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High Resolution Methods for Angle of Departure (AOD) and Angle of Arrival (AOA) Estimation in Bistatic Multiple-Input-Multiple-Output (MIMO) Radar Systems

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Abstract

Objectives: In this paper we make a comparative study between two adapted methods for the estimation of the angle of departure and the angle of arrival in a Bistatic multiple input multiple output radar system. **Methods:** We have adapted the high resolution methods known in the literature to the estimation of the parameters of the sources with a Bistatic radar equipped with an antennas array on the transmit station and on that of the receive one. The two methods are a Derived Multiple Signal Classification method (D-MUSIC) and the Derived Propagator method (D-Propagator). **Findings:** Through simulation results we have conducted a comparative study between the different adapted methods. We take advantage of each one for different pçolpscenarios in terms of accuracy and resolution.The two proposed methods showed good performances even for low signal-to-noise ratio. In addition, the parameters of the radar systems have a great influence on the performance of the estimators. For this reason, a thorough study on the precision of the high resolution methods according to these parameters was made. **Application:** By varying the Radar Cross Section we noticed the superiority of D-MUSIC method compared to D-Propagator method for a limited reflection surface what was not the case in the other scenarios where the propagator showed remarkable superiority.

Keywords: AOA, AOD, Bistatic Radar, -MUSIC Method, D-Propagator Method, MIMO

1. Introduction

Several works have dealt with high resolution methods and have shown their precision and performance in several areas of source localization[1–4].

These methods, for example, MUSIC, Propagator, Esprit, have been adapted to several applications, such as parasitic antennas[5], UWB sources[6] and military applications. This paper presents the adaptation of Propagator and MUSIC methods for Radar Bistatic applications.

The search in the field of estimating the angle of departure and the angle of arrival by a Bistatic radar system with multiple inputs and multiple outputs has drawn a great interest in the last decades[7–9].

In addition, since the Doppler phenomenon is an important parameter in radar systems, several works have proposed methods for estimating the Doppler frequency simultaneously with the estimation of the angles ofparrival[7–10].

In[11] authors used thepMUSIC method for angles of arrival estimation on Bistatic radar system and compared it to the ESPRIT method.

In[12] authors presented an AOD and AOA estimation method using a circular array structures. They transformed

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thesteeringvector of the UCA into a steering vector with a Vandermondep structure by using the Jacobi-Anger expansion.

In this paper, we present a comparative study between two high resolution methods for estimating angles of departure and angles of arrival in a bistatic MIMO radar system. To do so, we started by adapting the two methods to the new system and doing minimization and optimization to the new signal subsystem.

On the other hand, we have chosen two methods that treat the covariance matrix differently. The MUSIC method based on the eigenvalue decomposition of this matrix and the propagation method based on a linear processing of the covariance matrix.

The results given by simulation on MATLAB treat the resolution and the precision of the two methods in different scenarios. In addition, the influence of the parameters of the radar system, such as the RCS and the number of elements on the transmission and the reception antennas arrays, will be treated and conclusions will be drawn.

The rest of the paper is organized as follows: Section 2 presents the data model on the bistatic MIMO system. Sections 3 elaborate the proposed derived MUSIC and Propagator methods. Simulation results and discussion are provided in Section 4pand Section 5 concludes the paper.

These superscripts: (.)T, (.)*, (.)H and (.)-1 presents the matrix transpose, conjugate, hermitian and inverse operators, respectively.

2. Bistatic MIMO Radar Data Model

The bistatic MIMO radar is equipped by two linear antennas array for the transmit station and the receive one placed on the y axis. The number of elements in the antennas array for each station is M and N, respectively. The distance between two successive antennas is uniform and equal to d. We suppose have K targets on the y-z plane with AOD and AOA of the kth target are (θ_k, φ_k), respectively.

The output signal at the receiver can be expressed as:

$$y(t) = [a_r(\varphi_1) \otimes a_t(\theta_1), \ldots, a_r(\varphi_k) \otimes a_t(\theta_k) \ldots.., a_r(\varphi_K) \otimes a_t(\theta_k) s(t) + n(t) \quad (1)$$

Where are and at are the steering vectors of the receive station and the transmit station, respectively.

S (t) is the amplitudes and phases of the K sources at time t and n (t) is Gaussian white noise with zero mean.

⊗ Is the Kronecker product.

3. Derived Methods for AOD and AOA Estimation

3.1 Derived MUSIC Method

The MUSIC method[1] was invented by Schmidt in 1986. In this paper, AOD and AOA are determined from the Eigen values decomposition of the covariance matrix given as:

$$R = y(t) * y(t) H \quad (2)$$

The covariance Matrix R can also be expressed as:

$$R = E_S^H E_S D_S + E_n^h E_n D_n \quad (3)$$

H denotes the conjugate-transpose.

D_S and D_n are the diagonal matrixes containing the K highest eigenvalues and the MN-K lower eigenvalues, respectively. Es contains the eigenvectors of the K eigenvalues of R, while En represents the matrix including the remainder of eigenvectors. Note that Es and En can be regarded as the subspaces of the signal and the noise, respectively.

If we consider that the noise is zero the expression of the signal subspace becomes:

$$E_S = {}^\wedge F = [a_r(\varphi_1) \otimes a_t(\theta_1), \ldots\ldots a_r(\varphi_k) \otimes a_t(\theta_k)] \quad (4)$$

Where F is a full-scale K × K matrix.

From Equation (4) we obtain:

$$\wedge = E_S F^{-1} \quad (5)$$

The signal subspace will be structured as:

$$\hat{F}\hat{\wedge} = \arg\min || \wedge - \hat{E}_S F^{-1} ||^2 \quad (6)$$

$\hat{E}_S$ Is an estimation of ? ES

We can denote the subspace fitting as:

$$\hat{F}, \hat{\wedge} = \arg\min \operatorname{tr} (\wedge^H \Pi_{\hat{E}_S}^{\perp} \wedge) \quad (7)$$

Where $\Pi_{\hat{E}_S}^{\perp} = I_{MN} - \hat{E}_S(\hat{E}_S^H\hat{E}_S)^{-1}\hat{E}_S^H = I_{MN} - \hat{E}_S^H\hat{E}_S$.

The minimization of Equation (7) gives:

$$a_r(\varphi), a_t(\theta_1) = \arg\min \sum_{k=1}^{k}[a_r(\varphi_k)\otimes a_t(\theta_k)]^H \Pi_{\hat{E}_S}^{\perp} a_r(\varphi_k)\otimes a_t(\theta_k \quad (8)$$

The minimization for (8) can be obtained via searching the deepest K minimum in the following criterion:

$$\begin{aligned} V(\varnothing,\theta) &= [a_r(\varnothing)\otimes a_t(\theta)^H \Pi_{\hat{E}_S}^{\perp}[a_r(\varnothing)\otimes a_t(\theta)] \\ &= a_t(\theta)^H[a_r(\varnothing)\otimes I_M]^H \Pi_{\hat{E}_S}^{\perp}[a_r(\varnothing)\otimes I_M]a_t(\theta) \\ &= a_t(\theta)^H Q(\varnothing)a_t(\theta) \end{aligned} \quad (9)$$

Where $Q(\varnothing) = [a_t(\varnothing)\otimes I_M]^H \Pi_{\hat{E}_S}^{\perp}[a_r(\varnothing)\otimes I_M]$

We added the constraint that:

$$g^T a_t(\theta) = 1 \quad (10)$$

Where $g^T = [1,0,\ldots 0]^T \in R^{I\times 1}$

The optimization problem can be modeled with the linear constraint minimum variance solution.

$$\text{Min } a_t(\theta)^H Q(\varphi)a_t(\theta), g^T a_t(\theta) = 1 \quad (11)$$

The estimated angles of arrival are given as a solution of (11):

$$\hat{\varphi} = \arg\min \frac{1}{g^T Q(\varphi)^{-1} g} = \arg\min g^T Q(\varphi)^{-1} g \quad (12)$$

On the same way we can estimate the angles of departure.

$$\begin{aligned} V(\varnothing,\theta) &= [a_r(\varphi)\otimes a_t(\theta)^H \Pi_{\hat{E}_S}^{\perp}[a_r(\varphi)\otimes a_t(\theta)] \\ &= a_r(\theta)^H[I_N\otimes a_t(\theta)]^H \Pi_{\hat{E}_S}^{\perp}[I_N\otimes a_t(\theta)]a_r(\varphi) \\ &= a_r(\theta)^H P(\varphi)a_r(\theta) \end{aligned} \quad (13)$$

Where $P(\theta) = [I_N\otimes a_t(\theta)]^H \Pi_{\hat{E}_S}^{\perp}[I_N\otimes a_t(\theta)]$

3.2 Derived Propagator Method

The idea of adapting the high-resolution Propagator method[3] to the estimation of departure angles and angles of arrival in a Bistatic radar system is based on the extraction of the signal subspace with a linear partitioning of the covariance Matrix.

Then the same steps described above on the derived MUSIC method will be applied to the Propagator method to estimate the departure angles and the angles of arrival.

The idea is based on the partitioning of the covariance matrix:

$$R = \left[\frac{X1}{X2}\right]_{MN-H}^{H} \quad (14)$$

X1 is the non-singular matrix of dimension H * (H * MN) which contains H independent lines of R and X2 is the non-singular matrix of dimension H * (MN-H).

Then the Propagator operator is given as:

$$P*X1 = X2 \quad (15)$$

The extraction of the noise subspace is done by the creation of the flowing matrix:

$$Q = [P, Id\,[M\text{-}N]] \quad (16)$$

Where Id is the identity matrix

On the same way Equations (7) to (13) are applied to estimate the departure and the arrival angles.

4. Comparative Study Results

In this section, we will validate the good performances of the two methods via simulations on MATLAB. A comparative study will be made in different scenarios while varying parameters of the bistatic radar system. The transmitted signals (t) have the flowing expression:

$$s(t) = \beta_k e^{j2\pi fkt/f_s} \quad (17)$$

With βk, fk and fs being the RCS, Doppler frequency and the pulse repeat frequency, respectively.

We consider in all simulations a Doppler frequency fk = 500 Khz and a pulse repeat frequency fs = 2.4 GHz. Figures 1 and 2 present the estimation of the angle of departure and the angle of arrival of a source, respectively. The source is localized at an AOD = -20° and an AOA = 30°. In the simulation we use a snr = 10 dB, 100 realizations, 100 observations on the antennas arrays and 10 elements in every antennas array (M = N = 10). We can notice

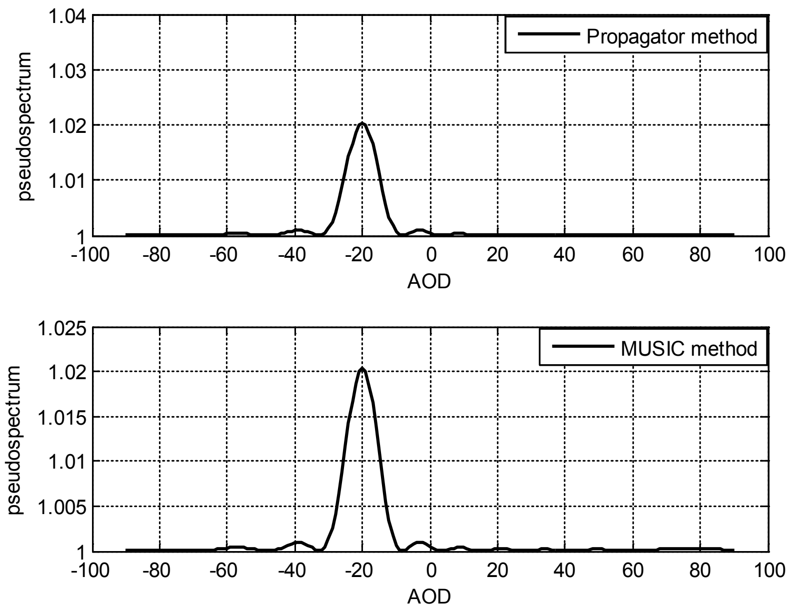

Figure 1. Estimation of the AOD of a source located at -20°.

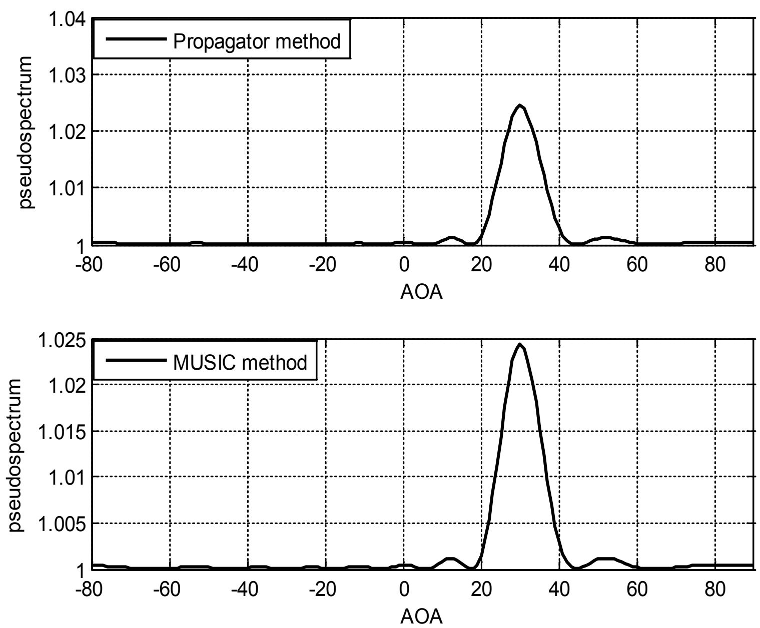

Figure 2. Estimation of the AOA of a source located at 30°.

that max of pseudo spectrums coincide very well with the real AOD and AOA of the source and that valid by consequence the good working of the two developed methods.

In Figures 3 and 4, we have minimized the number of elements constituting the antenna arrays in the reception and the transmission with M = N = 5. We can notice a superior performance of the D-Propagator method compared to the D-MUSIC method especially for low signal-to-noise ratios. We have retained the same parameters used in Figures 3 and 4.

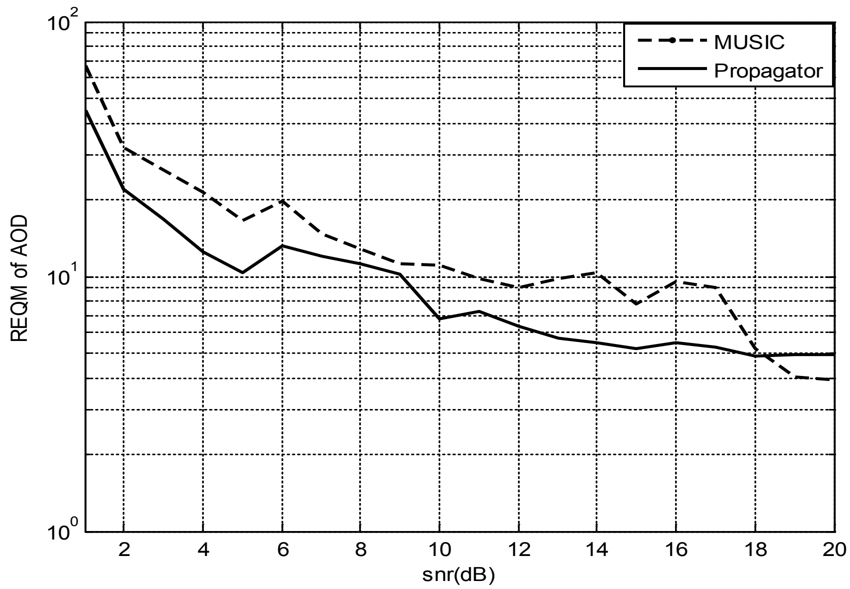

Figure 3. RMSE of estimation of AOD with various snr, M = N = 5, snaps = 500.

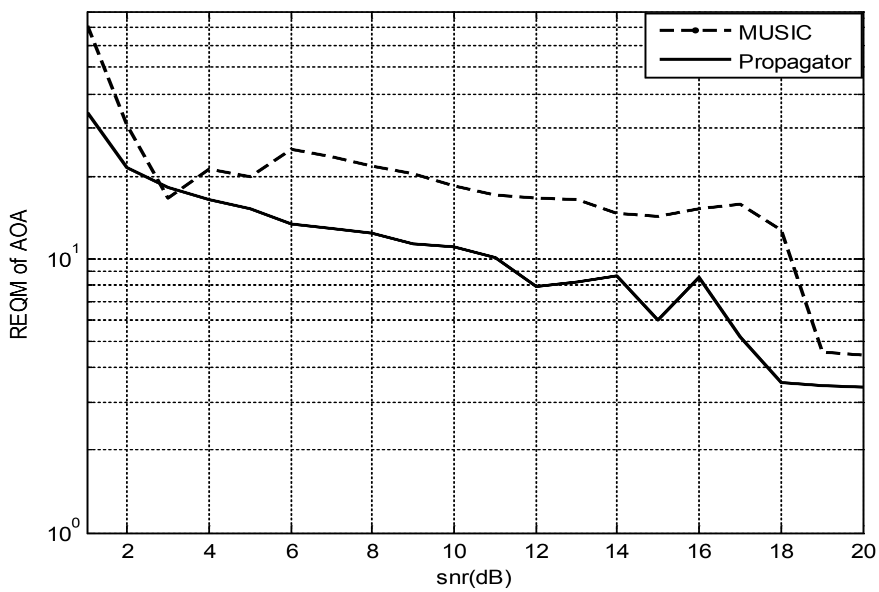

Figure 4. RMSE of estimation of AOA with various snr, M = N = 5, snaps = 500.

The Radar Cross Section (RCS) is an inherent physical property of objects indicating the relative importance of the reflection surface of an electromagnetic beam that it causes. In Figures 5 and 6 we will study the influence of this parameter on the quality of the estimation of the departure and arrival angles of a source located at an AOD = 20° and AOA = 60°. To do it, we varied the RCS from $0.01cm^2$ to 1 cm^2 in Figure 5 and from 1 cm^2 to 100 cm^2 in Figure 6. It is very clear the superiority of the D-MUSIC method compared to D-Propagator in the two scenarios.

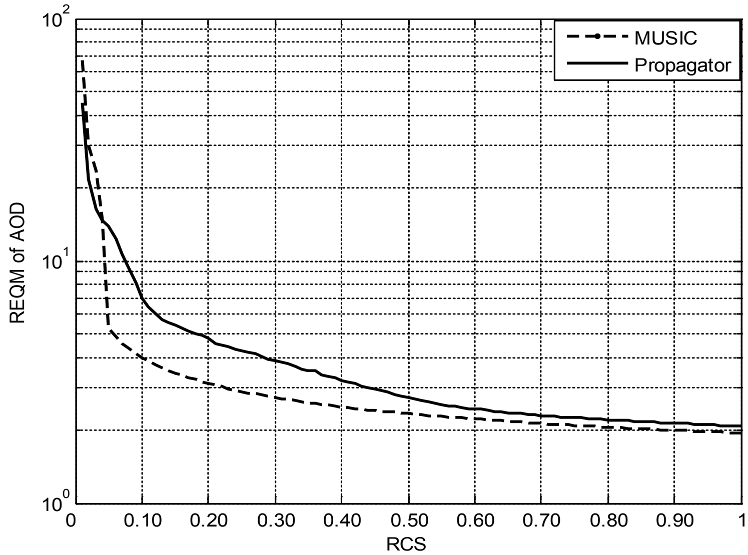

Figure 5. RMSE of estimation of AOD with various RCS, M = N = 5, snaps = 500.

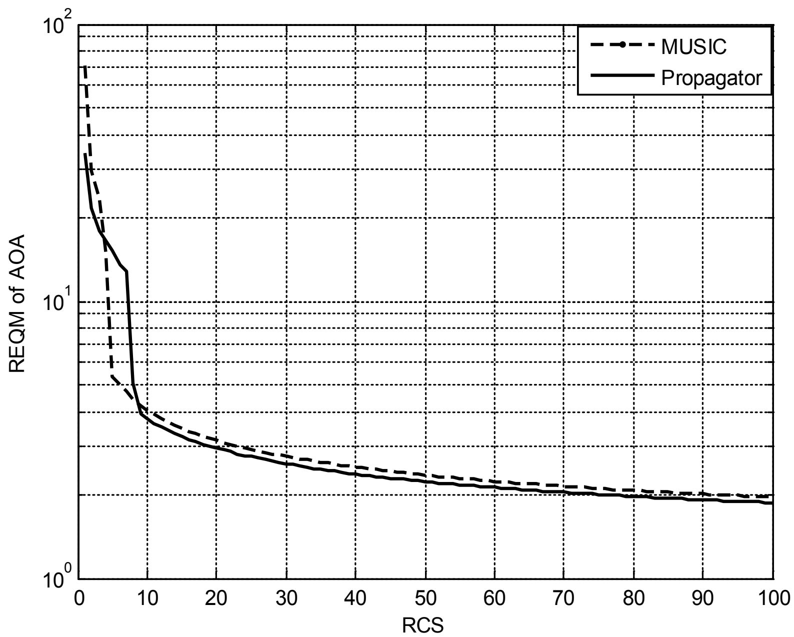

Figure 6. RMSE of estimation of AOA with various RCS, M = N = 5, snaps = 500.

5. Conclusions

In this paper we have presented the derivation of two high resolution methods for the estimation of the angles of departure and angles of arrival of sources using a bistatic radar system with multiple inputs and multiple outputs. The two methods have shown good accuracy in different scenarios via simulations. On the other hand, by varying the parameters of the system, we noticed the superiority of the D-Propagator method compared to D-MUSIC for a limited number of elements on the emission and reception antenna arrays. The situation changed by varying the RCS and we noticed the superiority of D-MUSIC method compared to D-Propagator method for a limited reflection surface.

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