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Wavelet-based Damage Detection Technique via Operational Deflection Shape Decomposition

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Abstract

Background/Objective: Early detection of damage in structures can prevent loss of lives and extend their service life. This has drawn considerable attention of researchers to applying Structural Health Monitoring (SHM) to detect structural damage. Wavelet Transform (WT) is a tool utilized to identify damage in structures by using structural responses. In this study, a WT damage detection method using Operational Deflection Shape (ODS) difference is presented. **Method:** A two-dimensional Continuous Wavelet Transform (CWT) is utilized to decompose the difference between the ODS of the intact and damaged structure to detect and locate damage. To demonstrate the ability of this technique, a numerical model of a steel plate is applied. **Findings:** The results show that the location of damage influences damage detectability when the decomposed signal is noisy. **Improvements:** This method eliminates the problem of border distortion experienced when conventional ODS is applied. The methodology shows the proposed techniques' simplicity while the results show accuracy.

Keywords: Cantilever Plate, Continuous Wavelet Transform, Damage Detection, Damage Severity Tree, Deflection Shape

1. Introduction

Civil infrastructures remain the most expensive assets of any society. These infrastructures are exposed to extreme weather and loading conditions causing degradation and deterioration at an alarming rate which may result to structural failure, thus costly repairs and/or loss of lives[1]. To prevent this catastrophic failure, early damage detection in structures like aircraft, high-rise buildings, bridges and offshore platforms is extremely important. In order to accomplish this task, several Structural Health Monitoring (SHM) techniques have been developed to provide solutions to these problems. Among these SHM techniques, Vibration-Based Damage Detection (VBDD) methods which apply the dynamic responses of structures have been the most attractive; due to its ease, cost effectiveness, time and applicability. The dynamic characteristics measured for application are natural frequency[2], frequency response function[3], mode shape[4], mode shape curvature[5,6], modal flexibility[7] or their combination[8].

The application of fundamental mode shape in damage detection can be attributed to its sensitivity and estimation convenience, although higher modes are more sensitive to small damage, problems and difficulties in estimating them in practice decreases their applicability[9]. An attractive alternative to mode shapes is Operational Deflection Shape (ODS) of the structure due to concentrated or uniformly distributed loadings[10]. The viability of this approach has been studied by researchers. For examples, in[11] applied laser ODS for damage detection in beams; In[12] identified crack in a beam using Wavelet Transform by measuring the static deflection profile. In[13] detected damage in a beam through the ODS measured by scanning laser vibrometer[14] applied spatial wavelet analysis to detect damage in an Euler-Bernoulli beam. In[15] identified damage in beams under static loads. In[16] detected crack in an elastic beam by measuring the static deflection. In[17] proposed a damage detection technique by applying ODS obtained from Frequency Response Function (FRF). Their technique applied the Global Fitting Method (GFM) to fit

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the smooth and analytic ODS to measured ODS. In[9] evaluated the sensitivity of deflection in cantilever structures for damage characterization.

Literatures show the application of deflection shape of structures for damage detection has focused on simple structures (mostly beams), i.e. one-dimensional structures. These studies have explored the application of one-dimensional deflection measurements. However, various important two-dimensional civil structures play vital role in our society i.e. plate, slab, bridge, frame, truss. This study proposes a Wavelet Transform (WT) method to detect damage through ODS measurement of a two-dimensional structure. Unlike previous studies where the traditional ODS is applied, this study involves application of the ODS difference of the plate structure. With the application of the ODS difference, the problem of boundary distortion which affects the detection of damage that are close to the boundaries is eliminated. The damage is detected and localized by utilizing Continuous Wavelet Transform (CWT) to decompose the ODS difference signal. The method is demonstrated using numerical model of rectangular plate. The damage is inflicted on the plate by decreasing thickness at the location of damage. A parametric study is presented to show the effect of noise and different damage percentages.

2. Continuous Wavelet Transforms

The application of WT for structural damage identification is the discontinuity introduced at the damage location. These discontinuities are not detectable by observation of the structural response, but are detectable by WT decomposition of the response. Wavelets are oscillations with amplitude which originates at zero, increases progressively and then decreases back to zero. The function ψ(x), which is the mother wavelet, is localised in space and frequency domain, creating a wavelet family ψu,s(x).

$$\psi_{us}(\mathbf{x}) = \frac{1}{\sqrt{s}}\psi\frac{(\mathbf{x}-\mathbf{u})}{\mathbf{s}} \tag{1}$$

S is scale and u position.

The CWT of a one-dimensional signal f(x) is the integral of the signal function product with complex conjugate $\psi(\mathbf{x})$ of the wavelet function. Taking the structure's deflection as one-dimensional signal f(x), therefore the CWT by[18] is:

$$Wf(u,s) = \frac{1}{\sqrt{s}}\int_{-\infty}^{+\infty} f(x)\,\psi\left(\frac{x-u}{s}\right)dx \tag{2}$$

$\mathbf{Wf(u,s)}$ represents wavelet coefficient of the wavelet $\psi_{\mathbf{u,s}}(\mathbf{x})$.

The n vanishing moment of a wavelet is vital in detecting singularity of signals, which is based on the Equation:

$$\int_{-\infty}^{+\infty} x^k\psi(x)dx = 0, \qquad k = 0,1,2,\ldots\ldots,n-1 \tag{3}$$

Rewriting Equation (2) as

$$Wf(u,s) = \frac{1}{\sqrt{s}}\int_{-\infty}^{+\infty} f(x)\psi\left(\frac{-(u-x)}{s}\right)dx = \frac{1}{\sqrt{s}}f * \psi\left(\frac{-u}{s}\right) = f * \bar{\psi}_s(u) \tag{4}$$

and

$$\bar{\psi}_s(u) = \frac{1}{\sqrt{s}}\psi\left(\frac{-x}{s}\right)\bar{\psi}_s(u) = \frac{1}{\sqrt{s}}\psi\left(\frac{-x}{s}\right) \tag{5}$$

The wavelet with n vanishing moment is re-written as nth order derivation of a smooth function $\theta(\mathbf{x})$ [18].This is given by a multiscale differential operator:

$$\begin{aligned} Wf(u,s) &= \frac{s^n}{\sqrt{s}}\int_{-\infty}^{+\infty}\frac{f(x)d^n}{dx^n}\theta\left(\frac{x-u}{s}\right)dx \\ &= \frac{s^n}{\sqrt{s}}\frac{d^n}{dx^n}\int_{-\infty}^{+\infty} f(x)\theta\left(\frac{-(u-x)}{s}\right)dx \\ &= \frac{s^n}{\sqrt{s}}\frac{d^n}{du^n}f * \theta\left(\frac{-u}{s}\right) = s^n\frac{d^n}{du^n}f * \bar{\theta}_s(u) \end{aligned} \tag{6}$$

$$\bar{\theta}_s(x) = \frac{1}{\sqrt{s}}\theta\left(\frac{-x}{s}\right)\bar{\theta}_s(x) = \frac{1}{\sqrt{s}}\theta\left(\frac{-x}{s}\right) \tag{7}$$

$\mathbf{f * \bar{\theta}_s}$ denotes convolution of function which is the average of f(x) over a domain proportional to the scale s.

In two-dimensional CWT, the function f(x, y) is given by[19]:

$$W^i f(u,v,s) = \frac{1}{s}\int_{-\infty}^{\infty}\Box\int_{-\infty}^{\infty} f(x)\psi^i\left(\frac{(x-u)}{s},\frac{y-v}{s}\right)dxdy \quad = \frac{1}{s}f * \psi^i\left(\frac{-u}{s},\frac{-v}{s}\right) = f * \bar{\psi}_s(u,v), \quad i = 1,2 \tag{8}$$

The horizontal wavelet $\psi^{\mathbf{1}}(\mathbf{x},\mathbf{y})$ and vertical $\psi^{\mathbf{2}}(\mathbf{x},\mathbf{y})$ are estimated by using products of scaling Ø and wavelet function ψ:

$$\psi^{\mathbf{1}}(x,y) = \boldsymbol{\varnothing}(x)\psi(y);\psi^{\mathbf{2}}(x,y) = \psi(x)\boldsymbol{\varnothing}(y) \tag{9}$$

The deflection shape displacement difference is taken to be a spatially distributed signal. A perturbation in the wavelet coefficients indicates the damage location.

3. Numerical Analysis

3.1 Plate Model

The square steel plate dimensions are: length l = 560 mm, breadth b = 560 mm and thickness h = 2 mm. The boundary conditions at the four sides of the plate are fixed. The plate is modeled with 784 rectangular shell elements of 20 mm by 20 mm. For the damage, it is assumed to be caused by corrosion and this is simulated by reducing the plate's thickness at the damage area location, this is displayed. Figures 1-2 show positions of damage at the plate's middle, side and corner. The material properties of the plate are: Poisson's ratio ν = 0.3, Young Modulus E = 200 Gpa and steel density ρ = 7850 kg/m^3. The damage is square of dimension 80 mm x 80 mm which is approximately 2.041% of the plate's area. Thickness at the point of damage is reduced by 25 % of the plate thickness, this corresponds to 0.5 mm reduction. The plate's deflection shape is computed using the commercial Finite Element Analysis (FEA) program SAP2000[20]. The deflection is measure at every 20 mm, with 28 measurement points on each side of the plate. Figure 3 shows a 3-dimensional deflection diagram of the undamaged plate.

3.2 Wavelet Damage Detection using Deflection Shape Difference

To analyse the deflection shape difference, wavelet toolbox in MATLAB[21] is utilised. To calculate the mode shape difference, a uniformly distributed load of 1 KN/m^2 is applied to the plate's surface, the displacement values of the damaged plate is then subtracted from the displacement values of the undamaged plate. The data estimated as the difference is decomposed using two-dimensional Continuous Wavelet Transform. To successfully implement WT, the appropriate mother wavelet, scale and angle need to be chosen, thus are important. The mother wavelet that provides the highest number of wavelet coefficient that is close to zero facilitates damage identification[22]. The identification of the most suitable mother wavelet for analysis is based on trial and error[23,24]. After several trials, the best mother wavelet is chosen for identifying damage. After series of trials and errors, Paul and Dog mother wavelets provided the best decomposed damage detection signal. In this study, the Paul mother wavelet Figure 4 shows adopted for damage identification with a combination of scale and angle 3 and 1 respectively.

The signals considered in this section are noise-free. Figures 5(a) – (c) shows the Wavelet Transform modulus of the mode shape difference of the plate for each damage location: Middle, corner and side points of the plate respectively.

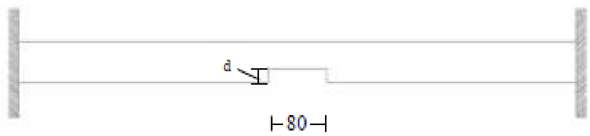


Figure1. Thickness reduction at damage location.

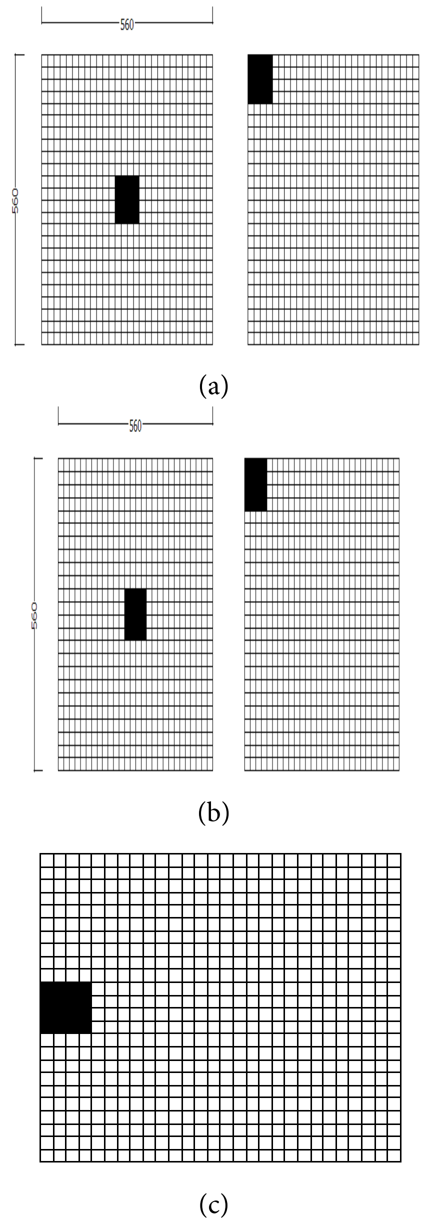


Figure 2. Damage location: (a) middle; (b) corner; (c) side.

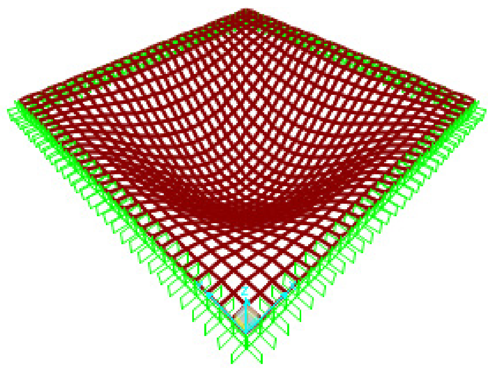


Figure 3. Deflection shape of the plate.

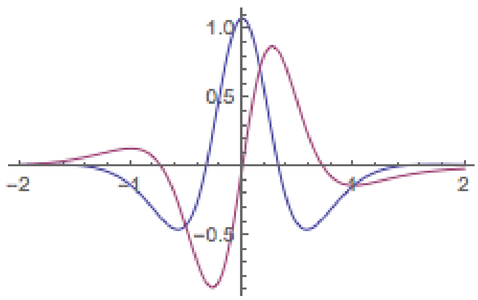
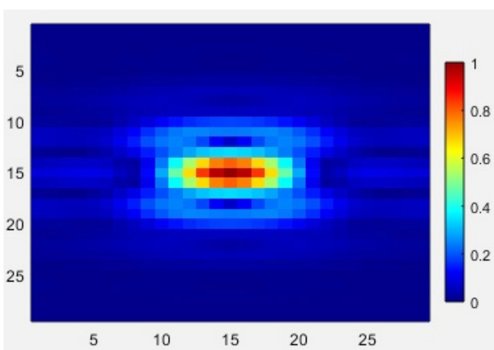
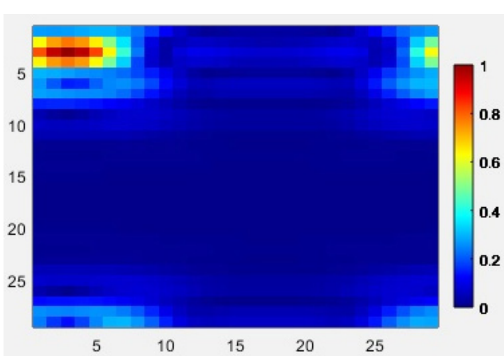


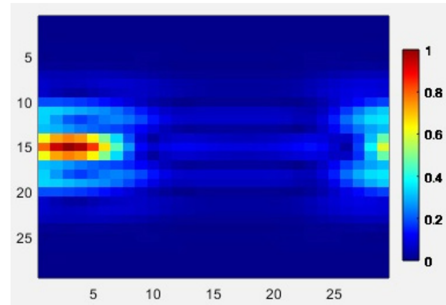
Figure 4. Basis function of Paul Wavelet shape.



(a)

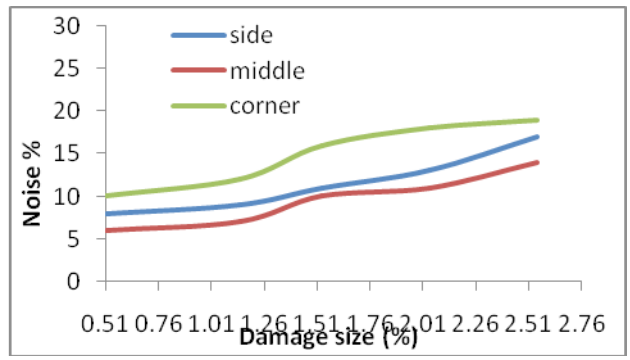


(b)

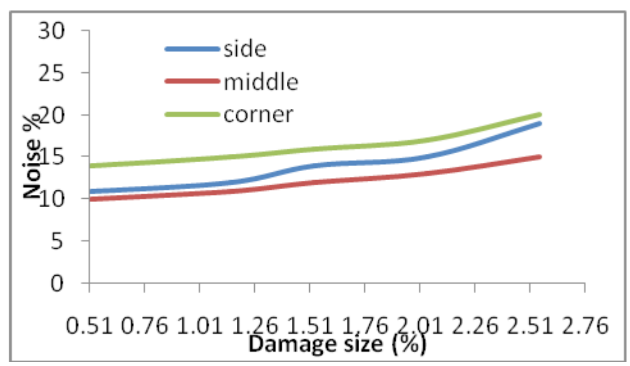


(c)

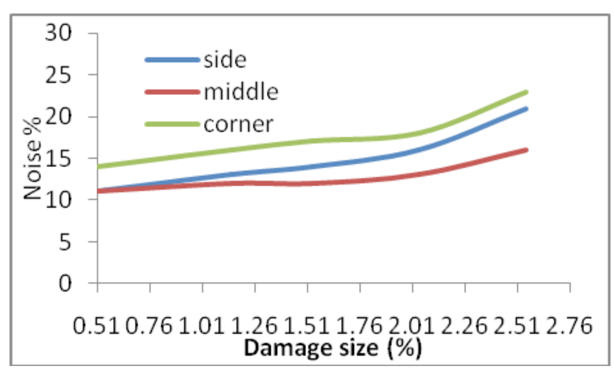
Figure 5. Damage detection and location: (a) middle; (b) corner; (c) side.



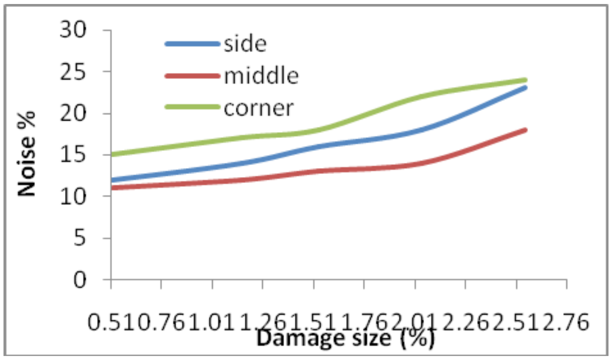
(a) Case 1



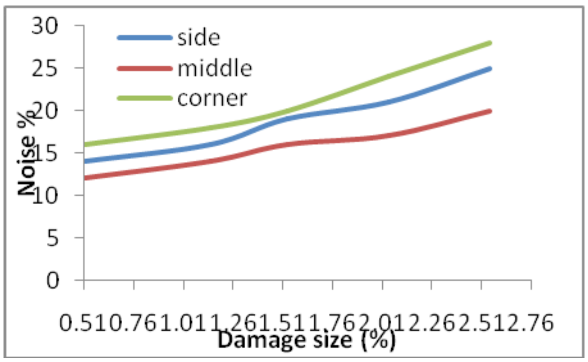
(b) Case 2



(c) Case 3



(d) Case 4



(e) Case 5

Figure 6. Damage detectable level with respect to noise.

4. Parametric Study

In practice and experimental analysis, the effects of noise are unavoidable. Therefore, this section provides more detailed study to evaluate the performance of this technique to different damage sizes, severities and noise levels at the three different locations.

For the damage size, the five sizes considered are: 2.55%, 2.04%, 1.53%, 1.14% and 0.51% area of plate. The intensity of damage is changed at all locations by changing the thickness. The damage severities considered are shown in Table 1. MATLAB software is utilized to add noise to the deflection shape difference data using Equation 10 and also for the decomposition of the data to estimate the allowable noise limit for damage size and severity.

$$e = 20 \log_{10}\left({}^{1}/_{n}\right) \tag{10}$$

Figure 6(a) shows the level of damage detectability is summarized at the three locations which correspond to Case 1 (5% thickness reduction or 1.90 mm). The damage is considered detectable if the decomposed signal clearly indicates the position of damage. The lines indicate this method's damage detectability for damage locations, namely; Middle, corner and side of the plate. Beneath and above sections of these lines constitute the detectable and undetectable noise and their corresponding size of damage. From this graph, it can be seen that the level of damage detectability increases with damage size increment. This is due to the distortion in the decomposed signal that gets larger as the damage size increases and this submerges and/or reduces the effect of noise. In other words, when the noise level is higher, larger size of damage is detectable. This situation is also observed in the three damage locations that are considered in this study.

For example, when damage is at the corner of the plate, the maximum noise level for damage detection of 2.55% plate area damage is 19%. This means that to detect corner damage with 5% (Case 1) damage severity and 2.55% damage area of plate, the noise level should not be more than 19%. Also, for the same severity (5%) and damage area of 1.53%, noise should not be above 16%. Figures 6(b) to (e) show the results for cases 2 to 5. It is observed from Figures 6(a) to (e) that the location of damage affects the damage detectability in presence of noise.

The detectability of damage with noisy data is in the ascending order of middle, side and corner and damages i.e. corner damage has the highest level of detectability when noisy data is applied. This is due to the corner damage close proximity to the boundaries (2 sides of the plate), resulting to have most effect on the plate's stiffness. The side damage comes next as it is also at a fixed boundary, while the middle damage comes last as it's the furthest to any fixed boundary.

Table 1. Damage severity in plate

Case	**1**	**2**	**3**	**4**	**5**
Thickness reduction (%)	5	25	50	75	87.5
Plate thickness *d* (mm)	1.90	1.50	1.00	0.50	0.25

5. Conclusions

This study focuses on identification of damage in plate structures by using Wavelet Transform technique applied to the plate's operational deflection. Damage is induced in the plate by reducing thickness at the points of damage. The damage locations considered in this study are at the plate's middle, side and corner. A numerical model is applied to estimate the mode shape which is analyzed with Wavelet Transforms. The locations of damage are established using pictorial representation of the spatial variation. A parametric study is presented to show the robustness of this technique in its performance in noisy data and difference damage scenarios. The Paul mother wavelet with scale 3 and angle 1 are combine in this study. From this study, the following conclusions are made:

- Detection of damage in plate is possible via decomposing operational deflection shape difference by using continuous Wavelet Transform.
- The location of damage plays a significant role in the tolerable noise level for damage identification.
- This technique performs efficiently for damage identification even in noisy data.
- The application of the deflection shape for damage detection via Wavelet Transform prevents the problem of border distortion as we have in application of mode shape.
- Selection of the appropriate mother wavelet influences the performance of this technique as an inappropriate mother wavelet leads to false results and poor performance.

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