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Entropy generation analysis of fully developed laminar forced convection in a confocal elliptical duct with uniform wall heat flux

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Abstract

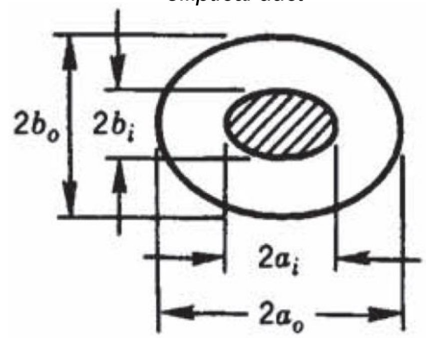
This paper presents the entropy generation and pumping power for developing forced convection in a confocal elliptical duct with constant heat flux. Entropy generation is obtained for various aspect ratio (ε), various ratio of the minor axis to the major axis (α), various wall heat flux and various Reynolds number. It is found that with the increasing aspect ratio (ε) values and decreasing ratio of the minor axis to the major axis (α), total entropy generation and pumping power at fixed Reynolds number increases; with increasing wall heat flux values, total entropy generation and pumping power decreases; with increasing cross-sectional area of duct values, total entropy generation and pumping power decreases.

Keywords: Entropy, Laminar flow, Confocal elliptical duct, Heat flux.

Introduction

In a thermodynamic process the loss of exergy is primarily due to the associated irreversibilities which generate entropy. Most convective heat transfer processes are characterized by two types of exergy losses: losses due to fluid friction and those due to heat transfer across a finite temperature difference. The above two interrelated phenomena are manifestations of thermodynamic irreversibility and investigation of a process from this standpoint is known as second law analysis. However, there exists a direct proportionality between the wasted power (the rate of available work lost) and the entropy generation rate (Bejan, 1994 & 1996). For efficient optimal thermodynamic design, entropy generation must be reduced. In this context, geometry of duct (cross-sectional area) is an important parameter on entropy generation. Various cross-sectional ducts are used in heat transfer devices due to the size and volume constraints to enhance heat transfer with passive method. Pressure drop and heat transfer analysis in various shaped ducts were summarized by Shah and London (1978).

Fig.1.Cross section of confocal elliptical duct



Sahin (1998) presented the second law analysis for different shaped duct such as triangular, sinusoidal etc, in laminar flow and constant wall temperature boundary conditions. He made a comparison between these ducts to find an optimum shape and found that the circular duct geometry is the favorable one among them. He also investigated the constant heat flux effects on these cross-sectional ducts without taking into account the viscosity variation in the analysis. The viscosity variation was considered for turbulent flow condition for circular ducts (Sahin, 2000).

Recently, Oztop (2005) made a study on entropy generation in semicircular ducts. Dagtekin *et al*. (2005) investigated the problem for circular duct with different shaped longitudinal fins for laminar flow using the similar methods of Sahin (1998). Oztop *et al*. (2009) made a study on entropy generation in rectangular ducts with semicircular ends ducts with two boundary conditions: constant wall temperature and constant wall heat flux. Jarungthammachote (2010) investigated entropy generation for hexagonal duct subjected to constant heat flux.

In the present study, the entropy generation for fully developed laminar convection through a confocal elliptical duct with constant heat flux is investigated. To the best of the author's knowledge the entropy generation in confocal elliptical duct s with constant heat flux has not yet been investigated. The present paper reports an analytical study of entropy generation in laminar flow. The effect of Reynolds number, heat flux and geometrical dimensions on entropy generation and pumping power are analyzed.

Physical model of problem

The physical model of confocal elliptical duct is depicted in Fig. 1.

The hydraulic diameter of any duct given by

$$D_h = \frac{4A_c}{p} \tag{1}$$

where A_c is cross-sectional area and P is perimeter. The hydraulic diameter for annular sector cross-sectional area can be written as:

$$D_h = \frac{2\sqrt{2}\,\sqrt{\alpha - \varepsilon\,\alpha\,\beta}}{\sqrt{\alpha^2 + 1} + \sqrt{\varepsilon^2\alpha^2 + \beta^2}}\,\sqrt{A_c} \quad (2)$$

In this equation some parameters can be written as

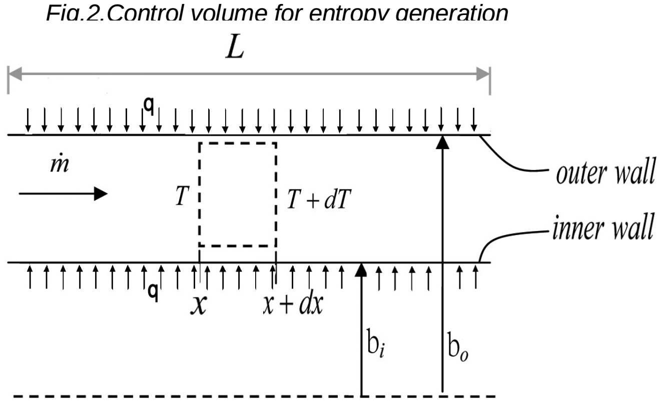
$$\beta = \varepsilon\alpha \frac{1 + \left(\dfrac{m^2}{\omega^2}\right)}{1 - \left(\dfrac{m^2}{\omega^2}\right)} \quad (3)$$

$$m = \sqrt{\frac{1-\alpha}{1+\alpha}} \quad (4)$$

$$\omega = \frac{\varepsilon\alpha + \sqrt{1 - \alpha^2 (1-\varepsilon)^2}}{1+\alpha} \quad (5)$$

Second law analysis

Fig.2.Control volume for entropy generation



The total entropy generation within a control volume of thickness dx, shown in Fig. 2, along the duct can be written as follows: $d\dot{S}_{gen} = \dot{m}\,ds - \dfrac{\delta\dot{Q}}{T_w}$ (6)

and

$$\delta\dot{Q} = \dot{m}C_P\,dT = qp\,dx \quad (7)$$

For an incompressible fluid we have

$$T\,ds = C_P\,dT - v\,dP \quad (8)$$

Pressure drop in Eq. 8 is given in the following equation.

$$dP = -\frac{f\rho U^2}{2D_h}dx \quad (9)$$

The bulk temperature variation of fluid along a duct can be written as follows: $T = T_0 + \left(\dfrac{4q}{\rho U D_h C_P}\right)x$ (10)

The non-dimensional entropy generation number Ns can be defined as

$$Ns = \frac{\dot{S}_{gen}}{\dot{Q}/\Delta T} = \frac{\dot{S}_{gen}}{\dot{m}C_P} \quad (11)$$

Integrating Eq. 6 along the duct length L and by using Eq. 7-10, the dimensionless total entropy generation becomes

$$Ns = \ln\left[(1 + 4St\,\tau\,\lambda)(1+\tau)/(1 + \tau + 4St\,\tau\,\lambda)\right] + \frac{f\,Ec}{8St}\ln(1 + 4St\,\tau\,\lambda) \quad (12)$$

In these equations some parameters can be made dimensionless as follows

$$St = \frac{h}{\rho U C_P} = \frac{Nu}{\mathrm{Re}\,\mathrm{Pr}} \quad (13)$$

$$Ec = \frac{U^2}{C_P(T_w - T_i)} \quad (14)$$

$$\tau = \frac{T_w - T_i}{T_w} = \frac{(q/h)}{T_i} \quad \lambda = \frac{L}{D_h} \quad (15)$$

$$\lambda = \frac{L}{D_h} \quad (16)$$

The values of $(f\,\mathrm{Re})$ and Nu for fully developed laminar flow in confocal elliptical duct is given by Shah and London (1978) for variety of duct geometries.

Pumping power to heat transfer ratio

The power required to overcome the fluid friction in the duct in dimensionless form is

$$P_r = \frac{A_c\,\Delta P U}{\dot{Q}} \quad (17)$$

For constant wall heat flux boundary conditions, the pumping power to heat transfer ratio for fully developed laminar flow becomes

$$P_r = \frac{\mu(f\,\mathrm{Re})}{8\rho^2 D_h q}\mathrm{Re}^2 \quad (18)$$

Results and discussion

Table 1.Thermophysical properties of water

$C_P\,(J/kg\,K)$	4182
Pr	7
$T_w\,(K)$	293
$\mu\,(Ns/m^2)$	9.93×10^{-4}
$\rho\,(kg/m^3)$	998.2

A second law analysis was conducted for confocal elliptical duct in laminar flow regime. Water has been used as working fluid. The thermophysical properties used are shown in Table1. Fig. 3 shows dimensionless

Fig.3.Variation of dimensionless entropy generation for different ε values and Reynolds number (α=0.4, τ=0.1)

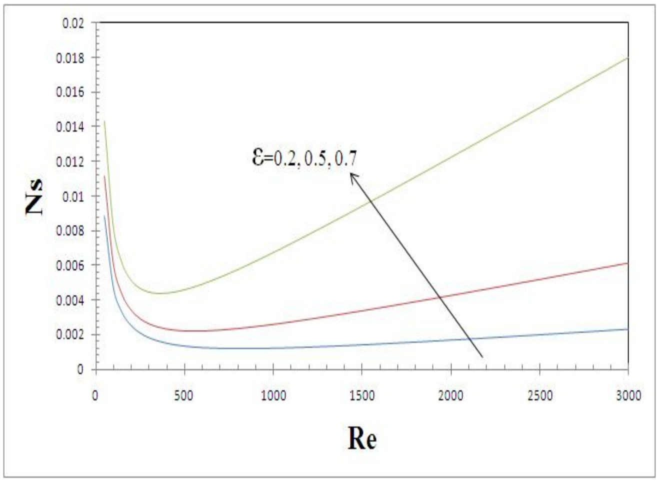


Fig.4.Variation of pumping power for different ε values and Reynolds number (α=0.4, τ=0. 1)

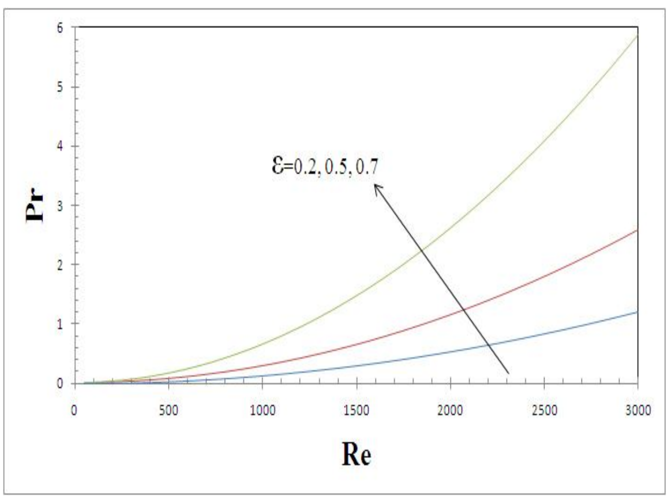


Fig.5.Variation of dimensionless entropy generation for different α values and Reynolds number (τ=0. 1 and ε=0.5)

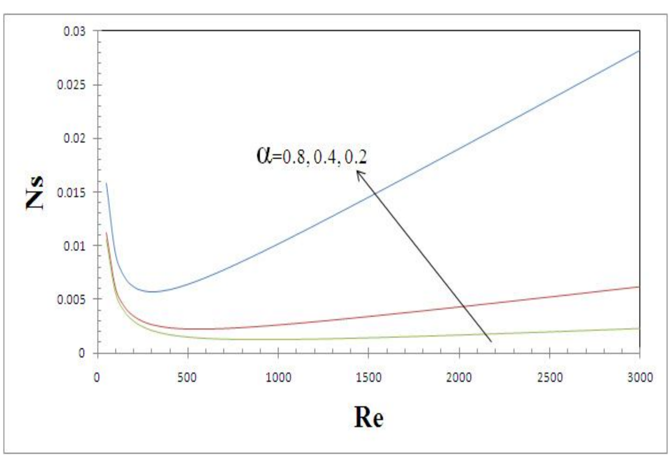


Fig.6.Variation of pumping power for different α values and Reynolds number (τ=0. 1 and ε=0.5)

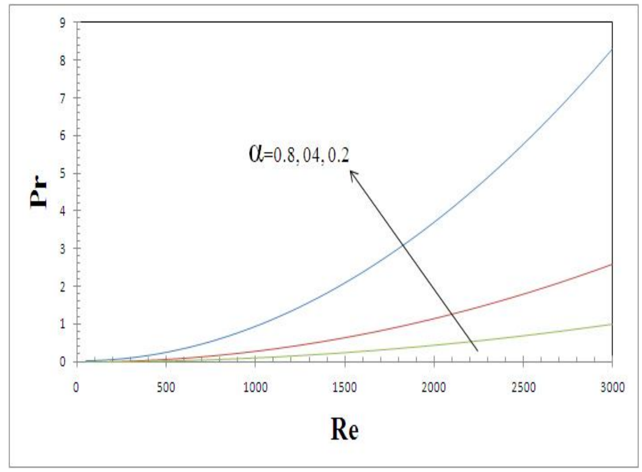


Fig.7.Variation of dimensionless entropy generation for various q values and Reynolds number (ε=0. 5 and α=0.4)

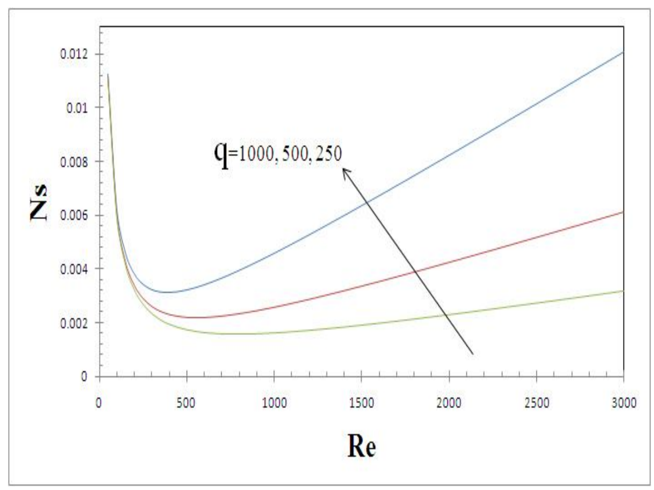
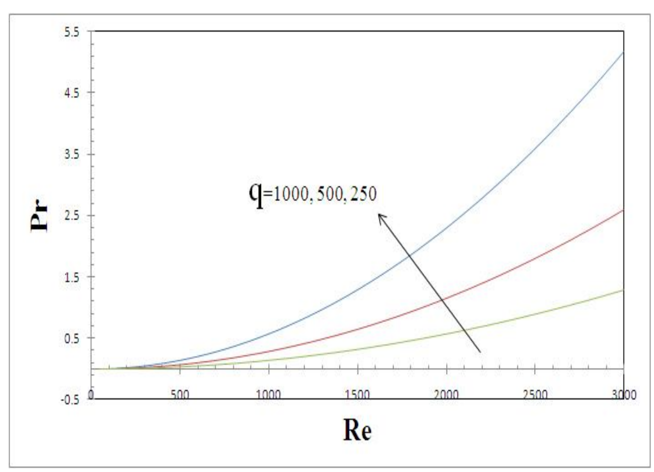


Fig.8.Variation of pumping power for various q values and Reynolds number (ε=0. 5 and α=0.4)



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entropy generation for different aspect ratio (ε) of confocal elliptical duct at different Reynolds number values. The total entropy generation increases considerably while the Reynolds number is increased. As the value of aspect ratio (ε) is increased total entropy generation increased for fixed Reynolds number. Fig. 4 shows variation of pumping power for different aspect ratio (ε) and Reynolds numbers. As aspect ratio (ε) is increased pumping power to heat transfer ratio increases. With increasing of Reynolds number, pumping power to heat transfer ratio values also increase. For a fixed Reynolds number as aspect ratio (ε) values increased, pumping power to heat transfer ratio values also increased, especially for higher values of Reynolds number. These results indicate that for larger aspect ratio, friction losses are higher than that of lower aspect ratio as expected. Fig.5 shows dimensionless entropy generation for different ratio of the minor axis to the major axis (α) at different Reynolds number values. As ratio of the minor axis to the major axis (α) increased, total entropy generation decreases for fixed Reynolds number. With increasing of Reynolds number, total entropy generation values also increase. For a fixed Reynolds number, as ratio of the minor axis to the major axis (α) values increased, total entropy generation values decreased, especially for lower values of Reynolds number. Fig. 6 shows variation of pumping power for different ratio of the minor axis to the major axis (α) and Reynolds numbers. As the ratio of the minor axis to the major axis (α) increased, pumping power to heat transfer ratio decreased. With increasing of Reynolds number, pumping power to heat transfer ratio values also increased. For a fixed Reynolds number, as minor axis to the major axis (α) increased, pumping power to heat transfer ratio values decreased, especially for higher values of Reynolds number. These results indicate that for lower ratio of the minor axis to the major axis (α) friction losses are higher than that of larger aspect ratios as expected. Fig.7 shows the effects of heat flux difference on entropy generation at different Reynolds numbers for a fixed aspect ratio $(\varepsilon = 0.5)$ and ratio of the minor axis to the major axis $(\alpha = 0.4)$, namely, for a fixed cross sectional area. Entropy generation is influenced by the heat flux. As Reynolds number increased, the dimensionless entropy generation increases, especially for lower heat flux values. Thus, lower entropy generation is obtained for higher value of q. The decrease of total entropy generation depends on the increase of heat flux as expected. Fig. 8 shows variation of pumping power to heat transfer ratio for various heat flux and Reynolds numbers. Increasing Reynolds number yields higher pumping power to heat transfer ratio values for fixed heat flux values. As the heat flux is increased pumping power to heat transfer ratio values decreases for fixed Reynolds number, especially for higher values of Reynolds number.

Fig.9.Variation of dimensionless entropy generation for various Ac values and Reynolds number (ε=0. 5, α=0.4 and q=500 W/m^2)

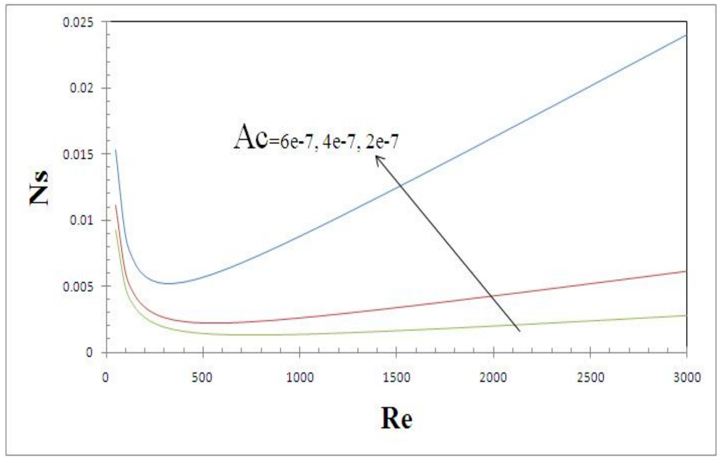

Fig.10.Variation of pumping power for various Ac values and Reynolds number (ε=0. 5, α=0. 4 and q=500W/m^2)

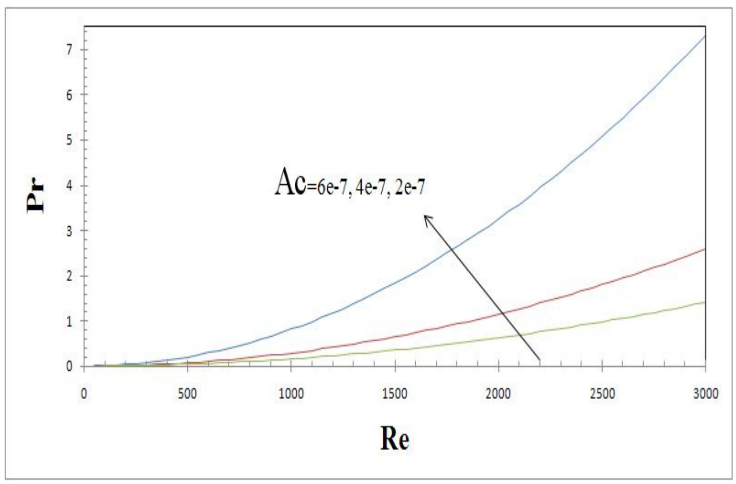

Fig. 9 shows the effects of cross-sectional area difference on entropy generation at different Reynolds numbers for a fixed aspect ratio $(\varepsilon = 0.5)$ and ratio of the minor axis to the major axis $(\alpha = 0.4)$. As the cross-sectional area increases, dimensionless entropy generation decreases. Lower value of entropy generation is obtained for larger A_c. Fig.10 shows the effects of cross-sectional area difference on pumping power to heat transfer ratio at different Reynolds numbers for a fixed aspect ratio $(\varepsilon = 0.5)$ and ratio of the minor axis to the major axis to the major axis $(\alpha = 0.4)$. As the cross-sectional area is increased pumping power to heat transfer ratio decreases.

Conclusion

The second law analysis of laminar flow, subjected to constant wall heat flux, has been obtained for confocal elliptical ducts. As Reynolds number is increased total entropy generation is increasing. With increasing value of aspect ratio (ε) and decreasing ratio of the minor axis to the major axis (α) there results increased total entropy generation for fixed Reynolds number. The pumping power ratio increases with the aspect ratio (ε), and decreases as ratio of the minor axis to the major axis (α) increased. When heat flux and cross-sectional area increased the total entropy generation and pumping power ratio decreased for fixed Reynolds number.

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