ISSN (Print) : 0974-6846
ISSN (Online) : 0974-5645
Indian Journal of Science and Technology, *Vol 10(22), DOI: 10.17485/ijst/2017/v10i22/95684, June 2017*

Contingency Approach to Mesonic Binding States by Boolean Matrix

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Abstract

Predicting the exact binding system is presented in this article based on colored particle characteristic in strong interactions via Graph-Matrix theory. Usage of this theory in particle physics needs complex computations in order to evaluation of various functions, so there are some powerful methods including field theory method, characteristic of interactions between elementary particles. Therefore, the main goal of this paper is preparing a primary generalization of mesonic graph, as a type of graphical models, to bounding case and applying in particle interactions in high energy physics. It is shown that by using Graph-Matrix theory with two colored particles, creation of exact mesonic binding state in the hadronic system can be predicted.

Keywords: Boolean Matrix, Colored Particles, Graph Theory, High Energy Physics, Mesonic Binding State, Strong Interaction

1. Introduction

The graph and matrix theories as a mathematical concept need to be clarified in the particle physics that in general referred to artificial formations of matrix elements, nodes and edges[1–3]. The first thing that should be described and presented connection between mathematical and physical characteristics is that the terms interactions and creation of bounding states in the particle physics. Therefore, the term connection and interaction are reserved for the graphs representing real interaction between colored objects in which the vertices present entities of the bounding system and the links present the interaction among them. So, we will refer to the binding system of individuals and their interactions as hadronic states. We start by defining a graph formally for the mathematical concepts of graph, the incidence and adjacency of matrix relations in graphs. In particle physics it is usual to describe binding state and hadronic systems by considering the interaction between two colored particles (mesonic state) whose behavior is determined by a Hamiltonian of the following form[4,5]:

$$\begin{gathered}
H\Psi(r) = E\Psi(r) \\
\left(\frac{\hat{p}^2}{2\mu_1} + \frac{\hat{p}^2}{2\mu_2} + W(r)\right)\psi(r) = E\psi(r) = (\mathrm{M} - \mathrm{m}_1 - \mathrm{m}_2)\psi(r) \Rightarrow \\
\left(\frac{1}{2\mu}\hat{P}^2 + V(r)\right)\Psi(r) = E\Psi(r) \\
\Psi(r) \to \Psi_{n\ell}(r,\theta,\varphi) = \psi_{n\ell}(r)\Upsilon_{n\ell}(\theta,\varphi) \\
\left(-\frac{1}{2\mu r^2}\left(\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) - \hat{L}^2\right) + V(r)\right)\psi_{n\ell}(r) = E\psi_{n\ell}(r) \\
\left(-\frac{1}{2}\left(\frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} - \frac{\ell(\ell+1)}{r^2}\right) + \mu V(r)\right)\psi_{n\ell}(r) = \mu E\psi_{n\ell}(r)
\end{gathered} \tag{1}$$

Where $V(r)$ is the potential describing the interaction

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between colored particles. The description of interactions should be determine in quantum field theory naturally conduces to the evaluation of Feynman integrals that are associated to Feynman graphs, which are graphs with vertices and links and some special characteristics that include in interactions and connected colored particles. In creation of binding states by hadrons for instance, the vertices play a fundamental role as they present the different colored particles, such as up or down quarks with different flavours. Interaction of colored particles is represented by simple lines. Therefore, we can find good communication between graph theory and particle physics as it finds long time ago between group theory and particle physics. The colored particles in hadronic physics are best known as the six flavours of quarks, i.e., at least two quarks are bounded to each other by strong interactions that named after mesons or mesonic system like Pion, Psion, Upsilon etc. Mesonic systems in graph theory are presented by colored particle (vertices) and interactions between colored particles (links)[6–9] with different flavour. If there are two vertices without a direct link, we could possibly take other routes based on possible strong interactions that related to colored particle mass and colored particlegenerations[10,11]. The most possible number of interactions need to be in own generation for colored particle mass in the range 2.3MeV~4.8MeV and could be on own and lower generation for mass in the range 4.2GeV~173GeV. Hence, the intuitive notion of a graph in particle physics is a diagram consisting of vertices and lines adjoining these points if they have interactions and could be created binding states as mesonic or baryonic systems. In this article author tries to present the new method to predict creation of hadronic states based on graph and matrix theories.

2. Colored Particle's Graph

The basic role of Matrix applications and graph theory in particle physics are visualizing some of specification and characteristics. Matrix-Graph methods (MGM) usually induct connections to the interactions between particles. This is what makes MGM interesting in physics[6,7]. Matrix applications and graph theory answer as mathematical models to analyze many concrete and certain problems in physics very successfully therefore these problems can be formulated in graph theory and equivalently, one can represent a given hadronic system creations by matrix. Some several difficulties of a theoretical and experimental nature of bounding states can be solved in the development of various topics in MGM models. The presentation of the particle's interactions and creations by MGM are very important. It needs deep results in high energy interactions and Quantum chromodynamics theory. Proofs of MGM results and methods are usually given in a completely combinatory form, but rather using the possibilities of visualization given by matrix-graphical presentations of MGM. One of the goals of MGM in particle physics understands which line of hadronic bounding states can be created based on quark's characteristics. In the particle physics approach for studying hadronic bounding states, the interaction between colored particles is not neglect able. Study creation of hadronic states by using MGM methods can be very useful and simple. Here, we concentrate our presentation on alternant conjugated colored and anti-colored states in which consider a hadronic system consists of quark and its own antiquark in the range of light and heavy masses: 2.3MeV up to 4.2Ge. The most problem in quantum chromodynamics and particle physics is how to define new hadronic systems with different flavour-colored bounding states. Some of graphical method includes in quantum field theory and Feynman graphs with too many equations based on theoretical and experimental results. One can use interesting communication between MGM and creation of hadronic systems like mesons, baryons, pion, psion, upsilon and etc[10–17]. All the time we try to find more fundamental and simpler methods based on strong interactions characteristics. Creation of hadronic binding states can be presented by MGM model. Based on MGM model each vertex shows a colored particle with specific flavour and each link shows high energy interactions between the two colored particles. So, we describe some of notation in graph theory. The intuitive notion of a graph is a figure consisting of points and lines adjoining them: a graph is a set of objects called vertices along with a set of unordered pairs of vertices called links. Note that each link in a graph has no direction associated with it. If we wish to specify an interaction in particle physics that makes hadronic binding state, we can us the notion of a directed graph or digraph. The definition of a digraph is the same as that of a graph, except the links is ordered pairs of links[2,3,7]. Now, let us consider a finite set for colored particle group i.e., six flavours of quarks $V=\{v_1,v_2,\ldots,v_6\}=\{u,d,s,c,b,t\}$of unspecified elements and let $V\otimes V$ be the set of all ordered pairs $(v_i,v_j)$of the elements of $V=\{v_1,v_2,\ldots,v_6\}=\{u,d,s,c,b,t\}$. A relation on the set V is any subset $E\subseteq V\otimes V$. The relation E is symmetric if $(v_i,v_j)\in E$ implies $(v_j,v_i)\in E$. The relation E is anti-reflexive if $(v_i,v_j)\in E$ implies $(v_i\neq v_j)$. Now we can define a simple graph as the pair $G=(V,E)$, where V is a finite set of nodes, vertices or points and E is a symmetric and anti-reflexive relation on $V=\{v_1,v_2,\ldots,v_6\}=\{u,d,s,c,b,t\}$, whose elements are known as the edges and their interactions known as links of the graph. Because of the quark's short lifetime, it is impossible to create all possible mesonic systems that we can link in quark's graph diagram, example we have not mesons consist of u-quark and t-antiquark and tt*-mesons are not expected to be found in nature. Now, let us consider a finite set $V=\{v_1,v_2,\ldots,v_6\}=\{u,d,s,c,b,t\}$of unspecified elements and let $V\otimes V$ be the set of all ordered pairs $(v_i,v_j)$of the elements of $V=\{v_1,v_2,\ldots,v_6\}=\{u,d,s,c,b,t\}$. A relation on the set $V=\{v_1,v_2,\ldots,v_6\}=\{u,d,s,c,b,t\}$is any subset $E\subseteq\{u,d,s,c,b,t\}\otimes\{u,d,s,c,b,t\}$. The relation E is symmetric if $(v_i,v_j)\in E$ implies $(v_j,v_i)\in E$. The relation E is anti-reflexive if $(v_i,v_j)\in E$ implies $(v_i\neq v_j)$. Now we can define a simple graph as the pair $G=(V,E)$, where $V=\{v_1,v_2,\ldots,v_6\}=\{u,d,s,c,b,t\}$ is a finite set of nodes, vertices or points and E is a symmetric and anti-reflexive relation on $V=\{v_1,v_2,\ldots,v_6\}=\{u,d,s,c,b,t\}$, whose elements are known as the edges or links of the graph. In a directed graph the relation E is non-symmetric. Formally, in hadronic physics quarks generation based on graph theory and diagram is a pair of sets $G=(V,E)$, where $V=\{v_1,v_2,\ldots,v_6\}=\{u,d,s,c,b,t\}$the set of vertices and E is the set of edges, formed by pairs of vertices. Elements of E can occur more than once so that every element has a multiplicity. So, we label the quark vertices with letters of colored and anti-colored particles as it shows in Figure 1"[7–10]. An edge of the form colored-colored particle is a null loop, i.e., E is empty. A graph can linked two nodes just if they are in the same generation or linked with nodes from the lower generations. Edges are adjacent if they share a common end vertex. Two vertices colored and anti-colored particle are adjacent if they are connected by an edge. There is no mesonic state with those nodes if include colored-colored or anti-colored_anti-colored particles[11–15]. Therefore, the bounding graph can be presented as a bipartite graph, which vertex set can be partitioned into two sets each edge has one vertex in $\{u,d,s,c,b,t\}$ and end vertex in $\{u^*,d^*,s^*,c^*,b^*,t^*\}$, so we can show contingency of bounding states (mesonic states: qq*).

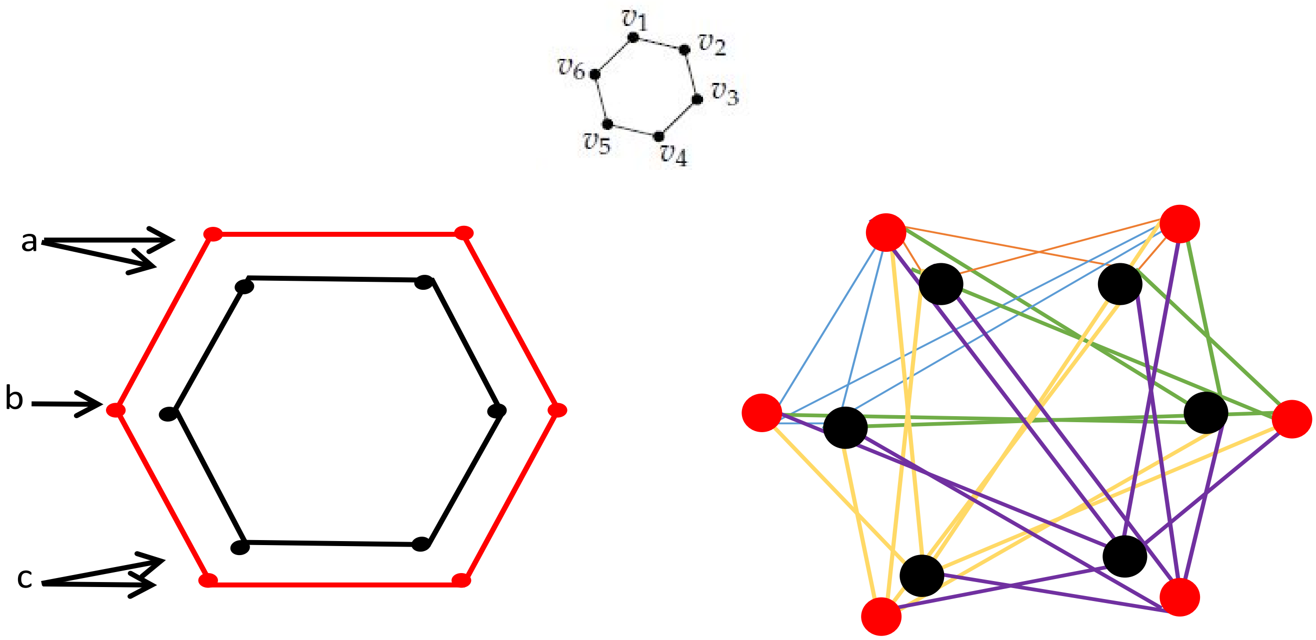

Figure 1. Black graph is quarks graph, red graph is anti-quark graph. (a) The first generation includes up and down quarks. (b) The second generation includes strange and charm quarks. (c) The third generation includes bottom and top quarks. (v_i is colored particle).

3. Matrix of Colored Interaction

Matrix theory is one of the branches of the mathematics that used in particle physics which describes the interactions, with the help of graph theory. A graph can be presented in interactions by using the adjacency and incidence matrices. One can define adjacency matrix and observed based on graph diagram of colored particle interactions and proves theorem of adjacency and incidence matrices. New progress has been made in the field of colored particles increase the data on hadronic spectra with binding states and un-binding states. These unstable states can be presented in graph theory by using definition of direct and indirect graph in particle physics and strong interactions. We can show that some colored particle's graph has special name depending on their specifications and particularity. When we talk about undirected graphs, really, all we are saying is whether the vertices in a graph are bidirectional or not. Most, but not all, colored particle's graphs have only one kind of vertices, i.e., dependence on their generations, they could be just on own generation or could be on own generation and lower generation. This low gets us to the meaning of directed or undirected graphs. Based on the strong interaction principles one can finally present interaction of two hadronic particles using graph theory and propounding it by matrices[1,2,8]. The interaction can be described by graph through matrix i.e. adjacency and incident matrices with two different description when we use adjacency matrix, that mean the connection between colored particle can be determined in accordance with the strong interactions and principles of generations. But, when we use incidence matrix: $M_{ij} = [q_i q_j^*]$, the elements of incidence matrix show us mesonic state that can be created via two colored particles and this biding state can exist in the real hadronic world. Let $V = \{v_1, v_2, \ldots, v_6\}$ a graph with 6 vertices: $V = \{v_1, v_2, ..., v_6\} = \{u, d, s, c, b, t\}$. The adjacency matrix of V with respect to colored particles in mesonic states of 6 vertices is the 6×6 matrix and denoted by $E \subseteq \{u, d, s, c, b, t\} \otimes \{u, d, s, c, b, t\}$ and defined as $AG_{ij} = [\,Q_{ij}\,]$ where Q_{ij} is the number of vertex joining colored particle. Elements of the adjacency matrix are either 0,1. Such a matrix we can call as a creation matrix. In the colored particle's adjacency matrix $AG_{ij} = [\,Q_{ij}\,]$:

$\{v_1, v_2, ..., v_6\} = \{u, d, s, c, b, t\} \otimes$

$\{v_1, v_2, ..., v_6\} = \{u^*, d^*, s^*, c^*, b^*, t^*\}$ and also $(AG_{ij})^* = [\,Q^*{}_{ij}\,]$

$\{v_1, v_2, ..., v_6\} = \{u^*, d^*, s^*, c^*, b^*, t^*\}$

$\otimes \{v_1, v_2, ..., v_6\} = \{u, d, s, c, b, t\}$ we have elements for:

$$AG_{ij} = [\,Q_{ij}\,] \text{ or } (AG_{ij})^* = [\,Q^*{}_{ij}\,]:$$

$$\begin{cases} 0 & \text{if there is imposible conection between } (vi, vj) \\ 1 & \text{if there is posible conection between } (vi, vj) \end{cases}$$

Two vertices is said to be *adjacent* or neighbor if it support at least one common vertex. Mesonic graph in that is shown in "Figure 1", has $6 \otimes 6$ vertices in two dimensions i.e., six vertices in each dimensions (x and y). Thus, we make adjacency matrix with 12 elements. To fill the adjacency of mesonic matrix, we look at the name of the vertex (colored particle) in row and column (anti-colored particle)[12,15] as shown in the below matrices[1,3,8]:

$$AG_{ij} = \begin{array}{c} \\ u \\ d \\ s \\ c \\ b \\ t \end{array} \begin{array}{c} \begin{array}{cccccc} u^* & d^* & s^* & c^* & b^* & t^* \end{array} \\ \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 \\ \tilde{1} & \tilde{1} & \tilde{1} & \tilde{1} & \tilde{1} & \tilde{1} \end{pmatrix} \end{array}$$

$$(AG_{ij})^* = \begin{array}{c} \\ u^* \\ d^* \\ s^* \\ c^* \\ b^* \\ t^* \end{array} \begin{array}{c} \begin{array}{cccccc} u & d & s & c & b & t \end{array} \\ \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 \\ \tilde{1} & \tilde{1} & \tilde{1} & \tilde{1} & \tilde{1} & \tilde{1} \end{pmatrix} \end{array}$$

Elements of the adjacency matrix are either 0 or 1 if we neglected predicting of interactions of colored particle with very short lifetime and give its connection with other colored particle as impossible links, this type of interaction in graph-matrix can be shown as non-bounding state that will present in the below. A logical interaction matrix that reaches binding state can be called as a binary matrix, relation matrix, Boolean matrix, or (0, 1) matrix is a matrix with entries from {0, 1}. Such a matrix can be used to represent a binary relation between a pair of finite colored particles for mesonic states. Therefore, adjacency of Boolean matrix for mesonic binding states:

$$AG_{ij} = \begin{array}{c} \\ u \\ d \\ s \\ c \\ b \\ t \end{array} \begin{array}{c} \begin{array}{cccccc} u^* & d^* & s^* & c^* & b^* & t^* \end{array} \\ \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

$$(AG_{ij})^* = \begin{array}{c} \\ u^* \\ d^* \\ s^* \\ c^* \\ b^* \\ t^* \end{array} \begin{array}{c} \begin{array}{cccccc} u & d & s & c & b & t \end{array} \\ \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

The connection between links and vertices in mesonic graph (Fig.1) is the best ways to represent interactions matrix between particles. We can make matrix that related vertices to links i.e. colored particles binding states (links). A vertex is said to be *incident* to an edge if the edge is connected to the vertex. If we have vertices $V = \{u, u^*, ..., t, t^*\}$

and edges: $E=\{e_k=q_i q_j^*\}$, incidence matrix: $M_{ij}=[q_i q_j^*]$

where $q_i q_j^*$ is the mesonic binding states or created mesonic system (elements "$\tilde{1}$" in incidence matrix show us mesonic state that because of colored particle's short lifetime it cannot be created in today's experimental data and maybe in future we can discover them)). In the colored particle's incidence matrix $M_{ij}=[q_i q_j^*]: \{u,d,s,c,b,t\}\otimes$

$\{u^*, d^*, s^*, c^*, b^*, t^*\}$and also $M_{ij}=[q_i^* q_j]$:

$\{u^*, d^*, s^*, c^*, b^*, t^*\}\otimes\{u,d,s,c,b,t\}$ we have elements for:

$M_{ij}=[q_i q_j^*]$or $M_{ij}^*=[q_i^* q_j]$:

0	if there is no binding state
$\neq 0$	if there is binding state
$\tilde{1}$	Cannot detect binding state

Elements of the incidence matrix are either 0 or some type of mesonsas shown in below[15-17]:

$$M_{ij}=\begin{array}{c|cccccc} & u^* & d^* & s^* & c^* & b^* & t^* \\ \hline u & \pi^0 & \pi^+ & 0 & 0 & 0 & 0 \\ d & \pi^- & \pi^0 & 0 & 0 & 0 & 0 \\ s & \kappa^- & \overline{\kappa}^0 & \phi^0 & 0 & 0 & 0 \\ c & \overline{D}^0 & D^+ & D_s^+ & J/\Psi & 0 & 0 \\ b & B^- & \overline{B}^0 & \overline{B}^{*0} & B_c^{*-} & Y & 0 \\ t & \tilde{1} & \tilde{1} & \tilde{1} & \tilde{1} & \tilde{1} & \tilde{1} \end{array}$$

$$M^*_{ij}=\begin{array}{c|cccccc} & u & d & s & c & b & t \\ \hline u^* & \pi^0 & \pi^- & 0 & 0 & 0 & 0 \\ d^* & \pi^+ & \pi^0 & 0 & 0 & 0 & 0 \\ s^* & \kappa^+ & \kappa^0 & \phi^0 & 0 & 0 & 0 \\ c^* & D^0 & D^- & D_s^- & J/\Psi & 0 & 0 \\ b^* & B^+ & B^0 & B^{*0} & B_c^{*+} & Y & 0 \\ t^* & \tilde{1} & \tilde{1} & \tilde{1} & \tilde{1} & \tilde{1} & \tilde{1} \end{array}$$

4. Conclusion

Graph theory is a branch of mathematics which developed slowly over the different fields of sciences. Graphical models have various applications in basic and engineering sciences which include statistical physics, solid state physics, bioinformatics, telecommunication and etc. Finding its origins in modern physics, this field would finally be applied strongly to particle, quantum physics and quantum computing. Usage of graph theory in particle physics needs complex computations in order to evaluation of various functions, so there are some powerful methods including field theory method, characteristic of interactions between elementary particles and etc. Quarkonium or mesonic bounding states by graphical models could be described and developed hadronic physics, which is possible by generalization of interactions probability theory to bound states characteristics. Indeed, graph theory has the advantage that it contains easily formulated issues that can be stated in the quantum chromodynamics theories. Finding possible interactions to any two of six quark flavours by using graph theory it is very important. In recent years, due to the extension of new hadronic systems the concepts and applications of the graph and matrix theories are being important to cover the advances made in strong interactions. Thus we proved the number of links from $v_i=\{u,d,s,c,b,t\}$to

$v_j=\{u^*, d^*, s^*, c^*, b^*, t^*\}$of a colored particle with adja-

cency and incidence in matrices based on Boolean matrix. The Boolean matrix elements for mesonic states are connected to each other by strong interactions and hadronic principles. Through adjacency matrix, we have drawn a possible interactions based on the lows of colored particle's interactions in quark generation lines and through incidence matrix, we have shown a possible interactions that give us mesonic binding states as pion, kaon, psion and etc.

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