ISSN (Print) : 0974-6846
Indian Journal of Science and Technology, Vol 10(25), DOI: 10.17485/ijst/2017/v10i25/104644, July 2017 ISSN (Online) : 0974-5645

Heat Transfer in a Non-Newtonian Jeffery Fluid from an Inclined Vertical Plate

C. H. Amanulla[1,2], N. Nagendra[1*], M. Surya Narayana Reddy[2], A. Subba Rao[1] and M. Sudhakar Reddy[1]

[1]Department of Mathematics, Madanapalle Institute of Technology and Science, Madanapalle –517325, Andhra Pradesh, India; nagsvu76@gmail.com
[2]Department of Mathematics, JNTU College of Engineering, Pulivendula – 516390, Andhra Pradesh, India

Abstract

In this manuscript, the scope of implicit finite difference method in solving a mathematical model for incompressible Jeffery's fluid past an inclined vertical plate. The influence of many multi-physical parameters in these variables is illustrated graphically and skin friction and Heat transfer rate are computed and discussed through the table. The results found that fluid flow is reduced with increasing Deborah number *De*, but temperature is accelerates. Increasing λ enhances flow of the fluid, but temperature reduced.

Keywords: Heat Transfer, Inclined Plate, Jeffrey's Fluid, Nusselt Number, Skin Friction

1. Introduction

Non-Newtonian fluids have the emerging importance in current industries and technology, and the mathematical modelling of non-Newtonian fluid flows and their understanding are of both fundamental and practical significance. The constitutive Jeffery fluid model explained[1–6]. Recently, Siva Reddy Sheri et al.,[7] considered natural convection flow from a vertical plate in the absence of MHD with ramped temperature. Ahmad et al.,[8] have studied mixed convection flow of a Jeffrey's non-Newtonian fluid over an exponentially stretched plate. Zin et al.,[9] discussed the heat transfer of unsteady flow past a vertical plate under the Influence of thermal radiation and MHD. Tripathy et al.,[10] explained the effects of chemical and hydromagnetic viscous flow over a vertical plate. Javed et al.,[11] discussed the radiation effect on MHD flow along a vertical impermeable wavy texture. Jayachandra Babu and Sandeep[12] have reported on MHD flow of Williamson fluid over a stretching sheet. Zeeshan and Majeed[13] performed of the flow, heat transfer of Jeffery fluid past a linearly stretching sheet with the presence of a magnetic dipole. Seth[14] obtained numerical solutions of the model MHD flow over semi-infinite plate through porous regime. Most of the studies are related to vertical plate, the articles on the heat transfer in Jeffrey's flow from an inclined vertical plate are very limited. Subba Rao et al.,[15] investigated non-similar solutions for the influence of thermal slipy flows of Casson viscoplastic fluid from an inclined vertical plate and he has used the Keller Box method.

Motivated by the above said research work, in the present paper, we investigate a numerical solution for two dimensional incompressible viscoelastic Jeffrey's non-Newtonian fluid flow from an inclined vertical plate. Numerical solutions for the velocity and the temperature are obtained using a powerful technique namely finite difference method (Keller-Box).

2. Mathematical Model

We examine steady buoyancy-driven convection flow of Jeffrey's non-Newtonian fluid over an inclined vertical plate. Figure 1 shows the flow model and associated coordinate system. The x - axis taken along the cone surface measured from the origin and the y - axis is measured normal to the surface.

* *Author for correspondence*

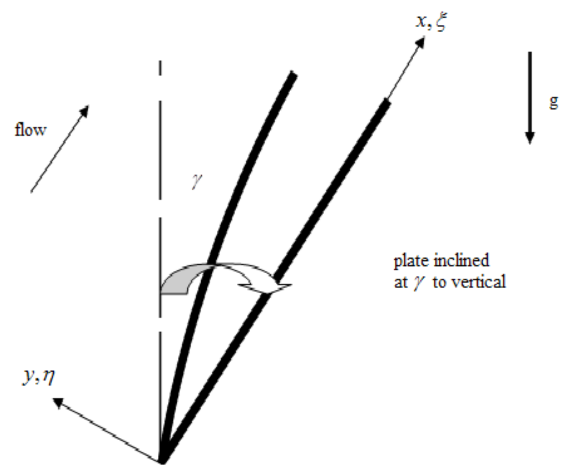


Figure 1. Non-Newtonian heat transfer over a plate.

For an incompressible Jeffrey's fluid, the continuity, momentum and energy equations are:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{v}{1+\lambda}\left[\frac{\partial^2 u}{\partial y^2}+\lambda_1\left(u\frac{\partial^3 u}{\partial x\partial y^2}-\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}+\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}+v\frac{\partial^3 u}{\partial y^3}\right)\right]+g\beta\left(T-T_\infty\right)\cos\gamma \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2} \tag{3}$$

The following spatial and temporal boundary conditions:

At $x=0,\ u=u_\infty,\ T=T_\infty$

$$\text{At } y=0,\ u=0,\ v=0,\ T=T_w \tag{4}$$

$As\ y\to\infty,\ u\to u_\infty,\ T=T_\infty$

Taking the following non-dimensional quantities:

$$\eta=y\left(\frac{u_\infty}{vx}\right)^{1/2},\ \xi=\xi(x),\ f(\xi,x)=\frac{\psi(x,y)}{(vu_\infty x)^{1/2}},\ \theta(\xi,x)=\frac{(T-T_\infty)}{(T_w-T_\infty)}$$

$$Gr=\frac{g\beta(T_w-T_\infty)x^3}{v^2},\ De=\frac{\lambda_1 U_\infty}{x},\ Pr=\frac{v}{\alpha},\ \mathrm{Re}_x=\frac{U_\infty x}{v} \tag{5}$$

Here $u=\frac{\partial\psi}{\partial y}$ and $v=-\frac{\partial\psi}{\partial x}$ are the velocity components in the x - and y - directions respectively, ψ -stream function, $v=\mu/\rho$ - the kinematic viscosity of the conducting fluid, k - the thermal conductivity, γ - inclination of the plate to the vertical, De - Deborah number, λ- the ratio of relaxation to retardation times, λ_1 - the retardation time. Pr - Prandtl number, Gr - Grashof number, T_∞ - the free stream temperature, Re_x - Local Reynolds number, $\xi=Gr/\mathrm{Re}^2{}_x$ - thermal buoyancy force parameter for forced convection.

In view of Eqs (5) the governing equations (2)-(3) and dropping primes yields the following dimensionless equations:

$$\frac{1}{1+\lambda}f'''+\frac{1}{2}ff''-\frac{De}{1+\lambda}\left(ff'''+\frac{1}{2}f''^2-\frac{1}{2}ff^{iv}\right)+\xi\theta\cos\gamma$$
$$=\xi\left(f'\frac{\partial f'}{\partial\xi}-f''\frac{\partial f}{\partial\xi}-\frac{De}{1+\lambda}\left(f'\frac{\partial f'''}{\partial\xi}-f'''\frac{\partial f'}{\partial\xi}+f''\frac{\partial f''}{\partial\xi}-f^{iv}\frac{\partial f}{\partial\xi}\right)\right) \tag{6}$$

$$\frac{\theta''}{\Pr}+\frac{1}{2}f\theta'=\xi\left(f'\frac{\partial\theta}{\partial\xi}-\theta'\frac{\partial f}{\partial\xi}\right) \tag{7}$$

The dimensionless form of the boundary conditions:

At $\eta=0,\quad f=0,\quad f'=0,\quad \theta=1$

$$\text{As } \eta\to\infty,\quad f'=1,\quad \theta=0 \tag{8}$$

The Skin friction coefficient and Nusselt number in the non-dimensional form is given by

$$\frac{1}{2}C_f\,\mathrm{Re}_x^{-1/2}=f''(\xi,0) \tag{9}$$

$$\frac{Nu}{\sqrt{\mathrm{Re}_x}}=-\theta'(\xi,0) \tag{10}$$

3. Computational Finite Difference Solutions

The finite difference method (Keller-Box) is employed to solve the transformed, coupled boundary layer problem defined by eqns. (6)-(7) under (8). Keller-Box method is the most versatile technique available for engineering analysis and equally adept at handling ordinary or partial differential equations as well as integral equations. Keller box method was originally settled for low speed aerodynamic problems and this system is established by Keller[16]. These include Casson slip boundary layer flows[17]. This method remains among the most powerful, versatile and accurate computational finite difference schemes employed in modern viscous fluid dynamics simulations. This method has been used extensively and effectively for over three decades in a large spectrum of nonlinear fluid mechanics problems. Keller's method provides unconditional stability and rapid convergence for strongly non-linear flows.

4. Results and Interpretation

The nonlinear boundary value problem solved in the previous section is dictated by an extensive number of thermal and hydrodynamic parameters. In order to gain a clear insight into the physical problem, numerical calculations for distribution of the velocity and temperature for different values of these parameters is conducted with graphical illustrations (Figure 2–4). For the purpose of our computation, we adopted the following default parameters: $\Pr = 0.71, De = 0.1, \lambda = 0.2, \xi = 1.0, \gamma = 70^0$ are initial values (unless otherwise stated). Table 1 shows that the effects of Deborah number *De* and λ on skin friction, Nusselt number, along with a difference in oblique coordinate, ξ. With increasing *De*, the Nusselt number (heat transfer rate) increases significantly and skin friction is reduced. A rising effect in λ is observed that enhance skin friction and Nusselt number.

In Figures 2(a)-2(b), present the evolution in the effect of the ratio of relaxation to retardation times i.e. λ on the velocity (f') and temperature (θ). Fluid flow is significantly increased with increasing λ. Conversely temperature is depressed slightly with increasing values of λ. In Figures 3(a)-3(b), illustrate the velocity (f') and temperature (θ) with a difference in Deborah number (*De*). Velocity component of the fluid flow (Figure 3a) is considerably reduced with increasing *De*. In Figure 2b, an increase Deborah number is seen to considerably enhance temperature all the way through the surface of plate. Figure 4(a)-4(b) presents the influence of the plate inclination on the dimensionless velocity and temperature. Taking angle of the plate ($\gamma < 0$) i.e. negative inclination, in Figure 4a, the velocity is reduced. Conversely in Figure 4b, with $\gamma < 0$ negative inclination the temperature decreases slightly. Further, more temperatures are improved marginally with positive inclination of the plate.

Table 1. Values of $f''(\xi, 0)$ and $-\theta'(\xi, 0)$ for different ξ, De, and λ

De	λ	ξ = 1.0		ξ = 2.0		ξ = 3.0	
		f"(ξ, 0)	-θ′(ξ, 0)	f"(ξ, 0)	-θ′(ξ, 0)	f"(ξ, 0)	-θ′(ξ, 0)
0.5	0.2	0.4040	0.3133	0.8956	0.3205	1.4699	0.3270
1.0		0.3977	0.3164	0.8844	0.3234	1.4546	0.3298
2.0		0.3864	0.3232	0.8645	0.3298	1.4241	0.3357
1.0	0.0	0.3725	0.3029	0.8223	0.3100	1.3446	0.3165
	1.0	0.5337	0.3332	1.1887	0.3413	1.9573	0.3485
	2.0	0.6583	0.3505	1.4732	0.3589	2.4348	0.3664

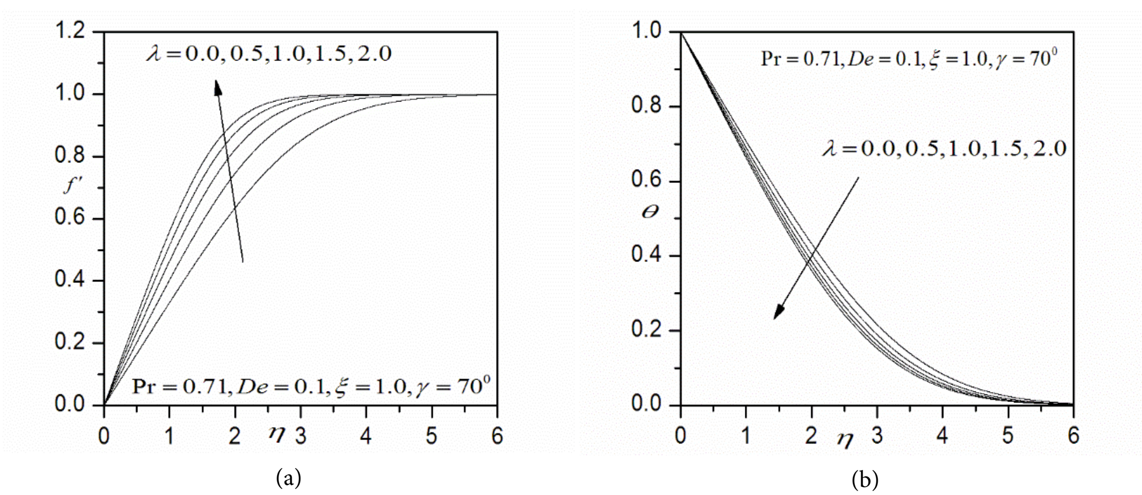
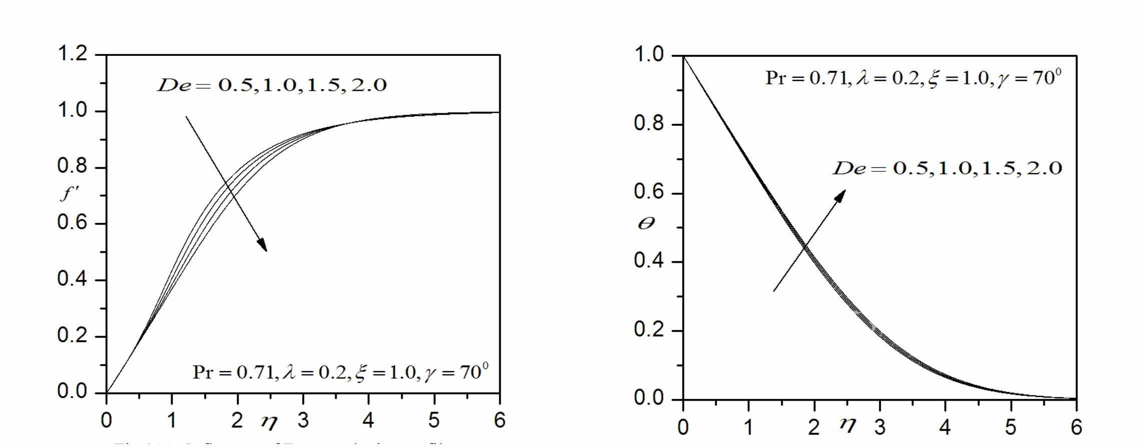
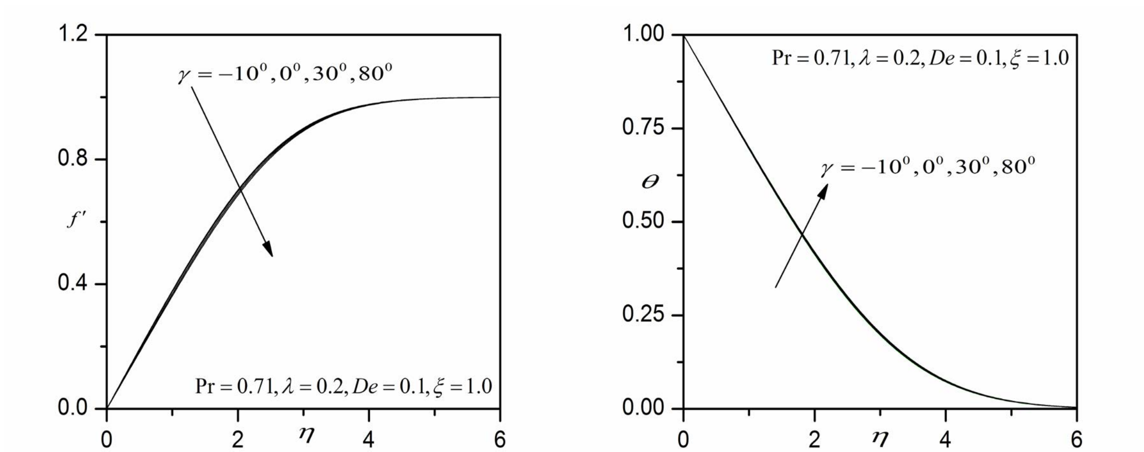


Figure 2. (**a**) Influence of λ on velocity profiles. (**b**) Influence of λ on temparature profiles.



(a) (b)

Figure 3. **(a)** Influence of De on velocity profiles. **(b)** Influence of De on temparature profiles.



(a) (b)

Figure 4. **(a)** Influence of γ on velocity profiles. **(b)** Influence of γ on temparature profiles.

5. Conclusion

In this work, motivated by applications in non-Newtonian fluid model has been developed for *heat transfer of Jeffery's fluid from an inclined vertical plate*. The governing equations are solved numerically the finite difference method. Numerical results are reported for various values selected parameters interest. When De takes the values larger than 0.5, the flow near plate decreases, skin friction- $f''(\xi, 0)$ and heat transfer rate $(-\theta'(\xi, 0)$ while enhances temperature. Raising the parameter ratio of relaxation and retardation times (λ), hike the velocity, skin friction coefficient $f''(\xi, 0)$, heat transfer rate $(-\theta'(\xi, 0)$ it reduces whereas temperature for all values of radial coordinate.

6. References

1. Hayat T, et al. Impact of Cattaneo-Christov Heat Flux in Jeffrey's Fluid Flow with Homogeneous- Heterogeneous Reaction. PLoS ONE. 2016; 11(2):e0148662. Crossref, PMid:26859675 PMCid:PMC4747563
2. Subba RA, Nagendra N, Prasad VR. Heat Transfer in a Non-Newtonian Jeffery's Fluid over a Non-Isothermal Wedge. Procedia Engineering. 2015; 127:775–82. Crossref
3. Abd AAM, Abo-Dahab SM. Rotation effect on peristaltic transport of a Jeffrey's fluid in an asymmetric channel with gravity field. Alex. Eng. J. 2016; 55:1725–35. Crossref
4. Das K, Acharya N, Kundu PK. Radiative flow of MHD Jeffrey's fluid past a stretching sheet with surface slip and melting heat transfer. Alex. Eng. J. 2015; 54:815–21. Crossref

5. Srinivas ANS, et al. Free Flow of a Jeffrey's Fluid between Two Long Vertical Thin Plates. International Journal of Pure and Applied Mathematics. 2015; 4:635–52.
6. Hayat T, Hussain T, Shehzad SA, Alsaedi A. Thermal and Concentration Stratifications Effects in Radiative Flow of Jeffrey Fluid over a Stretching Sheet. 2014. Crossref
7. Reddy SS, Chamkha AJ, Suram AK. Heat and Mass Transfer Effects on MHD Natural Convection Flow Past an Impulsively Moving Vertical Plate with Ramped Temperature. Columbia International Publishing American Journal of Heat and Mass Transfer. 2016; 3:129–48.
8. Ahmad K, Hanouf Z, Ishak A. Mixed convection Jeffrey's fluid flow over an exponentially stretching sheet with magneto hydrodynamic effect. AIP ADVANCES. 2016; 6:035024. Crossref
9. Nor AMZ, Khan I, Shafie S. Influence of thermal radiation on unsteady MHD free convection flow of Jeffrey's fluid over a vertical plate with ramped wall temperature. Mathematical Problems in Engineering. 2016; 6257071. Crossref
10. Tripathy RS, et al. Chemical reaction effect on MHD free convective surface over a moving vertical plate through porous medium. Alexandria Engineering Journal. 2015; 54:673–9. Crossref
11. Tariq J, Hussain A, Abuzar G. Influence of radiation on vertical wavy surface with constant heat flux: Using Keller box scheme. Alex. Eng. J. 2016. Crossref
12. Jayachandra BM, Sandeep N. MHD non-Newtonian fluid flow over a slendering stretching sheet in the presence of cross diffusion effects. Alexandria Engineering Journal. 2016. Crossref
13. Zeeshan A, Majeed A. Heat transfer analysis of Jeffery fluid flow over a stretching sheet with suction injection and magnetic dipole effect. Alex. Eng. J. 2016. Crossref
14. Seth GS, Sarkar S, Chamkha AJ. Unsteady Hydromagnetic Flow past a Moving Vertical Plate with Convective Surface Boundary Condition. Journal of Applied Fluid Mechanics. 2016; 4:1877–86
15. Subba RA, Prasad VR, Nagendra N, Reddy BN, Beg AO. Non-Similar Computational Solution for Boundary Layer Flows of Non-Newtonian Fluid from an Inclined Plate with Thermal Slip. Journal of Applied Fluid Mechanics. 2016; 2:795–807
16. Keller HB. A new difference method for parabolic problems, J. Bramble (Editor), Numerical Methods for Partial Differential Equations. New York: Academic Press; 1970.
17. Subba RA, Prasad VR, Harshavalli K, Beg AO. Thermal radiation effects on non-Newtonian fluid in a variable porosity regime with partial slip. J. Porous Media. 2016; 19(1):313–29. Crossref