ISSN (Print) : 0974-6846
Indian Journal of Science and Technology, Vol 11(6), DOI: 10.17485/ijst/2018/v11i6/117481, February 2018 ISSN (Online) : 0974-5645

Dynamic Parameter Estimation of Analog to Digital Converter with Multipoint Interpolation Technique

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Abstract

Objectives: The objective of this paper is to estimate the dynamic parameters of Analog to Digital Converter (ADC) with multi frequency sine signal as stimuli. **Methods/Statistical Analysis:** Traditionally the characteristics of a signal are estimated with its spectrum using Fast Fourier Transform (FFT) algorithm. Interpolated FFT technique gives more accurate results compared to the FFT technique. The signal is weighted with rectangular and Hanning window before the Discrete Fourier Transform (DFT) is calculated, frequency and magnitude of various components of input stimulus are estimated with this technique. Weighted signal is applied to ADC under test and the frequency spectrum of digital output is obtained. **Findings:** This technique is used to determine the frequency dependent parameters of ADC. The analytical formula for amplitude, frequency and phase estimation is simple to implement. The expression for the accurate estimation of the fractional part between the two largest spectrums of the recorded cycles by I_pDFT method is presented. Sampled input signal is weighted with rectangular and tapered window and averaged. A reduction in noise level can be observed in the spectrum. The obtained values of frequency, amplitude and phase which are more accurate as presented in literature. Comparative values of ideal and real ADC dynamic parameters are presented here. The research work carried can contribute significantly to ADC design and testing with multi frequency input signals. The novelty of this approach is that the accurate results for dynamic parameters of ADC can be characterized for a particular bandwidth in a single run. **Application/Improvements:** An ADC is a vital component in high-speed data communications video and radar systems. The overall accuracy of the system is dependent on the accuracy of the ADC. For such systems, dynamic testing of the device using standard signal is not useful. Therefore dynamic testing of ADC under application conditions is needed. The techniques discussed in the paper are very much useful in testing high-speed ADCs by the manufacturer as well as a circuit designer.

Keywords: Analog to Digital Converter, Dynamic Parameters, Multi Frequency Signal, Multipoint Interpolation

1. Introduction

Rapid advances in hardware technology in Very Large Scale Integration (VLSI) implementation of digital algorithms have encouraged the researchers in the field of instrumentation design and testing. Sine signal is considered as the fundamental element of all periodic and multi-frequency signal[1]. For analyzing the parameters of ADC with various algorithms, sine signal is primarily used. Combination of sine signals with different amplitudes and frequencies can also be considered as an input signal for ADC testing. Many methodologies have been suggested in the literature to evaluate the parameters of multi-frequency signals[2] it can be estimated either in the time domain or in the frequency domain. Time-domain models provide a high spectral resolution but require a great computational effort. Frequency domain methods employ fast Fourier algorithms. The main dif-

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ficulty encountered in this approach is the spectral leakage. It occurs even in the presence of tapered window, this means that every spectral component is spread over the whole frequency axis. A frequency domain technique for accurate real-time signal parameter measurement is I_pFFT[3]. This method provides a very accurate frequency, amplitude, and phase estimation and its best performances are obtained using the maximum side lobe decay windows. Many of the FFT based estimation methods assume coherent sampling, which is on the accurate synchronization of the signal and the ADC sampling rate. In the real situation for testing the ADC sampling may be non-coherent sampling, and may affect the estimation over tones.

An ideal ADC or quantize experience only quantization error, its spectrum comprises of spectral lines of stimulus and noise only due to quantization error called noise floor. While the actual ADC has a dc offset spectral line at zero frequency, spectral lines of stimulating signals, noise due to quantization error, harmonics due to presented integral nonlinearity and inter modulation distortion in case of multi-frequency stimulus[4]. A frequency domain procedure often used to compensate both spectral leakage and picket fence effect is I_pFFT. This method provides a very accurate estimation of the ratio of signal frequency to the sampling frequency by processing the DFT of the acquired data weighted by a suitable window sequence[5]. A combination of interpolation algorithm with tapered time window is implemented to overcome the deficiency of short range and long range leakage[6]. In this paper, a multi-frequency sine signal is simulated and applied to ideal and real life ADC. Amplitude and frequency of all the components presented in the input signal are estimated with FFT and 2-point IPFFT. The signal is weighted with suitable window and averaged spectrum is analyzed at the output end to estimate SNR, SINAD, and THD.

The paper is organized as follows: Section 2 presents the theoretical perspective for characterization of ADC under test and input signal to be applied. Section 3 describes the mathematical formulation of the parameters estimations using interpolation techniques with the windowed signal. Estimation of dynamic parameters of ADC has been shown in section 4 and simulation results are presented in section 5. Based on the results obtained conclusion is discussed in section 6.

2. Theoretical Prerequisites

2.1 Input Signal

Multi-frequency signal can be analytically expressed as the sum of a finite number of sine signals. In order to estimate the characteristics of this signal, spectrum using FFT is attained. Multi-frequency signal simulates the operating conditions of the real world. Mathematically it can be expressed as:

$$x(t) = x_1(t) + x_2(t) \ldots x_m(t) \tag{1}$$

Where m=1,2,3….M is number of sine components presented in the stimulus. This signal is uniformly sampled with the sampling frequency of f_s. Sampling time interval will be defined as: $\Delta t=1/f_s$. Thereforeeq. 1may be written as:

$$x(k\Delta t) = \sum_{m=1}^{M} A_m sin(2\pi f_m k\Delta t + \varnothing_m) \tag{2}$$

Where f_m,A_m and $\varnothing_{mare}$ fundamental frequency, amplitude and phase of m the component of the signal respectively. Total number of components is M, total number of samples is N, andK=0,1,2,….N-1.

When a Multi-frequency signal is considered, the individual frequency should be selected in a manner to get minimum interference errors on each component due to the contribution of leakage from all other components[7]. Selected sampled frequency should be at least double of the maximum frequency component of the input signal as per sampling theorem.

2.2 Analog to Digital Converter

An ADC is characterized as function of obtained digital output code and applied input signal. It can be modeled as the conversion of the analog signal into digital data with finite precision, called quantization. Finite precision itself persuades error known as quantization error. In ideal ADC, transition voltages are equally spaced with a measure equal to the least significant bit, which is not the case with actual ADC[8]. Known values of Integral Nonlinearity (INL) are inserted into ideal ADC[9]. These nonlinearities are the source of curvature in the ADC transfer characteristics. Nonlinearity creates harmonic distortion in the output spectrum. Figure 1 shows the characteristics of

ADC with nonlinearities. It presents the relation between the applied input signals and obtained the digital output. Arbitrary values of INL are introduced in the ideal ADC to make it real life ADC. In Multi-frequency signal relation between different frequencies and sampling frequency in absence of coherent relationship lead to spectral leakage. It can be reduced by weighing the discrete time domain sample with a sequence that is selected to be a real and symmetric function to obtain a symmetric reduction of the leakage called windowing[10]. Sampling the input signal in time domain to finite numbers of data points is equivalent to multiplying that signal by a rectangular time window with time width equal to total sampling time. Multiplication in the time domain is same as a convolution of the corresponding transforms in the frequency domain[11]. Since the leakage is directly related to discontinuities of the rectangular time window, a window with smoother edges will give better results. Hence the Hanning window can be considered due to its simplicity. Another advantage of using Hanning windows as compared to rectangular is that the roll-off rate at the side lobes is higher; therefore, the spectral leakage is limited to a smaller frequency range[12].

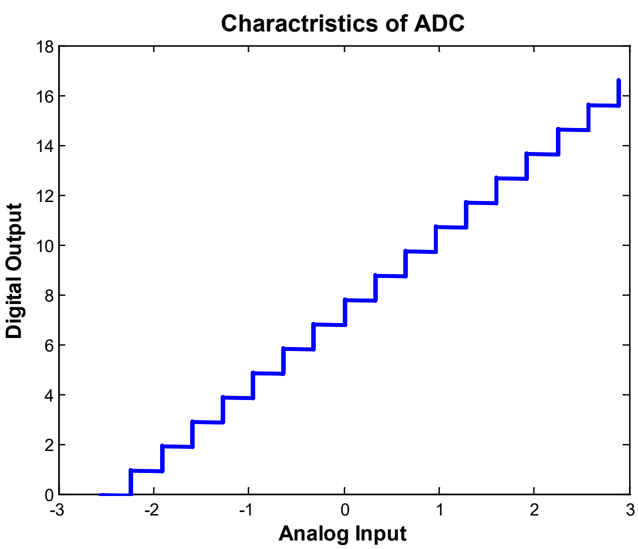

Figure 1. Characteristics of 5-bit real life ADC.

3. Estimation of Signal Parameters with Interpolation Technique and Windowing

Leakage due to discontinuities can be reduced by selecting proper window function. Along with this, characteristics of the applied signal should also be accurately determined. Special methods have been suggested to estimate the frequency, amplitude, and phase of the signal[3,6]. The Multi-frequency input signal considered in eq. (2) can also rewrite as:

$$x\left(\frac{K}{f_s}\right) = A_0 + \sum_{(m=1)}^{M} A_m sin\left(2\frac{f_m}{f_s}K + \varnothing_m\right) \tag{3}$$

$$K = 0,1,2\ldots N-1$$

Weighted signal is attained by multiplying the input signal with time domain window.

$$x_w(K\Delta t) = x(K\Delta t) \times x_w(K\Delta t) \tag{4}$$

DFT of weighted signal x_w (kΔt)at spectral line n is given by eq. (5).

$$\begin{aligned} X_w(k\Delta t) = A_0 W(n) - j0.5[A_m e^{j\varnothing_m} W((\lambda_m - n) f_0 \\ + A_m e^{-j\varnothing_m} W(\lambda_m - n) f_0)] \end{aligned} \tag{5}$$

Where W(f) is the spectrum of the selected time domain window, f_0 is defined as the frequency resolution and can be evaluated as the ratio of sampling frequency to the total number of samples $f_{s/N}$ and λ_m is the ratio of fundamental frequency to the frequency resolution.

$$\lambda_m = \frac{f_m}{f_0} = L_m + \delta_m \tag{6}$$

L_m is an integer which signifies the number of recorded m the component cycle. δ_m is the fractional part and its value lies in the interval $0 \le \delta_{m \le 1}$. It is measure of displacement due to non-coherency. The interval (L_m,$L_{(m+1)}$)covers for different fundamental frequencies, so that the largest two spectrum lines are and S(L_m) and S($L_{(m+1)}$)[12].

3.1 Rectangular Window

Spectrum of rectangular window:

$$W_R(f_m) = \frac{sin(\pi\lambda_m)}{sin\left(\frac{\pi\lambda_m}{N}\right)} e^{j\pi\frac{N-1}{N}\lambda_m} \tag{7}$$

Assume that,

$$a = \frac{\pi(N-1)}{N}$$

λ_m,L_m,δ_m have been calculated from eq. (6). Now, interpolation between two largest spectral components is to be carried out.

$$s(L_m) = -j0.5A_m \left[e^{j(a\delta_m+\varphi_m)} \frac{\sin\pi\delta_m}{\sin\frac{\pi\delta_m}{N}} \right] \quad (8)$$

$$s(L_m+1) = -j0.5A_m \left[e^{j(a(\delta_m-1)+\varphi_m)} \frac{\sin\pi\left(\delta_m-1\right)}{\sin\frac{\pi\left(\delta_m-1\right)}{N}} \right] \quad (9)$$

If a number of samples are equal to or greater than 1024, eq. (10) and eq. (11) can be approximated by replacing sine in the denominator by their respective arguments[13].

$$s(L_m) = 0.5A_m \left[\frac{|\sin\pi\delta_m|}{\frac{\pi\delta_m}{N}} \right] \quad (10)$$

$$s(L_m) = 0.5A_m \left[\frac{|\sin\pi\delta_m|}{\frac{\pi\delta_m}{N}} \right] \quad (11)$$

The ratio of two magnitudes can be expressed as:

$$\alpha_m = \frac{|s(l_m+1)|}{|s(l_m)|} \quad (12)$$

$$\delta_m = \frac{\alpha_m}{\alpha_m+1} \quad (13)$$

3.1.1 Frequency (f)

Frequency components of Multi-frequency signal can be computed as:

$$f_m = (L_m+\delta_m)f_0 \quad (14)$$

Frequency of h[th] harmonic of m[th] fundamental frequency can be considered as:

$$f_{hm} = h \times f_m \quad (15)$$

h = 1, 2, 3…..
From eq. (16) frequency of harmonics can be calculated.

$$f_{hm} = \lambda_{hm} f_0 \quad (16)$$

Where, $\lambda_{hm} = h\lambda_h$ value of λ_{hm} lies between h and h+1 and frequency can be calculated with the same procedure.

3.1.2 Amplitude (Am)

From eq. (17) and eq. (18) amplitudes of signal components can be calculated respectively,

$$A_m = \frac{2\pi\delta_m}{N} \frac{|s(L_m)|}{|\sin\pi\delta_m|} \quad (17)$$

$$A_m = \frac{2\pi(1-\delta_m)}{N} \frac{|s(L_m+1)|}{|\sin\pi(1-\delta_m|} \quad (18)$$

Eq. (18) is recommended to get more accurate results only if the spectral line at L_{m+1} is greater than L_m.

3.1.3 Phase (Øm)

The phase of fundamental frequencies can be calculated using eq. (19) and eq. (20).

$$\varnothing_m = \text{phase}(S(L_m)) - a\delta_m + \frac{\pi}{2} \quad (19)$$

$$\varnothing_m = \text{phase}(S(L_m)) - a\delta_m + \frac{\pi}{2} \quad (20)$$

Phase and amplitudes of harmonics can be computed with the same procedure as calculated for the fundamental.

3.2 Hanning Window

Hanning window is a good compromise between main lobe width, which determines frequency resolution and side lobe level, which governs spectral leakage. Spectral leakage is the main drawback of the rectangular window.

More accurate results can be expected with the combination of the tapered time window for long range leakage and Interpolation algorithm for reduction of short-range leakage[14]. From such a combined approach an accurate determination of the characteristics of a Multi-frequency signal from its sampled image is expected[15-17].

Spectrum of Hanning window:

$$\varnothing_m = \text{phase}(S(L_m)) - a\delta_m + \frac{\pi}{2} \quad (21)$$

Where δ_m is $[\![0 \le \delta]\!]_{m<1}$ corresponding to the symmetrical shape of the Hanning window, a relative maximum in the discrete spectrum is located at L_m Δf if δ_m<0.5 and $[\![(L]\!]_{m+1})\Delta f$ if δ_m>0.5[8].

$$\alpha_H = \frac{|S_H(L_m+1)\Delta f}{|S_H(L_m)\Delta f} \quad (22)$$

$$\delta_H = \frac{(2\alpha_H - 1)}{\alpha_H + 1} \quad (23)$$

3.2.1 Frequency (fm)

Frequency components of Multi-frequency signal can be computed as (eq.24):

$$f_m = (L_m \times \delta_m) f_0 \quad (24)$$

Frequency of h[th] harmonic of m[th] fundamental frequency can be considered as (eq. 25):

$$f_{hm} = h \times f_m \quad (25)$$

From (6)

$$f_{hm} = \lambda_{hm} f_0 \quad (26)$$

3.2.2 Amplitude

Amplitude can be calculated from either of the equations written below, but for more accuracy spectral line corresponding to the larger one is recommended.

$$A_{mH} = \frac{2\pi\delta_m(1-\delta_m)}{\sin(\pi\delta_m)} e^{\pi i \delta_m}(1+\delta_m) S_H(L_m \Delta f) \quad (27)$$

$$A_{mH} = \frac{2\pi\delta_m(1-\delta_m)}{\sin(\pi\delta_m)} e^{\pi i \delta_m}(1+\delta_m) S_H(L_m+1)\Delta f \quad (28)$$

3.2.3 Phase ($\varnothing_m$)

The technique as described for the rectangular window is also used to determine the phase for Hanning window.

$$\varnothing_m = \text{phase}(S_H(L_m)) - \text{a}\,\delta_m - 1 + \frac{\pi}{2} \quad (29)$$

$$\varnothing_m = \text{phase}(S_H(L_m+1)) - \text{a}(\delta_m - 1) + \frac{\pi}{2} \quad (30)$$

4. ADC Parameters Estimation

The output spectrum displays amplitude of the various harmonics and noise components of a digitized signal. The harmonics of the input signal can be distinguished from other distortions by their location in the frequency spectrum and ADC parameters based on the obtained values are estimated.

4.1 Signal to Noise Ratio

SNR can be defined as the ratio of the Root Mean Square (RMS) of signal amplitude to the mean value of the root-sum-squares RSS) of all other spectral components, excluding the harmonics and DC. It is a comparison of a level of signal power to the level of noise power and is most often expressed as a measurement of decibels (dB) Reference No. 14. Mathematically it can be defined as:

$$\text{SNR} = (\text{RMS}_{\text{signal}})/(\text{RMS}_{\text{noise}}) \quad (31)$$

4.2 Total Harmonic Distortion

THD is a measure of the degree of harmonic content in the signal. In case of the Multi-frequency signal, it can be defined as Total Harmonic Distortion (THD) is the ratio of RMS value of the fundamental signal to the mean value of the root sum square of its harmonics[15].

$$THD^2 = \sum_{(j=1)}^{m} \sum_{(n=2)}^{L} \frac{|Y_j(n)|^2}{|Y_j(1)|} \quad (32)$$

Where j is the number of components of the input signal, n is the number of harmonics considered. Y is the Fourier transform of the corresponding signal.

4.3 Signal to Noise and Distortion

Signal-to-Noise-and-Distortion (SINAD) is the ratio of the RMS of signal amplitude to the mean value of the RSS of all other spectral components, including harmonics, but excluding dc. SINAD is a good measure of the overall dynamic performance of an ADC because it includes all components which make up noise and distortion.

5. Simulation Results

To estimate the dynamic parameters of ADC a Multi-frequency signal is taken. Test frequencies are selected that guarantees the phases of the sampled values uniformly distributed between 0 and 2π. High-frequency multi-tone signal with a sampling frequency of 1MHz and a test frequency of components equal to 26377.9142Hz, 102533.8366Hz, have been simulated[16,17]. 8-bit ideal and real life bipolar ADC is simulated with full-scale voltage of 5.12V. Total number of samples is 4096. Rectangular and Hanning window of size 1024 samples are simulated, hence total number of windows will be 4. Initial phases of all components are assumed to be zero.

Figure 2 shows the frequency spectrum of ideal ADC with rectangular windowed multi-frequency input signals. It is averaged for all four windows to reduce the noise. Peak values are a measure of amplitudes of tones of the input signal with FFT. More accurate estimation of amplitude and frequency can be obtained with I_pFFT technique. Figure 3 presents the spectrum with Hanning windowed signal. Less amount of leakage can be observed in Figure 4-6 shows the average spectrum for actual ADC with rectangular and Hanning windowed input signal. Compared to the spectrum of ideal ADC, more harmonics are observed with actual ADC. These harmonics are presented due to inserted nonlinearity in the actual ADC. Using these spectrums frequency, amplitude and phase of signal and harmonics have been estimated with 2- point interpolation method and shown in Table 1. Based on the estimated values of frequency and amplitude, dynamic parameters of ADC are estimated as mentioned in Table 2.

Figure 2. Test set up.

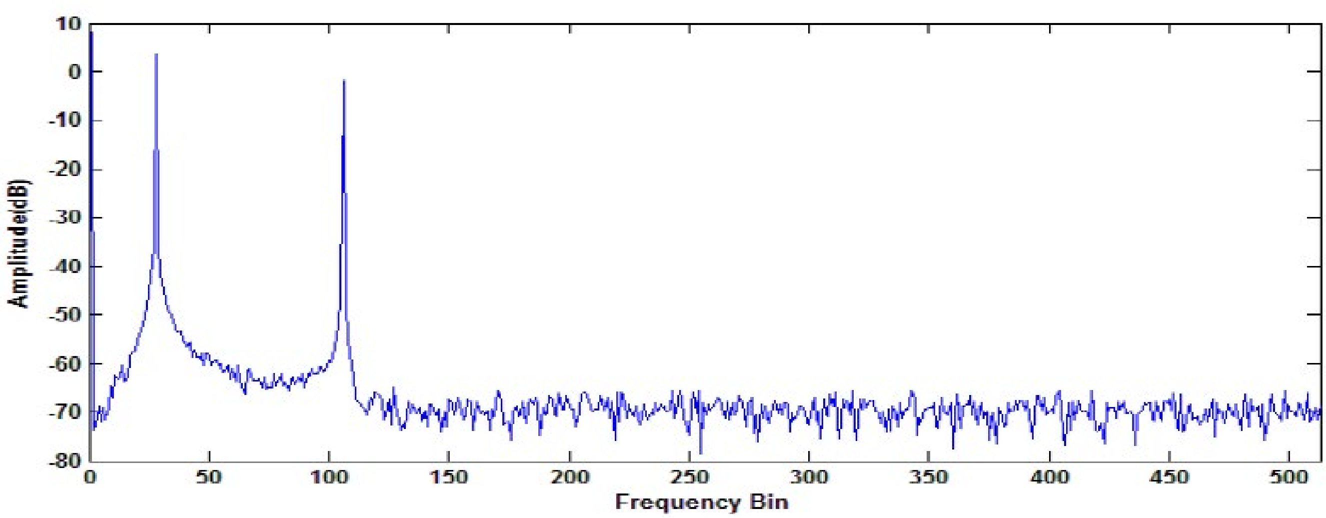

Figure 3. Frequency spectrum of Ideal ADC stimulated with rectangular windowed multi frequency signal.

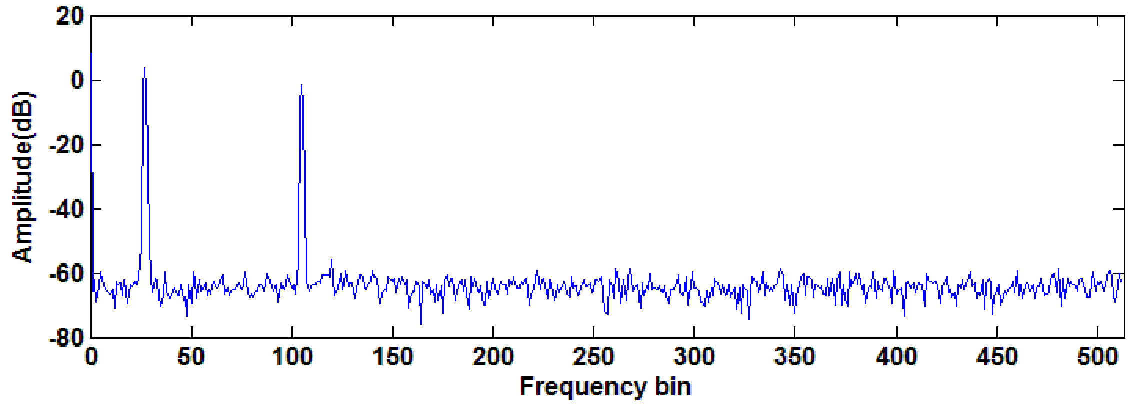

Figure 4. Frequency spectrum of Ideal ADC stimulated with hanning windowed multi frequency signal

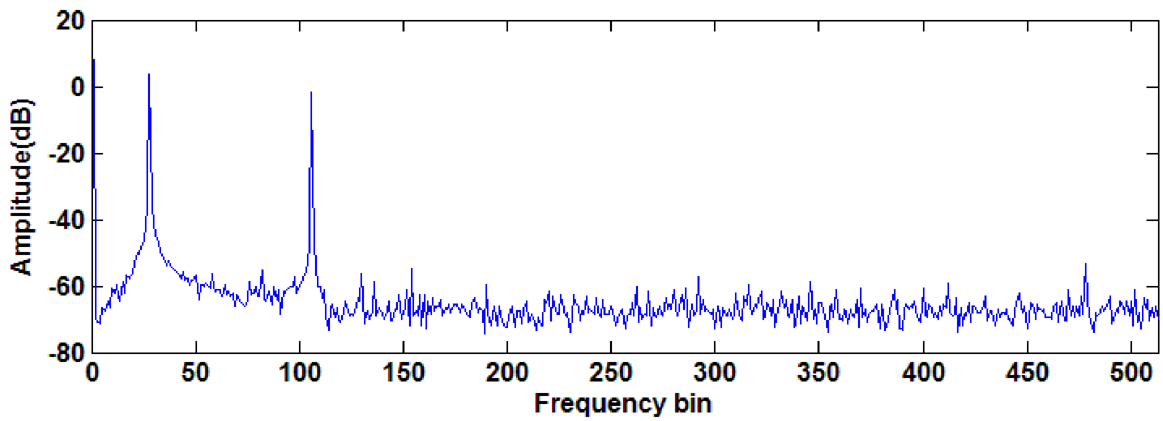

Figure 5. Frequency spectrum of actual ADC stimulated with rectangular windowed multi frequency signal

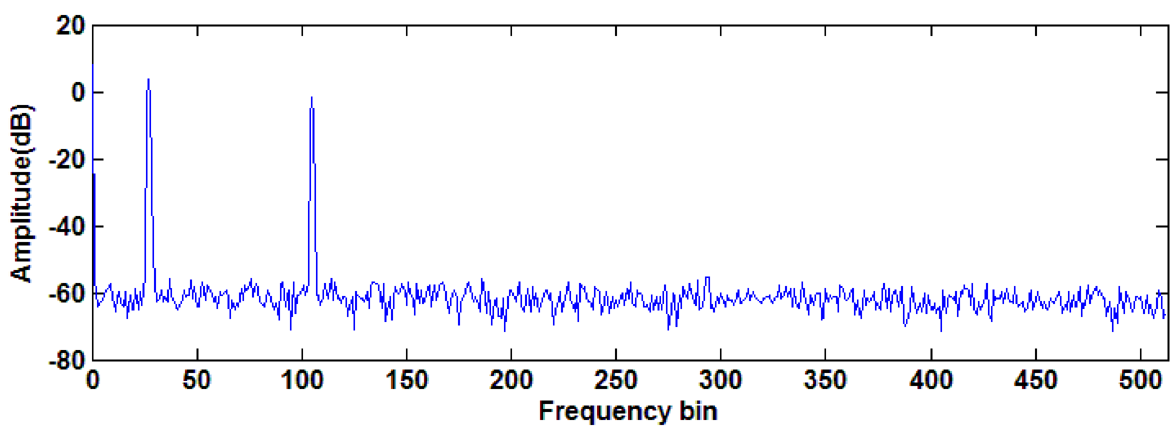

Figure 6. Frequency spectrum of actual ADC stimulated with Hanning windowed multi frequency

Table 1. Estimated values of input parameters using FFT and IPFFT

Sl. No.	Parameters	Assumed values	Estimated values interpolated method	
			Rectangular window	**Hanning window**
1.	Frequency	26377.9142	26367.1875	26367.1875
		102533.8366	102539.0625	102539.0625
2.	Amplitude	1.5 V	1.49938	1.498418
		0.8V	0.79966	.79920418
3.	Phase	0	-1.6052	-1.5338
		0	-1.5538	-1.5875

Table 2. ADC dynamic parameters estimation

	Ideal ADC		Real life ADC	
	Rectangular	**Hanning**	**Rectangular**	**Hanning**
THD (dB)	-53.12	-50.620	-49.028	-46.024
SNR (dB)	71.389193	68.8886	66.9168	59.1807
SINAD (dB)	71.08491	68.5843	67.1071	59.3694

6. Conclusion

Testing of ADC is the process of characterization of ADC, which includes the estimation of static and dynamic parameters of the device. Static parameter Gain, offset and nonlinearity are estimated with histogram method. For estimation of dynamic parameters, frequency domain based methods are used. In this paper interpolation method is used for dynamic parameters estimation of ADC. SNR, THD, and SINAD are important parameters of an ADC. The interpolated method offers more accurate results compared to FFT method with Hanning window. The overall work is carried out has made a significant contribution to ADC testing. The results presented here are based on computer simulation. The algorithm developed is equally suitable for real-life dynamic testing of ADC. The nonlinearity can contribute to harmonic distortion, while the sampling jitter, the quantization noise, and the thermal noise contribute to the noise floor of the digitizing system.

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