FEM Analysis of Ultrasonic Transducer with Square and Circular Diaphragm

Rashmi Sharma^{1*}, Rekha Agarwal¹, Ashwani Kumar Dubey² and Anil Arora³

¹Department of Electronics and Commuication, Amity School of Engineering & Technology Bijwasan – 110061, New Delhi, India; rashmiapj@gmail.com, ragarwal@amity.edu ²Department of Electronics and Communication, Amity University, Noida – 201303, Uttar Pradesh, India; akdubey@amity.edu ³Department of Electronics and Communication, Thapar University, Patiala – 147004, Punjab, India; anil16m@gmail.com

Abstract

The objective of the paper is to design a Microelectronic Mechanical Systems(MEMS) based Capacitive Micromachined Ultrasonic Transducer (CMUT) which has been found to be superior in terms of bandwidth, transduction efficiency and array fabrication as compared to conventional piezoelectric transducers. With the years CMUT has been shown with different element geometries and fabrication techniques, however the optimization techniques and the application areas for the device persist. In this paper CMUT is simulated in COMSOL with Square and Circular diaphragm. FEM analysis is being carried out considering the same area for both the geometries in 3D.CMUT dynamics have been modeled by combining the Electrostatic module, Solid Mechanics Module and Mesh modules in COMSOL. The distributed stress and electric field are measured as function of time. This gives the exact comparison for the Eigen frequency, pull in voltage, deflection with applied DC bias and deflection of the diaphragm with AC superimposed on DC considering the isotropic Silicon as the diaphragm material. This simulation work provides results for the researcher to conclude on the geometry of the device prior to fabrication.

Keywords: Circular, Deflection, Diaphragm, Eigen Frequency, FEM Analysis, Square, Ultrasonic Transducer

1. Introduction

Ultrasound has applications in various fields like medical imaging, distance measuring, Nondestructive testing etc¹. Many devices have been used to generate the ultrasound like piezoelectric transducers, and Micro Electrical Mechanical Systems (MEMS) based capacitive transducers. Capacitive micro-machined ultrasonic transducer (CMUT) provides better bandwidth, transduction efficiency and high resolution as compared to piezoelectric transducer². The piezoelectric transducer working principle is different from CMUT³. CMUT consists of a substrate which acts as the bottom electrode on which a membrane is overlaid with a cavity filled with air. The membrane acts as the top electrode of the capacitor and the cavity acts as the dielectric. The CMUT works both as a transmitter and receiver. To work as a transmitter the DC voltage is applied between bottom and top electrode which results in the deflection of the membrane. The AC bias is superimposed over DC bias which results in vibration of membrane. The vibrating membrane generates ultrasound. For making

*Author for correspondence

CMUT as a receiver the ultrasound is introduced onto the membrane which results in capacitance change^{4.5}.

The resolution of the received signal can be increased through the use of a flexible capacitive membrane. This situation is compensated while designing a CMUT, with the use of a DC bias generating strong attractive forces and reducing the displacement within the capacitive plates. The DC bias is applied of such a magnitude that it is marginally less than the collapse voltage of the capacitor which results in the high gains with a small magnitude of the AC bias^{6.7}.

2. Theory of Operation

2.1 Working Principle

The capacitive transducer can be modeled using a capacitor, a mass and a spring. The force due to mass is basically the sum of spring force and the capacitive force^{4.8}.

$$F_{Cap} + F_{Spring} = F_{Mass} \tag{1}$$

 F_{cap} , the capacitive force is the electrostatic force resulted with the application of the DC bias (V),

$$F_{Cap} = -\frac{d}{dx} \left(\frac{CV^2}{2} \right) = \frac{\epsilon_0 AV^2}{2(g_0 - w)^2}$$
(2)

Where, A is the area of the electrode, g_0 appears as the initial distance among capacitor plates, and w is the displacement of the membrane after the applied DC bias⁹.

Furthermore, Hooke's law is used to solve for the F_{spring} . F_{mass} is obtained from the Newton's second law of motion. Putting these values in Equation (1) the following expression is obtained:

$$m\frac{d^{2}x(t)}{dt^{2}} - \frac{\epsilon_{0} AV(t)^{2}}{2(g_{0} - W(t))^{2}} + kw(t) = 0 \qquad (3)$$

For force balance, the electrostatic force is equal to the mechanical forces in the membrane, the deflection can be calculated using the force balance:

$$\frac{\epsilon_0 A V^2}{2(g_0 - w)^2} = kw \tag{4}$$

As the applied DC bias is increased the effective gap height decreases which results in the increment of the spring force.

The Silicon material is used as the membrane which is anisotropic material. Its properties like young's modulus, refractive index, thermal and electrical conductivity. Hence the geometries are explained with a single Young's modulus which are used to calculate the stress and strain acting upon the material. These are calculated using Hooke's law, given by Equation 5.

$$\sigma = \mathbf{E} \boldsymbol{\epsilon}$$

(5)

In Equation 5, σ is the stress, \in is the strain, and E is the Young's modulus of the membrane material.

2.2 Analytical Solution

For finding the analytical solution of the CMUT device the general analytical plate deflection equation can be written as

$$\Delta p = \rho h \bar{w} + \nabla^2 \bot D \nabla^2 \bot w + \sigma_0 h \nabla^2 \bot w \qquad (6)$$

Where, ρ corresponds to the mass density of the capacitor plate, Δp is the applied pressure which acts as load, h represents the height of the plate, σ_0 corresponds to the pre-stress to which the plate is subjected, and w is the displacement of plate in the z direction. $\nabla^2 \bot$ is the two-dimensional Laplacian operator given in Equation 6 whereas the flexural rigidity coefficient, D, is given in equation 8. Here v corresponds to the Poisson ratio of the membrane material.

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

$$F b^3$$
(7)

$$D = \frac{12[(1 - \nu]^2)}{12[(1 - \nu]^2)} \tag{8}$$

The Circular and Square CMUT modeled in COMSOL were not pre-stressed and consequently the zero value is assigned to σ_0 , which results in the following Equation 6 to:

$$\Delta p = \rho h \bar{w} + \nabla^2 \bot D \nabla^2 \bot w \tag{9}$$

For circular membranes the solutions to this plate equation have been found while for square CMUT membranes only estimations have been obtained.

The displacement of the circular membrane along radius w(r) can be solved considering the thin and the thick membranes given by equation (10) placed in the geometry of the device¹⁰. For thin plate membranes the displacement equation is given by Equation 10a. In this

the maximum deflection w_0 is of magnitude which is higher than the thickness represented by h. For finding the displacement of the thick membranes Equation 10b can be used in which the maximum displacement is of the order of magnitude lower than the thickness of the membrane.

In the below mentioned equations r is the distance of a given point from the center of the membrane plate whereas R corresponds to the total radius of the membrane circular plate.

$$w(r)_{thin} = w_0 \left[1 - \frac{r^2}{R^2} \right]$$
 (10a)

$$w(r)_{thick} = w_0 \left[1 - \frac{r^2}{R^2} \right]^2$$
 (10b)

For the displacement at center of the circular membrane the analytical solution is given by equation written as¹¹:

$$w_0 = \frac{\Delta p r^4}{64D_i} \tag{11}$$

Where Di corresponds to the isotropic flexural rigidity, given by Equation 8. When we consider the square membranes, the solution is an infinite Fourier series which is not a closed form solution¹². For finding the exact solution for the maximum and central displacement of the square membranes the total potential energy of the system can be solved given by the equation $12^{13.14}$.

$$\frac{\partial U}{\partial w_0} = \frac{\partial}{\partial w_0} \left[\left\{ \int \left[\frac{D_i}{2} \left((\nabla^2 \bot w)^2 \right) - p_0 w \right] dy \right\} dx \right] = 0 \quad (12)$$

3. Fem Simulations

The COMSOL finite element analysis is being done to analyze the displacement profile of both the circular and square membranes with the isotropic Silicon as the membrane material. To calculate the maximum deflection with the DC bias along with the suitable boundary conditions. Model is validated with the analytical results for the circular membranes and collapse voltages and the resonant frequencies are calculated for both the circular and square membranes. The geometry for capacitive micro machined ultrasonic transducer with square and circular diaphragm is Figure 1. The simulation parameters are summarized in Table 1.

Description	Value
	[µm]
Radius of Circular	56.4
Side of Square	100
Gap Thickness	1
Diaphragm Thickness	0.5
Substrate Thickness	2

Table 1. Parameters used for simulation



Figure 1. Capacitive micromachined ultrasonic transducer with square and circular diaphragm.

To model the dynamics the Electrostatics, Solid mechanics and Moving mesh physics are being coupled. Moving mesh is added to have the mesh deforming with the membrane. This is imperative, owing to the state of the membrane which will evidently affect the electrostatic force between the moving membrane and the substrate. Each physics has its own set of boundary conditions. The Electrostatics module is employed to estimate the electric field distribution inside the air filled cavity of the CMUT. With this the electric displacement and potential distribution within dielectrics can be calculated under varying charge distributions. The geometry designed is stationary but coupled with other physics various studies like Eigen frequency, small signal analysis, frequency domain and time domain design finds a platform in all space dimensions. The 3D structural analysis is carried out with the Solid Mechanics interface of COMSOL. Parameters like stress, strains and displacements are results of solving Navier's equation^{15,16}. The geometry changes shape due to the applied boundary conditions and the biases to model the same the Moving Mesh interface is used. Moving mesh interface is involved in COMSOL for both the time dependent and stationary studies

in which the geometry shape is changed due to the dynamics involved in the system^{17,18}.

4. Results and Discussion

The CMUT was simulated for resonant frequency and membrane deflection with applied DC and AC bias. Eigen frequency analysis was used to simulate the resonant frequencies of the membrane structure for both the geometries. The boundaries of the membrane and the substrate are fixed and the boundary conditions are applied. For the square and circular membrane the Eigen frequencies were found to be7.3768E5 Hz and 6.6304E5Hz respectively as shown in the Figure 2 and 3 respectively. Figure 4 shows the displacement of both the membranes along frequency.



Figure 2. Eigen frequency of square diaphragm.



Figure 3. Eigen frequency of circular diaphragm.



Figure 4. Displacement with frequency.

The top electrode i.e. the membrane collapses with the sustrate due to increased DC bias; the electrostatic forces of the substrate overcomes the mechanical restoring forces of the membrane. As the DC bias is decreased further the membrane lifts-off as the membrane force exceeds the electrostatic force. For computation of pull in i.e collapse FEMmesh structures were oragnised. Figure 5 shows the deflection of diaphragm with aplied DC bias. The pull in occurs when the deflection reaches 1/3 of the gap height. The pull in calculated were 43V and 48V for the circular and square diaphragm respectively. The time dependent study was performed after solving the eigen frequency. After the application of DC bias the device was superimposed with the AC voltage and the displacement is shown in Figure 6 for both the diaphragms with the same area and same diaphragm material.



Figure 5. Deflection of Diaphragms with DC Bias.



Figure 6. Deflection of Diaphragms with AC Superimposed on DC Bias.

5. Conclusion

A 3D model was made in COMSOL with both circular and square geometry for the FEM analysis. Various boundary conditions were applied which were validated by the analytical solutions for circular and aquare CMUTs. Comparison was made for the displacement of Square and circular membranes with the isotropic material. The comparison proclaimed that device with circular membrane are better in contrast to the square membranes covering all aspects. The analytical model is required for the validation of the model. An improved design of CMUT will be recognized which can be further developed for fabrication. Smart existing materials with improved mechanical and electrical properties can be reflected for the diaphragms.

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