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A Novel Generation of OFDM 4096-QAM Sequences with Reduced Peak to Average Power Ratio

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Abstract

Objective: To do analytical investigations on PAPR reduction using Golay sequences and to develop mathematical expressions for finding PAPR. **Methods/Statistical Analysis:** The Coding method offers low PAPR as compared to the other prevalent techniques in present day wireless systems. Golay sequences used to explore for reduction in the PAPR without using extra hardware required for implementation of PAPR reduction techniques. **Findings:** This paper introduces a novel sequence of 4096-QAM having six QPSK sequences with the lowest PAPR that can be used for OFDM systems. The proposed analytical investigation results into a PAPR of 7.645 dB having better bandwidth efficiency. **Application/Improvements:** The 4096-Quadrature Amplitude Modulation (QAM) sequences provides efficient spectrum with high speed of OFDM signal with Low Bit Error Rate (BER) and reduced PAPR.

Keywords: QAM, PAPR, Golay sequences, OFDM, QPSK

1. Introduction

The Orthogonal Frequency Division Multiplexing (OFDM) technique has come to the forefront of technology in the area of Wireless communication system. It was first introduced by Robert W. Chang in 1966. In his work, he presented a scheme of simultaneously transmitting signals over a band-limited channel without ISI and ICI[1]. In this technique various orthogonal sub-channels are assigned to the users. In today's technology oriented world we need high data rates for the communication that can be achieved by using OFDM System. The OFDM is a useful technique for high data rates but the use is limited because of its high Peak to Average Power Ratio. To reduce this effect of PAPR few techniques are implemented to reduce the PAPR[2]. This paper is further organized into following sections. Section 2 briefly analyzes the mathematical equations of OFDM systems. In section 3, the Golay Complementary sequences are explained, section 4 provides the generation of 4096-QAM OFDM codes and section 5 concludes the paper.

2. Mathematical Analysis of OFDM

The Peak to Average Power Ratio is one of the important practical problems of the uncoded OFDM signals. A complex Signal $S_b(t)$ in OFDM transmitter[3] is represented as:

$$S_b(t)\sum_{k=0}^{n-1} b_k e^{i2\pi(f_r+kf_s)t}\text{, for}\quad 0\le t\le\frac{1}{f_s} \tag{1}$$

Where f_r is the carrier frequency and f_s is the bandwidth.

Its peak envelope power is[3] written as:

$$PEP=\max_{0\le t\le\frac{1}{f_s}}\left|S_b(t)\right|^2 \tag{2}$$

and the mean envelope power is calculated as:

$$P_{av}=\frac{1}{|S|}\sum_{b\in S}\|b\|^2 \tag{3}$$

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So the PAPR for the given signal $S_b(t)$[3] can be defined as:

$$PAPR = \frac{PEP}{P_{av}} \tag{4}$$

3. Golay Complementary Sequences

To design a new OFDM sequence, the brief introduction of Golay Complementary Sequences is provided in this section. The complementary golay codes have very good PAPR reduction properties and error correcting capabilities. These codes can be combined with another block codes for an efficient method of PAPR reduction. We know that if a sequence is a member of the Golay complementary pair then it is called a Golay Complementary Sequence (GCS).

Thus

$$A_a(u) + A_b(u) = \begin{cases} 0, \text{ at } u \neq 0 \\ 2n, \text{ at } u = 0 \end{cases} \tag{5}$$

The construction for an arbitrary length Golay Sequences proposed by Davis and Jedwabin[4].

4. Proposed Novek OFDM 4096-QAM Sequence

To design the given sequence with Low PAPR, let OFDM 4096-QAM sequence is defined as the sum of six QPSK symbols:

$$A_{4096-QAM} = \frac{1}{\sqrt{2}} S_{QPSK} + \sqrt{2} S_{QPSK} + 2\sqrt{2} S_{QPSK} + 4$$

$$\sqrt{2} S_{QPSK} + 8\sqrt{2} S_{QPSK} + 16\sqrt{2} S_{QPSK} \tag{6}$$

Let Six QPSK $A_x(t)$, $A_y(t)$, $A_z(t)$, $A_u(t)$ $A_w(t)$, and $A_v(t)$ OFDM sequences. Suppose $d = (d_0, d_1, \ldots, d_{n-1})$ where $d_i \in S_{4096-QAM}$ that is associated with six QPSK sequences[5,6].

$$x = (x_0, x_1, \ldots, x_{n-1}) \in Z_4^n$$

$$y = (y_0, y_1, \ldots, y_{n-1}) \in Z_4^n$$

$$z = (z_0, z_1, \ldots, z_{n-1}) \in Z_4^n$$

$$u = (u_0, u_1, \ldots, u_{n-1}) \in Z_4^n$$

$$w = (w_0, w_1, \ldots, w_{n-1}) \in Z_4^n$$

$$v = (v_0, v_1, \ldots, v_{n-1}) \in Z_4^n$$

$$d_k = (\frac{1}{\sqrt{2}} j^{x_k} + \sqrt{2} j^{y_k} + 2\sqrt{2} j^{z_k} + 4\sqrt{2} j^{u_k}$$

$$d_k = (\frac{1}{\sqrt{2}} j^{x_k} + \sqrt{2} j^{y_k} + 2\sqrt{2} j^{z_k} + 4\sqrt{2} j^{u_k} + 8\sqrt{2} j^{w_k} + 16\sqrt{2} j^{v_k}) e^{j\pi/4} \tag{7}$$

$$A_x(t) = \sum_{k=0}^{n-1} (j^{x_k} e^{j\pi/4} . e^{\frac{j2\pi kt}{T}}) \tag{8}$$

$$A_y(t) = \sum_{k=0}^{n-1} j^{y_k} e^{j\pi/4} . e^{\frac{j2\pi kt}{T}} \tag{9}$$

$$A_z(t) = \sum_{k=0}^{n-1} j^{z_k} e^{j\pi/4} . e^{\frac{j2\pi kt}{T}} \tag{10}$$

$$A_u(t) = \sum_{k=0}^{n-1} j^{u_k} e^{j\pi/4} . e^{\frac{j2\pi kt}{T}} \tag{11}$$

$$A_w(t) = \sum_{k=0}^{n-1} j^{w_k} e^{j\pi/4} . e^{\frac{j2\pi kt}{T}} \tag{12}$$

$$A_v(t) = \sum_{k=0}^{n-1} j^{v_k} e^{j\pi/4} . e^{\frac{j2\pi kt}{T}} \tag{13}$$

A 4096-QAM OFDM signal may be written as the weighted sum of six QPSK OFDM signals by[5,6]

$$A_d(t) = \sum_{k=0}^{n-1} d_k . e^{\frac{j2\pi kt}{T}} \tag{14}$$

$$A_d(t) = \sum_{k=0}^{n-1} (\frac{1}{\sqrt{2}} j^{x_k} + \sqrt{2} j^{y_k} + 2\sqrt{2} j^{z_k} + 4$$

$$\sqrt{2} j^{u_k} + 8\sqrt{2} j^{w_k} + 16\sqrt{2} j^{v_k}) e^{j\pi/4} . e^{\frac{j2\pi kt}{T}}$$

$$A_d(t) = \sum_{k=0}^{n-1} (\frac{1}{\sqrt{2}} j^{x_k} e^{j\pi/4} . e^{\frac{j2\pi kt}{T}} + \sum_{k=0}^{n-1} \sqrt{2} j^{y_k} e^{j\pi/4} . e^{\frac{j2\pi kt}{T}} + \sum_{k=0}^{n-1} 2\sqrt{2} j^{z_k} e^{j\pi/4}$$

$$.e^{\frac{j2\pi kt}{T}}+\sum_{k=0}^{n-1}4\sqrt{2}j^{u_k}e^{j\pi/4}.e^{\frac{j2\pi kt}{T}}$$

$$+8\sqrt{2}j^{w_k}e^{j\pi/4}.e^{\frac{j2\pi kt}{T}}+16\sqrt{2}j^{v_k}e^{j\pi/4}.e^{\frac{j2\pi kt}{T}})$$

$$A_d(t)=\frac{1}{\sqrt{2}}\sum_{k=0}^{n-1}(j^{x_k}e^{j\pi/4}.e^{\frac{j2\pi kt}{T}})+\sqrt{2}\sum_{k=0}^{n-1}(j^{y_k}e^{j\pi/4}.e^{\frac{j2\pi kt}{T}})+$$

$$2\sqrt{2}\sum_{k=0}^{n-1}(j^{z_k}e^{j\pi/4}.e^{\frac{j2\pi kt}{T}})+4\sqrt{2}\sum_{k=0}^{n-1}(j^{u_k}e^{j\pi/4}.e^{\frac{j2\pi kt}{T}})$$

$$+8\sqrt{2}\sum_{k=0}^{n-1}(j^{w_k}e^{j\pi/4}.e^{\frac{j2\pi kt}{T}})+16\sqrt{2}\sum_{k=0}^{n-1}(j^{v_k}e^{j\pi/4}.e^{\frac{j2\pi kt}{T}})$$

$$\therefore A_d(t)=\frac{1}{\sqrt{2}}A_x(t)+\sqrt{2}A_y(t)+2\sqrt{2}A_z$$

$$(t)+4\sqrt{2}A_u(t)+8\sqrt{2}A_w(t)+16\sqrt{2}A_v(t) \tag{15}$$

Instantaneous envelope power is defined by:

$$P_d(t)=\left|A_d(t)\right|^2 \tag{16}$$

$$=\left|\frac{1}{\sqrt{2}}A_x(t)+\sqrt{2}A_y(t)+2\sqrt{2}A_z(t)+4\sqrt{2}R_u(t)+8\sqrt{2}R_w(t)+16\sqrt{2}R_v(t)\right|^2$$

$$\le\left|\frac{1}{\sqrt{2}}.\sqrt{2n}+\sqrt{2}.\sqrt{2n}+2\sqrt{2}.\sqrt{2n}+4\sqrt{2}.\sqrt{2n}+8\sqrt{2}.\sqrt{2n}+16\sqrt{2}.\sqrt{2n}\right|^2$$

$$\le\left|\sqrt{n}+2\sqrt{n}+4\sqrt{n}+8\sqrt{n}+16\sqrt{n}+32\sqrt{n}\right|^2$$

$$\le\left|63\sqrt{n}\right|^2$$

$$\therefore P_d(t)=3969n \tag{17}$$

Now, average energy can be calculated as[5,6]

$$\mathrm{d}_k=\sum_{k=0}^{n-1}\left|d_k\right|^2 \tag{18}$$

$$=\left(\sqrt{\left(\frac{1}{\sqrt{2}}\right)^2+\left(\sqrt{2}\right)^2+\left(2\sqrt{2}\right)^2+\left(4\sqrt{2}\right)^2+\left(8\sqrt{2}\right)^2+\left(16\sqrt{2}\right)^2}\right)$$

$\therefore$ Average power $P_{av}=682.5n$

Therefore using the formula for PAPR given by equation no. 4 it can be calculated as:

$$\mathrm{PAPR}=\frac{3969n}{682.5n}$$

$$\mathrm{PAPR}(\mathrm{dB})=10\log_{10}5.815$$

$$\mathrm{PAPR}(\mathrm{dB})=7.645\mathrm{dB}$$

5. Conclusion

In this paper, the analytic investigations on Golay-coded OFDM 4096-QAM sequences with Six QPSK symbols were carried out. For this purpose Golay Complementary Sequences have been used as the building blocks for the construction of 4096-QAM sequences. The PAPR of our proposed sequence is 7.645 dB with better bandwidth efficiency as it will ensure the higher capacity in wireless systems. In the future scope, the code rate and Euclidean distance can be computed and the graphs would be compared based upon the 4096-Quadrature Amplitude Modulation (QAM) codes which further provides efficient spectrum with high speed of OFDM signal having Low Bit Error Rate (BER) and low PAPR.

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