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An algorithm for radial distribution power flow in Complex mode including voltage controlled buses

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Abstract: This paper presents an efficient algorithm to solve the radial distribution power flow problem in complex mode. The relationship between the complex branch powers and complex bus powers is derived as a non singular square matrix known as element incidence matrix. The power flow equations are rewritten in terms of a new variable as linear recursive equations. The linear equations are solved to determine the bus voltages and branch currents in terms of new variable as complex numbers. The advantage of this algorithm is that it does not need any initial value and easier to develop the code since all the equations are expressed in matrix format. It is tested on the distribution systems available in the literature. This proposed method could be applied to distribution systems having voltage-controlled buses also. The results prove the efficiency of the proposed method.

Keywords: radial distribution power flow, element incidence matrix, transmission loss, linear recursive equations.

Notations

N-no of buses

I_{ij} -Branch current flowing through element ij

I_j -Bus current of node j

V_j -Bus voltage of node j

S_{ij} -Complex power flowing from node i to node j

S_{ji} -Complex power flowing received at node j from node i

S_j -Specified Bus power at bus j

Z_{ij} -Impedance of element ij

TL_{ij} -Transmission loss of element ij

Introduction

The distribution systems are characterized by their prevailing radial nature and high R/X ratio. This renders the load flow problem ill conditioned. So many methods [1-8] have been developed and tested ranging from sweep methods, to conic programming formulation. Early research indicated that standard load flow methods fail to converge for ill-conditioned test systems [9]. The basis for the all the sweep methods is that they need an initial value (normally flat) for the voltages and the updating is done in forward and backward way implementing the kirchoff's equations. Expósito and Ramos [8] have proposed a radial load flow technique based on solving a system of equations in terms of new variables and using the Newton approach. Conic programming formulation is [9] to model the power flow problem as a conic optimization problem in terms of new variables. Both the methods the number of variables to be determined is 3N for an N bus radial system.

This paper exploits the radial structure of the distribution network and the relationship between the bus powers and branch powers is expressed as a non-singular square matrix known as element incidence matrix. The paper is organized into four sections. The first section describes the conventional radial distribution power flow problem, the second section derives the proposed method for PQ buses, third section deals with treatment of voltage-controlled buses and the fourth section discusses the simulation results. Radial systems of 12, 28, and 69 buses are considered for simulation.

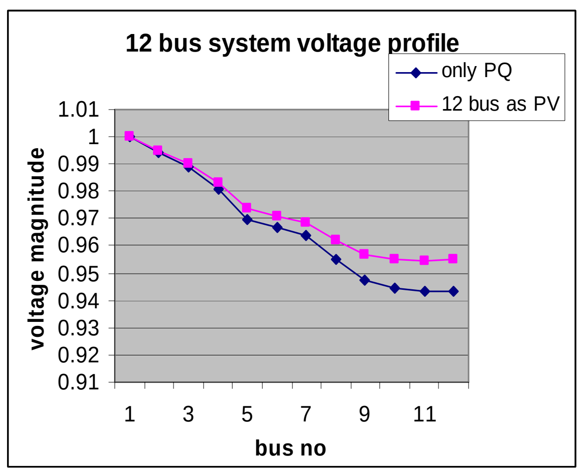

Fig. 1. 12 bus system voltage profile

Distribution power flow

The power flow equations for a radial distribution system are derived as the relationship between the specified complex bus powers and the bus voltages. Let S_{ij} is the complex power flowing from bus 'i' to bus'j'

$$S_{ij} = P_{ij} + iQ_{ij} = V_i (V_i^* - V_j^*) Y_{ij}^* \quad (1)$$

The 'i'th bus powers are expressed as

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$$P_i + Q_i = \sum_{i \in k(i)} P_{ij} + iQ_{ij} = \sum_{i \in k(i)} V_i (V_i^* - V_j^*) Y_{ij}^* \qquad (2)$$

Table.1. 12 bus Solution

	(Only PQ)		12 bus as PV	
Node	Voltage magnitude	Voltage angle Degrees	Voltage magnitude	Voltage angle Degrees
1	1	0	1	0
2	0.994333	0.116247	0.994904	0.090972
3	0.989031	0.223391	0.990219	0.170369
4	0.980579	0.402206	0.982855	0.299179
5	0.969825	0.628698	0.973743	0.447837
6	0.966538	0.697903	0.971015	0.489922
7	0.963751	0.75836	0.968739	0.525348
8	0.955312	1.011361	0.96219	0.656731
9	0.947279	1.242232	0.956566	0.730573
10	0.944463	1.317986	0.954967	0.725723
11	0.943565	1.341679	0.954699	0.707383
12	0.943356	1.348756	0.955	0.680377

k(i) is the set of nodes connected to node i, and Pi /Qi denote the real/reactive power at node i. The complex non linear equations (2) are to be solved to determine the bus voltages. The real and imaginary parts of the equations are separated and solved using numerical methods

Formulation of proposed method for load buses

The basis for the proposed method is that an N bus radial distribution network has only N-1 lines (elements) and the branch currents (powers) can be expressed in terms of bus currents (powers). For an element ij connected between nodes ‘i’ and ‘j’ the bus current of node j can be expressed as a linear equation.

$$I_j = I_{ij} - \sum I_{jk(j)} \qquad (3)$$

k(j) is the set of nodes connected to node j. For the slack bus the power is not specified so it is excluded and the relationship between bus currents and branch currents are derived as a non-singular square matrix.

$$I_{bus} = K.I_{branch} \qquad (4)$$

$$I_{bus} = [I_{b2} \ \mathrm{I}_{b3} \ .. \ \mathrm{I}_{bn}]^T \qquad (5)$$

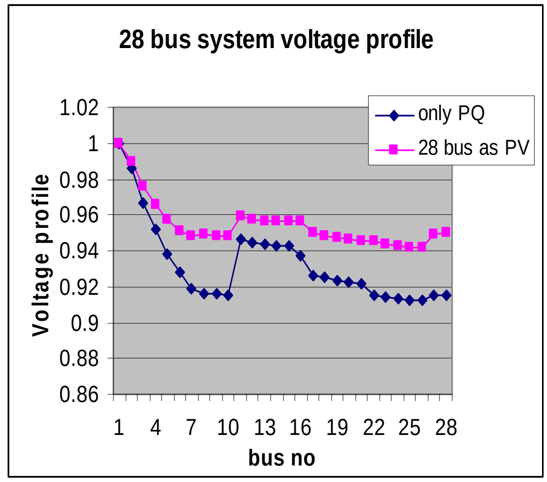

Fig. 2. Voltage profile of 28-bus system

The matrix K is element incidence matrix. It is a non singular square matrix of order N-1.The elemental incidence matrix is constructed in a simple way same like bus incidence matrix. In this matrix K each row is describing the element incidences. The elements are numbered in conventional way i.e. the no of element ‘ij’ is j-1.

1. The diagonal elements of matrix K are one. The variable j is denoting the element number.

K(j, j) =1

2. For each ‘j’th element let m (j) is the set of element numbers connected at its receiving end.

K(j, m(j)) = -1

3. All the remaining elements are zero. It can be observed that all the elements of matrix K below the main diagonal are zero

$$I_{branch} = K^{-1} I_{bus} \qquad (6)$$

The relationship between the branch currents and bus currents can be extended to complex branch powers and bus powers. The sending end power and the receiving end powers are not same due to transmission loss. The transmission loss is included as the difference between the sending end/receiving end powers.

The relationship between branch powers and bus powers is established in same way of bus/branch currents. Multiplying both sides by element incidence matrix K

$$S_{bus} = K[.S_{brnach}^{sending} - TL_{branch}] \qquad (7)$$

$$S_{branch} = K^{-1}.S_{bus} + TL_{branch} \qquad (8)$$

The power flow equations are complex quadratic equations .A new variable Rij is introduced for each element ‘ij’ and the equations becomes recursively linear.

$$R_{ij} = V_i (V_i^* - V_j^*) \qquad (9)$$

The branch power of 'ij' th element is expressed in terms of Rij

Table 2. Power flow solution of 28 bus system

	Only PQ buses		28 bus as PV	
Node	Voltage magnitude	Voltage angle (Degrees)	Voltage magnitude	Voltage angle (Degrees)
1	1	0	1	0
2	0.986219	0.146014	0.9899639	-0.042667
3	0.966452	0.361889	0.9758035	-0.118909
4	0.952347	0.520975	0.9657681	-0.179095
5	0.938183	0.685113	0.9572787	-0.331986
6	0.927651	0.810164	0.9513973	-0.472848
7	0.91848	0.921138	0.9479441	-0.696673
8	0.916023	0.951358	0.9489848	-0.875462
9	0.915742	0.95482	0.9487143	-0.872236
10	0.915497	0.957847	0.9484774	-0.869415
11	0.946157	0.678832	0.9596653	-0.025623
12	0.944387	0.724373	0.9579209	0.018643
13	0.94333	0.751628	0.9568787	0.0451318
14	0.94305	0.758847	0.9566028	0.0521482
15	0.94281	0.76505	0.9563657	0.058176
16	0.937053	0.714454	0.9561718	-0.3038
17	0.925866	0.856874	0.949657	-0.42844
18	0.924889	0.882505	0.9487047	-0.40408
19	0.92322	0.926466	0.9470776	-0.36231
20	0.922361	0.949121	0.9462404	-0.34077
21	0.921726	0.965878	0.9456218	-0.32485
22	0.915598	0.997398	0.9451531	-0.62509
23	0.914061	1.038243	0.9436643	-0.58676
24	0.912885	1.069554	0.9425252	-0.55739
25	0.912625	1.076475	0.9422737	-0.5509
26	0.912459	1.080897	0.9421134	-0.54675
27	0.915528	0.96455	0.9495815	-0.9629
28	0.915404	0.967851	0.95	-1.0096

$$S_{ij} = P_{ij} + iQ_{ij} = R_{ij}Y_{ij}^{*} \quad (10)$$

$$R_{ij} = S_{ij}Z_{ij}^{*} \quad (11)$$

The proposed method is summarized as follows:

1. For the first iteration transmission losses are initialized as zero for each element.
2. From the bus powers specified the branch powers are determined as per equation (6&7).
3. The variable Rij. is determined for each element using equation (10).
4. The bus voltage, branch current and bus current are determined from Rij.

$$V_j = V_i - \frac{R_{ij}^{*}}{V_i^{*}} \quad (12)$$

$$I_{ij} = \frac{R_{ij}^{*}}{V_i^{*}} Y_{ij} \quad (13)$$

5. The bus currents are determined from (2) and bus powers are calculated. Since the transmission losses are neglected in the first iteration there will be mismatch between the specified powers and calculated powers. The mismatch is a part of the transmission loss. TLijr is the transmission loss part for 'ij'th element for 'r'th iteration. Transmission loss of each element is the summation of the transmission loss portions of all previous iterations.

$$TL_{ij} = \sum {}^{r}TL_{ij} \quad (14)$$

Where r is the iteration count.

$$TL_{ij}^{r} = S_j^{spec} - {}^{r-1}V_j \,.\, {}^{r-1}I_j^{*} \quad (15)$$

$$S_{ji} = S_{ij} - TL_{ij} \quad (16)$$

$$S_{branch}^{receiving} = S_{branch}^{sending} - TL_{loss} \quad (17)$$

$$\max(TL_{ij}^{r}) \le \varepsilon \quad (18)$$

It can be concluded that the power flow solution always exists for a distribution system irrespective of the R/X ratio if it is having connectivity from the source (slack bus) to all the nodes. The limitations of the algorithm are being investigated [10] in view of voltage stability limit. For system having less transmission loss the algorithm will perform faster. The convergence criteria is that the 'r'th iteration of the transmission loss part of each element should be less than the tolerance value.

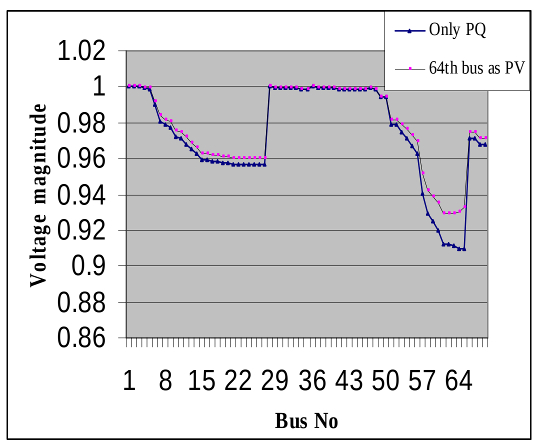

Fig. 3. Voltage profile of 69-bus system

Treatment of voltage controlled buses

If power is fed from multiple ends of the radial system, other feeding buses except slack bus are treated as voltage controlled buses. The equation (9) is modified for the j th voltage controlled bus.

$$real(S_{ij}) = P_{ij} = real(R_{ij}Y_{ij}^{*}) \quad (19)$$

$$R_{ij} = X_{ij} + iY_{ij} \quad (20)$$

$$P_{ij} = real((X_{ij} + iY_{ij})(G_{ij} + iB_{ij})) \quad (21)$$

$$P_{ij} = G_{ij}\left(|V_i|^2 - |V_i||V_j|\cos(\phi_{12})\right) - B_{ij}|V_i||V_j|\sin(\phi_{12}) \quad (22)$$

$$\frac{G_{ij}|V_i|^2 - P_{ij}}{|V_i||V_j|} = G_{ij}\cos(\phi_{12}) - B_{ij}\sin(\phi_{12}) \quad (23)$$

The trigonometric equations are to be solved to get the phase angle of each PV bus j and the reactive power can be updated as

$$Q_{ij} = B_{ij}\left(|V_i|^2 - |V_i||V_j|\cos(\phi_{12})\right) + G_{ij}|V_i||V_j|\sin(\phi_{12}) \quad (24)$$

Table 4 Power flow solution of 69 bus system

Node no	Voltage Magnitude		Node no	Voltage Magnitude	
	Only PQ	64 bus as PV		Only PQ	64 bus as PV
1	1	1	36	0.999919	0.9999321
2	0.999967	0.999973	37	0.999747	0.9997603
3	0.999933	0.9999459	38	0.999589	0.9996018
4	0.99984	0.9998717	39	0.999543	0.999556
5	0.999021	0.9992259	40	0.999541	0.9995538
6	0.99009	0.9916345	41	0.998843	0.9988561
7	0.9808	0.9837384	42	0.998551	0.9985637
8	0.978584	0.9818602	43	0.998512	0.9985251
9	0.977451	0.9809071	44	0.998504	0.9985168
10	0.972451	0.975925	45	0.998405	0.9984183
11	0.971349	0.9748275	46	0.998405	0.9984178
12	0.968188	0.9716775	47	0.999789	0.9998216
13	0.965262	0.9687626	48	0.998544	0.9985758
14	0.962363	0.965874	49	0.994699	0.9947311
15	0.959493	0.9630145	50	0.994154	0.9941861
16	0.95896	0.9624832	51	0.978549	0.9818249
17	0.958079	0.9616058	52	0.978539	0.9818154
18	0.95807	0.9615969	53	0.974666	0.9787527
19	0.957605	0.9611336	54	0.971424	0.9762473
20	0.957306	0.9608357	55	0.966952	0.9728057
21	0.956824	0.9603553	56	0.962585	0.9694584
22	0.956817	0.9603484	57	0.940117	0.9513583
23	0.956745	0.9602768	58	0.929059	0.9424364
24	0.956589	0.960121	59	0.924781	0.9389776
25	0.95642	0.9599526	60	0.919757	0.9349312
26	0.95635	0.9598831	61	0.91236	0.9293557
27	0.95633	0.9598636	62	0.91207	0.9293951
28	0.999926	0.999939	63	0.911683	0.9294964
29	0.999854	0.9998673	64	0.909783	0.93
30	0.999733	0.9997462	65	0.909208	0.9329186
31	0.999712	0.9997248	66	0.971293	0.974771
32	0.999605	0.9996179	67	0.971292	0.9747703
33	0.999349	0.9993617	68	0.967858	0.9713488
34	0.999013	0.9990263	69	0.967857	0.9713477
35	0.998946	0.9989588			

Then the same procedure described for the PQ buses is carried out till the convergence.

Simulation results

The proposed algorithm is implemented on some test systems available in the literature. The test systems considered are 12[4], 28[4] and 69[7] bus systems. Two case studies for these systems with and without PV buses are carried out. The results are given in tables. This algorithm is coded in MATLAB 7.01 and implemented on Pentium IV 800 MHZ systems. It is performing well in terms of speed and accuracy. The maximum time required is only .016 seconds for distribution system of 69 buses [5] for a tolerance value of 10^{-4}. In order to incorporate the PV bus the 12th node of the 12 bus system is chosen. It is observed that reactive power injection of 46.255 MVAR is required to maintain the 12th node voltage as .955 for the 12 bys system. For the 28 bus radial system the reactive required to make the 28th to make the 64th bus voltage, as .93 is determined as 710.64 MVAR.

Conclusion

In this paper an efficient methodology has been presented to solve the radial distribution power flow. The advantage of this method is that it does not require a flat start. The formulation can be extended to unbalanced three-phase networks. The reactive power injections at multiple ends can be effectively calculated to improve the voltage profile.

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