\Gamma, \upsilon_\Gamma \rangle$ and is a subcover for α. Thus $\Delta(\alpha) \leq \Delta(\beta)$.

Theorem 3.6

Let $\varphi : (X, \tau_{(\zeta_\Gamma, \upsilon_\Gamma)}) \to (X, \tau_{(\zeta_\Gamma, \upsilon_\Gamma)})$ be a RIC map,

$\theta = \{\langle \zeta^{\alpha}, \upsilon^{\alpha} \rangle : \alpha \in I, \ \langle \zeta^{\alpha}, \upsilon^{\alpha} \rangle \in \tau_{(\zeta_\Gamma, \upsilon_\Gamma)}\}$ be a RIO cover for $\langle \zeta_\Gamma, \upsilon_\Gamma \rangle$ then

$$\varphi^{-1}(\theta) = \{\langle x, \varphi^{-1}(\zeta^{\alpha})(x), \varphi^{-1}(\upsilon^{\alpha})(x) \rangle \cap \langle x, \zeta_\Gamma(x), \upsilon_\Gamma(x) \rangle : \langle \zeta^{\alpha}, \upsilon^{\alpha} \rangle \in \theta\}$$

$$\varphi^{-1}(\theta) = \{\langle x, \varphi^{-1}(\zeta^{\alpha})(x), \varphi^{-1}(\upsilon^{\alpha})(x) \rangle \cap \langle x, \zeta_\Gamma(x), \upsilon_\Gamma(x) \rangle : \langle \zeta^{\alpha}, \upsilon^{\alpha} \rangle \in \theta\}$$

is a RIO cover for $\langle \zeta_\Gamma, \upsilon_\Gamma \rangle \cap \varphi^{-1}(\langle \zeta_\Gamma, \upsilon_\Gamma \rangle)$.

Proof Consider,

$x \in X$, $\langle x, \varphi^{-1}(\zeta_\Gamma)(x), \varphi^{-1}(\upsilon_\Gamma)(x) \rangle \cap \langle x, \zeta_\Gamma(x), \upsilon_\Gamma(x) \rangle =$

$\langle x, \inf\{\zeta_\Gamma(x), \zeta_\Gamma(\varphi(x))\}, \sup\{\upsilon_\Gamma(x), \upsilon_\Gamma(\varphi(x))\} \rangle$.

θ is a RIO cover for $\langle \zeta_\Gamma, \upsilon_\Gamma \rangle$ so

$\zeta_\Gamma(x) = \sup_{\alpha \in I}\{\zeta^{\alpha}(x)\}$ and $\upsilon_\Gamma(x) = \inf_{\alpha \in I}\{\upsilon^{\alpha}(x)\}$.

We have $\zeta_\Gamma(\varphi(x)) \leq \sup_{\alpha \in I}\{\zeta^{\alpha}(\varphi(x))\}$ and

$\upsilon_\Gamma(\varphi(x)) \geq \inf_{\alpha \in I}\{\upsilon^{\alpha}(\varphi(x))\}$.

Thus

$\inf\{\zeta_\Gamma(x), \zeta_\Gamma(\varphi(x))\} \leq \inf\{\zeta_\Gamma(x), \sup_{\alpha \in I}\{\zeta^{\alpha}(\varphi(x))\}\} \leq$

$\sup_{\alpha \in I}\{\inf\{\zeta_\Gamma(x), \varphi^{-1}(\zeta^{\alpha})(x)\}\}$

and

$\sup\{\upsilon_\Gamma(x), \upsilon_\Gamma(\varphi(x))\} \geq \sup\{\upsilon_\Gamma(x), \inf_{\alpha \in I}\{\upsilon^{\alpha}(\varphi(x))\}\} \geq$

$\inf_{\alpha \in I}\{\sup\{\upsilon_\Gamma(x), \varphi^{-1}(\upsilon^{\alpha})(x)\}\}$

therefore

$$\bigcup_{\alpha \in I}(\langle x, \zeta_\Gamma(x), \upsilon_\Gamma(x) \rangle \cap \langle x, \varphi^{-1}(\zeta^{\alpha})(x), \varphi^{-1}(\upsilon^{\alpha})(x) \rangle)$$

$$= \langle x, \sup_{\alpha \in I}\{\inf\{\zeta_\Gamma(x), \varphi^{-1}(\zeta^{\alpha})(x)\}\},$$

$$\inf_{\alpha \in I}\{\sup\{\upsilon_\Gamma(x), \varphi^{-1}(\upsilon^{\alpha})(x)\}\} = \langle \zeta_\Gamma, \upsilon_\Gamma \rangle \cap \varphi^{-1}(\langle \zeta_\Gamma, \upsilon_\Gamma \rangle)(x).$$

So $\varphi^{-1}(\theta)$ is an open cover for $\langle \zeta_\Gamma, \upsilon_\Gamma \rangle \cap \varphi^{-1}(\langle \zeta_\Gamma, \upsilon_\Gamma \rangle)$.

Theorem 3.7

If $\varphi : (X, \tau_{(\zeta_\Gamma, \upsilon_\Gamma)}) \to (X, \tau_{(\zeta_\Gamma, \upsilon_\Gamma)})$ is a RIC map and θ is a RIO cover for $\langle \zeta_\Gamma, \upsilon_\Gamma \rangle$ then $H(\varphi^{-1}(\theta)) \leq H(\theta)$.

Proof If $\theta' = \{\langle \zeta^i, \upsilon^i \rangle : i = 1, ..., m(\theta)\}$ is a subcover of θ with the smallest cardinality, then

$$\varphi^{-1}(\theta') = \{\langle \zeta_\Gamma, \upsilon_\Gamma \rangle \cap \varphi^{-1}(\langle \zeta^i, \upsilon^i \rangle) : i = 1, ..., m(\theta)\}$$

is a RIO cover for $\langle \zeta_\Gamma, \upsilon_\Gamma \rangle \cap \varphi^{-1}(\langle \zeta_\Gamma, \upsilon_\Gamma \rangle)$. Hence

$$\Delta(\varphi^{-1}(\theta)) \leq \Delta(\varphi^{-1}(\theta')) \leq \Delta(\theta).$$

Corollary 3.8

If $\varphi : (X, \tau_{(\zeta_\Gamma, \upsilon_\Gamma)}) \to (X, \tau_{(\zeta_\Gamma, \upsilon_\Gamma)})$ is a homeomorphism and θ is a RIO cover for $\langle \zeta_\Gamma, \upsilon_\Gamma \rangle$ then

$H(\theta) \quad H(\varphi \quad (\theta))$.

Proof $H(\varphi^{-1}(\theta)) \leq H(\theta) = H(\varphi(\varphi^{-1}(\theta))) \leq H(\varphi^{-1}(\theta))$

Definition 3.9

Consider $\theta = \{\langle \zeta^{\alpha}, \upsilon^{\alpha} \rangle : \alpha \in I, \langle \zeta^{\alpha}, \upsilon^{\alpha} \rangle \in \tau_{(\zeta_\Gamma, \upsilon_\Gamma)}\}$ and

$\sigma = \{\langle \zeta^{\beta}, \upsilon^{\beta} \rangle : \beta \in J,$

$\langle \zeta^{\beta}, \upsilon^{\beta} \rangle \in \tau_{(\zeta_\Gamma, \upsilon_\Gamma)}\}$

which are two RIO covers. The join refinement of θ and σis defined by:

$$\theta \vee \sigma = \left\{\langle \zeta^{\alpha}, \upsilon^{\alpha} \rangle \cap \langle \zeta^{\beta}, \upsilon^{\beta} \rangle : \langle \zeta^{\alpha}, \upsilon^{\alpha} \rangle \in \theta, \langle \zeta^{\beta}, \upsilon^{\beta} \rangle \in \sigma\right\}$$

Theorem 3.10

Consider two RIO covers $\theta = \{\langle \zeta^{\alpha}, \upsilon^{\alpha} \rangle : \alpha \in I, \langle \zeta^{\alpha}, \upsilon^{\alpha} \rangle \in \tau_{(\zeta_\Gamma, \upsilon_\Gamma)}\}$ and

$\sigma = \{\langle \zeta^{\beta}, \upsilon^{\beta} \rangle : \beta \in J, \langle \zeta^{\beta}, \upsilon^{\beta} \rangle \in \tau_{(\zeta_\Gamma, \upsilon_\Gamma)}\}$ then θ ∨ σ is a RIO cover for $\langle \zeta_\Gamma, \upsilon_\Gamma \rangle$

Proof

$$\bigcup_{\alpha \in I\, \beta \in J} (\langle \zeta^{\alpha}, \upsilon^{\alpha} \rangle \cap \langle \zeta^{\beta}, \upsilon^{\beta} \rangle) = (\bigcup_{\alpha \in I} \langle \zeta^{\alpha}, \upsilon^{\alpha} \rangle) \cap (\bigcup_{\beta \in J} \langle \zeta^{\beta}, \upsilon^{\beta} \rangle) = \langle \zeta_\Gamma, \upsilon_\Gamma \rangle.$$

Theorem 3.11

Let θ and σ be two RIO covers for $\langle \zeta_\Gamma, \upsilon_\Gamma \rangle$ then $H(\theta \vee \sigma) \le H(\theta) + H(\sigma)$.

Proof Let $\theta' \subseteq \theta, \sigma' \subseteq \sigma$, be subcovers of θ and σ and with the smallest cardinality *m* and *n*. Thus $H(\theta) = \log m \ and\ H(\sigma) = \log n$ $\theta' \vee \sigma'$ is a subcover of $\theta \vee \sigma$ so $H(\theta \vee \sigma) \le \log mn = \log m + lon = H(\theta) + H(\sigma)$.

4. Relative Intuitionistic Topological Entropy on Relative Intuitionistic Dynamical System

We begin this section with a definition of Relative Intuitionistic Topological entropy on relative intuitionistic dynamical system.

Definition 4.1

Consider RIT space $(X, \tau_{(\zeta_\Gamma, \upsilon_\Gamma)})$ and RIC map $\varphi : (X, \tau_{(\zeta_\Gamma, \upsilon_\Gamma)}) \to (X, \tau_{(\zeta_\Gamma, \upsilon_\Gamma)})$. $((X, \tau_{(\zeta_\Gamma, \upsilon_\Gamma)}), \varphi)$ is denominated a RI dynamical system.

Theorem 4.2

Let $\{(a_i)\}_{i=1}^{\infty}$ be a sequence of nonnegative member such that $a_{r+s} \le a_r + a_s$ for each *r,s* = 1,2,..., then $\lim_{n \to \infty} \frac{1}{n} a_n$ exists[16].

Theorem 4.3

Suppose that $\theta = \{\langle \zeta^{\alpha}, \upsilon^{\alpha} \rangle : \alpha \in I, \langle \zeta^{\alpha}, \upsilon^{\alpha} \rangle \in \tau_{(\zeta_\Gamma, \upsilon_\Gamma)}\}$ and $\sigma = \{\langle \zeta^{\beta}, \upsilon^{\beta} \rangle : \beta \in J, \langle \zeta^{\beta}, \upsilon^{\beta} \rangle \in \tau_{(\zeta_\Gamma, \upsilon_\Gamma)}\}$ are two RIO covers for $\langle \zeta_\Gamma\ \upsilon_\Gamma \rangle$ and $\varphi : (X, \tau_{(\zeta_\Gamma, \upsilon_\Gamma)}) \to (X, \tau_{(\zeta_\Gamma, \upsilon_\Gamma)})$ is a RIC map, then

$$H(\varphi^{-1}(\theta \vee \sigma)) = H(\varphi^{-1}(\theta) \vee \varphi^{-1}(\sigma))$$

Proof Let $\zeta^{\alpha,\beta} = \inf\{\zeta^{\alpha}, \zeta^{\beta}\}$ and $\upsilon^{\alpha,\beta} = \sup\{\upsilon^{\alpha}, \upsilon^{\beta}\}$

$$\{\langle x, \inf\{\zeta^{\alpha}, \zeta^{\beta}\}(\varphi(x)), \sup\{\upsilon^{\alpha}, \upsilon^{\beta}\}(\varphi(x))\rangle \cap \langle x, \zeta_\Gamma(x), \upsilon_\Gamma(x)\rangle\} =$$

$$\{\langle x, \inf\{\zeta^{\alpha}, \zeta^{\beta}\}(\varphi(x)), \sup\{\upsilon^{\alpha}, \upsilon^{\beta}\}(\varphi(x))\rangle \cap \langle x, \zeta_\Gamma(x), \upsilon_\Gamma(x)\rangle\} =$$

$$\{\langle x, \inf\{\zeta^{\alpha} \circ \varphi, \zeta^{\beta} \circ \varphi, \zeta_\Gamma\}(x), \sup\{\upsilon^{\alpha} \circ \varphi, \upsilon^{\beta} \circ \varphi, \upsilon_\Gamma\}(x)\rangle\} =$$

$$\{\langle x, \inf\{\zeta^{\alpha} \circ \varphi, \zeta_\Gamma\}(x), \sup\{\upsilon^{\alpha} \circ \varphi, \upsilon_\Gamma\}(x)\rangle\} \cap \{\langle x, \inf\{\zeta^{\beta} \circ \varphi, \zeta_\Gamma\}(x), \sup\{\upsilon^{\beta} \circ \varphi, \upsilon_\Gamma\}(x)\rangle\}$$

$$\{\langle x, \inf\{\zeta^{\alpha} \circ \varphi, \zeta_\Gamma\}(x), \sup\{\upsilon^{\alpha} \circ \varphi, \upsilon_\Gamma\}(x)\rangle\} \cap \{\langle x, \inf\{\zeta^{\beta} \circ \varphi, \zeta_\Gamma\}(x), \sup\{\upsilon^{\beta} \circ \varphi, \upsilon_\Gamma\}(x)\rangle\}$$

Theorem 4.4

Let $((X, \tau_{(\zeta_\Gamma, \upsilon_\Gamma)}), \varphi)$ be a RI dynamical system and θ be a RIO cover for $\langle \zeta_\Gamma, \upsilon_\Gamma \rangle$ then

$$h(\varphi, \theta) := \lim_{n \to \infty} \frac{1}{n} H(\bigvee_{i=0}^{n-1} \varphi^{-i}\theta)$$

exists.

Proof Let $x_n = H(\bigvee_{i=0}^{n-1} \varphi^{-i}\theta)$. For every $m, n \in \mathrm{N}$, we have:

$$x_{n+m} = H(\bigvee_{i=0}^{m+n-1} \varphi^{-i}\theta) \le H(\bigvee_{i=0}^{m-1} \varphi^{-i}\theta) + H(\varphi^{-m}(\bigvee_{i=0}^{n-1} \varphi^{-i}\theta))$$

$$\le H(\bigvee_{i=0}^{m-1} \varphi^{i}\theta) + H(\bigvee_{i=0}^{n-1} \varphi^{-i}\theta) = x_m + x_n$$

Thus $\{x_n\}_{n \in \mathrm{N}}$ is a sub additive sequence so $\lim_{n \to \infty} \frac{1}{n} x_n$ exists.

Definition 4.5

$h(\varphi) = \sup\{h(\varphi, \theta) : \theta$ is a RIO cover for $\langle \zeta_\Gamma, \upsilon_\Gamma \rangle\}$ is called RIT entropy of φ.

Definition 4.6

Two RIC maps $\varphi:(X,\tau_{(\zeta_\Gamma,\upsilon_\Gamma)})\to(X,\tau_{(\zeta_\Gamma,\upsilon_\Gamma)})$ and $\phi:(X,\tau_{(\zeta_\Gamma,\upsilon_\Gamma)})\to(X,\tau_{(\zeta_\Gamma,\upsilon_\Gamma)})$ are said $\langle\zeta_\Gamma,\upsilon_\Gamma\rangle$-conjugate if there exists a RI homeomorphism $\psi:(X,\tau_{(\zeta_\Gamma,\upsilon_\Gamma)})\to(X,\tau_{(\zeta_\Gamma,\upsilon_\Gamma)})$ such that $\psi\circ\varphi=\phi\circ\psi$.

Theorem 4.7

If $\varphi:(X,\tau_{(\zeta_\Gamma,\upsilon_\Gamma)})\to(X,\tau_{(\zeta_\Gamma,\upsilon_\Gamma)})$ and $\phi:(X,\tau_{(\zeta_\Gamma,\upsilon_\Gamma)})\to(X,\tau_{(\zeta_\Gamma,\upsilon_\Gamma)})$ are $\langle\zeta_\Gamma,\upsilon_\Gamma\rangle$-conjugate, then $h(\varphi)=h(\phi)$.

Proof Let α be a RIO cover for $\langle\zeta_\Gamma,\upsilon_\Gamma\rangle$.

$$h(\phi,\alpha)=\limsup_{n\to\infty}\frac{1}{n}H(\overset{n-1}{\underset{i=0}{\vee}}\phi^{-i}\alpha)=$$

$$=\limsup_{n\to\infty}\frac{1}{n}H(\overset{n-1}{\underset{i=0}{\vee}}\varphi^{-i}(\psi^{-1}\alpha))=h(\varphi,\psi^{-1}(\alpha))$$

$$=\limsup_{n\to\infty}\frac{1}{n}H(\overset{n-1}{\underset{i=0}{\vee}}\varphi^{-i}(\psi^{-1}\alpha))=h(\varphi,\psi^{-1}(\alpha))$$

Since ψ is a homeomorphism thus $h(\varphi)=h(\phi)$.

Example 4.8

Let $X=\mathbb{R},\zeta_\Gamma:\mathbb{R}\to[0,1],\ \upsilon_\Gamma:\mathbb{R}\to[0,1]\ \textit{and}\ \phi:\mathbb{R}\to\mathbb{R}$, and such that:

$$\zeta_\Gamma(x)=\begin{cases}\frac{1}{2} & x\in\mathbb{Q}\\ 1 & x\in\mathbb{Q}^C\end{cases},\qquad \upsilon_\Gamma(x)=\begin{cases}\frac{1}{3} & x\in\mathbb{Q}\\ 0 & x\in\mathbb{Q}^C\end{cases},$$

$$\phi(x)=\begin{cases}x & x\in\mathbb{Q}^C\\ 0 & x\in\mathbb{Q}\end{cases}.$$

Consider $\tau_{(\zeta_\Gamma,\upsilon_\Gamma)}=\{\langle\zeta_\Gamma,\upsilon_\Gamma\rangle,\chi_\varnothing\}\cup\{\langle\zeta_i,\upsilon_i\rangle\subset\langle\zeta_\Gamma,\upsilon_\Gamma\rangle:\zeta_i(x)=0,\ \upsilon_i(x)=1\ \textit{if}\ x\in\mathbb{Q}\}$. We prove that $\tau_{(\zeta_\Gamma,\upsilon_\Gamma)}$ is a RIT space and ϕ is a RIC map.

- If $\langle\zeta_1,\upsilon_1\rangle$ and $\langle\zeta_2,\upsilon_2\rangle$ belong to $\tau_{(\zeta_\Gamma,\upsilon_\Gamma)}$ then $\langle\zeta_1,\upsilon_1\rangle\cap\langle\zeta_2,\upsilon_2\rangle=\langle\inf\{\zeta_1,\zeta_2\},\sup\{\upsilon_1,\upsilon_2\}\rangle\subseteq\langle\zeta_\Gamma,\upsilon_\Gamma\rangle$ and *if* $x\in\mathbb{Q}$ then $\inf\{\zeta_1\wedge\zeta_2\}(x)=0$ and $\sup\{\upsilon_1,\upsilon_2\}(x)=1$ thus $\langle\zeta_1,\upsilon_1\rangle\cap\langle\zeta_2,\upsilon_2\rangle\in\tau_{(\zeta_\Gamma,\upsilon_\Gamma)}$.
- If $\{\langle\zeta_i,\upsilon_i\rangle:i\in I\}\subseteq\tau_{(\zeta_\Gamma,\upsilon_\Gamma)}$ then $\underset{i\in I}{\cup}\langle\zeta_i,\upsilon_i\rangle=\langle\sup_{i\in I}\zeta_i,\inf_{i\in I}\upsilon_i\rangle\subseteq\langle\zeta_\Gamma,\upsilon_\Gamma\rangle$ and If $x\in\mathbb{Q}$ then $(\sup_{i\in I}\zeta_i)(x)=0\ \textit{and}\ (\inf_{i\in I}\upsilon_i)(x)=1$. Therefore, $\underset{i\in I}{\cup}\langle\zeta_i,\upsilon_i\rangle\in\tau_{(\zeta_\Gamma,\upsilon_\Gamma)}$.
- Let $\langle\zeta_i,\upsilon_i\rangle\in\tau_{(\zeta_\Gamma,\upsilon_\Gamma)}$. For any,

$$x\in\mathbb{R},(\phi^{-1}(\langle\zeta_i,\upsilon_i\rangle)\cap\langle\zeta_\Gamma,\upsilon_\Gamma\rangle)(x)=\langle\inf\{\zeta_i(\phi(x)),\zeta_\Gamma(x)\},$$

$$\sup\{\upsilon_i(\phi(x)),\upsilon_\Gamma(x)\}\rangle=\begin{cases}\langle\zeta_i(x),\upsilon_i(x)\rangle & x\in\mathbb{Q}^C\\ \langle\zeta_i(0),\upsilon_i(0)\rangle & x\in\mathbb{Q}\end{cases}$$

$$=\langle\zeta_i(x),\upsilon_i(x)\rangle$$

thus ϕ is a RIC map and $(\mathbb{R},\tau_{(\zeta_\Gamma,\upsilon_\Gamma)},\phi)$ is a RI dynamical system.

Now let we compute the entropy of RIO covers. Let $\theta=\{\langle\zeta^\alpha,\upsilon^\alpha\rangle:\alpha\in I\}$ be a RIO cover for $\langle\zeta_\Gamma,\upsilon_\Gamma\rangle$. If $\langle\zeta_\Gamma,\upsilon_\Gamma\rangle$ does not belong to θ then for $x\in\mathbb{Q}$ we have $(\underset{\alpha\in I}{\cup}\langle\zeta^\alpha,\upsilon^\alpha\rangle)(x)=$

$$\langle\sup_{\alpha\in I}\zeta^\alpha(x),\inf_{\alpha\in I}\upsilon^\alpha(x)\rangle=\langle0,1\rangle\neq\langle\zeta_\Gamma(x),\upsilon_\Gamma(x)\rangle=\langle\frac{1}{2},\frac{1}{3}\rangle,$$

and a contradiction thus $\langle\zeta_\Gamma,\upsilon_\Gamma\rangle\in\theta\ \textit{and}\ H(\theta)=0.$

$$H(\theta\vee\phi^{-1}(\theta)\vee...\vee\phi^{-n+1}(\theta))\leq nH(\theta)=0\ \textit{thus}\ h(\phi,\theta)=0$$

and since θ is arbitrary, therefore $h(\phi)=0$.

5. Concluding Remarks

In this study some properties of RIT entropy, RIT entropy on RI dynamical systems were investigated. Computing RIT entropies could bean interesting subject. The generators might help us in computing entropies, so trying to define generators for these entropies could be an interesting open problem.

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