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Application of Chebyshev polynomials for solving nonlinear Volterra-Fredholm integral equations system and convergence analysis

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Abstract
In this paper, we solve the nonlinear Volterra-Fredholm integral equations system by using the Chebyshev polynomials. First we introduce the Chebyshev polynomials and approximate functions via their application. Then, we use Chebyshev polynomials as a collocation basis to change the nonlinear Volterra-Fredholm integral equations system to a system of nonlinear algebraic equations. Finally, the convergence analysis is considered, and numerical examples given to illustrate the efficiency of this method.

Keywords: Volterra-Fredholm; System of integral equations; Chebyshev polynomials; Operational matrix.

Introduction

System of nonlinear Volterra-Fredholm integral equations are defined as follows:

$$f_i(s) = g_i(s) - (\sum_{j=0}^{n}\int_{-1}^{s} k_{ij}(s,t)[g_j(t)]^{p_j}\,dt) - (\sum_{j=0}^{n}\int_{-1}^{1} k'_{ij}(s,t)[g_j(t)]^{q_j}\,dt),$$

$$i = 0,1,2,...,n, \quad s \in [-1,1], \qquad (1)$$

where, for $i, j = 0,1,2,...,n$ the functions $f_i(s), k_{ij}(s,t)$ and $k'_{ij}(s,t)$ are known and $g_i(s)$ is the unknown functions to be determined, also $p_i, q_i \geq 1$ are positive integers. Equation (1) introduces a system of $n+1$ equations and $n+1$ unknowns.

Up to now several methods have been proposed for solving Volterra-Fredholm equations and it's systems. Yalsinbas (2002) used Taylor polynomials to approximate Volterra-Fredholm integral equations. Also Maleknejad and Mahmodi (2003) applied Taylor polynomials for solving high-order Volterra-Fredholm integro-differential equations. Rabbani *et al.,* (2007) solved Volterra-Fredholm integral equations system using an expansion method. Jumarhan and Mckee (1996) presented a numerical solution method based on integration to solve the nonlinear Volterra-Fredholm integral equations system. Solving the system of Volterra-Fredholm integral equations by Adomian decomposition method is considered in (Maleknejad & Fadaei Yami, 2006). Chuong and Tuan (1996) used Spline-collocation method for solving nonlinear Volterra-Fredholm equations system. Brunner (1990) solved the nonlinear Volterra-Fredholm integral equations by using collocation method. Maleknejad *et al.* (2007) solved nonlinear Volterra integral equations using Chebyshev polynomials. Also Cerdik-Yaslan & Akyuz-Dascioglu (2006) applied Chebyshev polynomials for solving Volterra-Fredholm integro-differential equations. Very recently, we used Chebyshev polynomials for solving nonlinear Volterra-Fredholm integral equations (Ezzati & Najafalizadeh, 2011).

Chebyshev polynomials of the first kind of degree n are defined as follows (Chihara, 1978):

$$T_n(s) = \cos(n\theta), \;\; \theta = \arccos(s), \quad n \geq 0.$$

Also we'll have the following recursive formula for these polynomials (Chihara, 1978):

$$T_0(s) = 1,$$

$$T_1(s) = s,$$

$$T_{n+1}(s) = 2sT_n(s) - T_{n-1}(s), \quad n = 1,2,3,... \quad (2)$$

Inner product in the interval [-1,1] for Chebyshev polynomials is defined by (Chihara, 1978):

$$< T_i(s), T_j(s) >= \int_{-1}^{1} T_i(s)T_j(s)\omega(s)ds \quad (3)$$

where

$$\omega(s) = (1-s^2)^{\frac{-1}{2}}.$$

With respect to the inner product which is defined in (3) Chebyshev polynomials are orthogonal (Chihara, 1978):

$$(T_i(s), T_j(s)) = \begin{cases} \pi, \; i = j, \\ \frac{\pi}{2}\delta_{ij}, \; i \neq j. \end{cases} \quad (4)$$

where

$$\delta_{ij} = \begin{cases} 1, \; i = j, \\ 0, \; i \neq j. \end{cases}$$

In this paper, we approximate functions by using Chebyshev polynomials and we present operational matrices for integration of vectors. Then Chebyshev polynomials defined in (2) are used as a collocation basis to solve system (1) and reduce it to a system of algebraic equations. The generated algebraic system, which according to the type of system (1) would be either linear

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or nonlinear. Newton's iterative method can be used for solving nonlinear algebraic system. Finally, we introduce two theorems and proofs for convergence analysis.

Approximation the function by using a series of Chebyshev polynomials

If $f(s)$ be a function in $[a,b]$ and $\{v_i\}_{i=0}^{\infty}$ be orthogonal on this interval, then $f(s)$ can be shown as follows:

$$f(s) = \sum_{i=0}^{\infty} \alpha_i v_i(s), \tag{5}$$

where α_i are Fourier coefficients that are as [11,12]:

$$\alpha_i = (f(s), v_i(s)), \tag{6}$$

As we mentioned above, we also can write the above series for the Chebyshev orthogonal basis, if $f(s)$ is defined in the interval [-1,1], by using Chebyshev polynomials of the first kind, relation (5) can be written as follows:

$$f(s) = \sum_{i=0}^{\infty} c_i T_i(s), \tag{7}$$

if the infinite series in (7) is truncated, then we'll have:

$$f(s) = \sum_{i=0}^{N} c_i T_i(s) = C^T T(s), \tag{8}$$

where C and T are $(N+1)\times 1$ definite vectors as follows:

$$C = [c_0, c_1, c_2, ..., c_N]^T, \tag{9}$$

$$T(s) = [T_0(s), T_1(s), T_2(s), ..., T_N(s)]^T. \tag{10}$$

Coefficients c_i are given as (6) where inner product with the weight function $\omega(s) = (1-s^2)^{-1/2}$ is:

$$c_i = (f(s), T_i(s)) = \begin{cases} \dfrac{1}{\pi}\displaystyle\int_{-1}^{1} \omega(s) f(s)ds, & i = 0, \\ \dfrac{2}{\pi}\displaystyle\int_{-1}^{1} \omega(s) T_i(s) f(s)ds, & i > 0. \end{cases}$$

((11)

For the positive integer powers of a function $f(s)$, we have:

$$[f(s)]^p = [C^T T(s)]^p = C_p^{*T} T(s), \tag{12}$$

where C and T are defined vectors in (9), (10), and C_p^* is a column vector and it's elements are nonlinear combinations of the elements of vector C. C_p^* is called operational vector of p th power. Maleknejad *et al.* (2006) compute the second and third product operational vector by using Chebyshev polynomials as follows:

$$C_2^* = \frac{1}{2}\begin{pmatrix} 2c_0^2 + c_1^2 + c_2^2 + c_3^2 \\ 4c_0c_1 + 2c_1c_2 + 2c_2c_3 \\ c_1^2 + 4c_0c_2 + c_1c_3 \\ 2c_1c_2 + 4c_0c_3 \end{pmatrix},$$

also

$$C_3^* = \frac{1}{4}\begin{pmatrix} 4c_0^3 + 6c_0c_1^2 + 3c_1^2c_2 + 6c_0c_2^2 + 6c_1c_2c_3 + 6c_0c_3^2 \\ 12c_0^2c_1 + 3c_1^3 + 12c_0c_1c_2 + 6c_1c_2^2 + 3c_1^2c_3 + 12c_0c_2c_3 + 3c_2^2c_3 + 6c_1c_3^2 \\ 6c_0c_1^2 + 12c_0^2c_2 + 6c_1^2c_2 + 3c_2^3 + 12c_0c_1c_3 + 6c_1c_2c_3 + 6c_2c_3^2 \\ c_1^3 + 12c_0c_1c_2 + 3c_1c_2^2 + 12c_0^2c_3 + 6c_1^2c_3 + 6c_2^2c_3 + 3c_3^3 \end{pmatrix}.$$

Similarly, regarding a function $k(s,t)$, with two variables, which is defined on [-1,1], we'll have:

$$k(s,t) = \sum_{i=0}^{N}\sum_{j=0}^{N} T_i(s) k_{ij} T_j(t), \tag{13}$$

where

$$K_{ij} = (T_i(s), (k(s,t), T_j(t))). \tag{14}$$

By choosing $T(s)$ as (10) and K as a $(N+1)\times(N+1)$ matrix with elements of k_{ij}, equation (13) can be written as follows:

$$k(s,t) = T^T(s) K T(t). \tag{15}$$

The operational matrices for integration

In this section we present the operational matrix as P for computing the integral of vector (10), (Rao, 1983) We have the following relation about Chebyshev polynomial:

$$\int_{-1}^{s} T_{N-1}(t)dt = \frac{1}{2N} T_N(s) - \frac{1}{2(N-2)} T_{N-2}(s) + \frac{(-1)^{N-1}}{1-(N-1)^2} T_0(s), \quad N \geq 3.$$

(16)

also for $T_0(s)$ and $T_1(s)$ we have:

$$\int_{-1}^{s} T_0(t)dt = T_0(s) + T_1(s),$$

$$\int_{-1}^{s} T_1(t)dt = \frac{-1}{4} T_0(s) + \frac{1}{4} T_2(s). \tag{17}$$

Equations (16) and (17) allow us to write:

$$\int_{-1}^{s} T(t)dt = PT(s), \tag{18}$$

$$\int_{-1}^{1} T(t)dt = PT(1), \tag{19}$$

where P is the $(N+1)\times(N+1)$ operational matrix as follows:

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$$P=\begin{pmatrix} 1 & 1 & 0 & 0 & . & . & . & 0 & 0 \\ -\frac{1}{4} & 0 & \frac{1}{4} & 0 & . & . & . & 0 & 0 \\ -\frac{1}{3} & -\frac{1}{2} & 0 & \frac{1}{6} & . & . & . & 0 & 0 \\ \frac{1}{8} & 0 & -\frac{1}{4} & 0 & . & . & . & 0 & 0 \\ . & . & . & . & . & & & . & . \\ . & . & . & . & & . & & . & . \\ . & . & . & . & & & . & . & . \\ \frac{(-1)^{N-1}}{1-(N-1)^2} & 0 & 0 & 0 & . & . & . & 0 & \frac{1}{2N} \\ \frac{(-1)^N}{1-N^2} & 0 & 0 & 0 & . & . & . & -\frac{1}{2(N-1)} & 0 \end{pmatrix}. \quad (20)$$

For Chebyshev polynomials we have:

$$T(s)T^T(s)C=\tilde{C}^T T(s), \quad (21)$$

where C is a vector in (9) and $\tilde{C}$ is a $(N+1)\times(N+1)$ square matrix as follows:

$$\tilde{C}=\frac{1}{2}\begin{pmatrix} 2c_0 & c_1 & . & . & . & c_i & . & . & . & c_{N-1} & c_N \\ 2c_1 & 2c_0+c_2 & . & . & . & c_{i-1}+c_{i+1} & . & . & . & c_{N-2}+c_N & c_{N-1} \\ . & . & . & & & . & & & & . & . \\ . & . & & . & & . & . & . & . & . & . \\ . & . & & & . & . & & & & . & . \\ 2c_i & c_{i-1}+c_{i+1} & . & . & . & 2c_0+c_{2i} & . & . & . & c_{N-i-1} & c_{N-i} \\ . & . & & & & . & . & & & . & . \\ . & . & & & & . & & . & & . & . \\ . & . & & & & . & & & . & . & . \\ 2c_{N-1} & c_{N-2}+c_N & . & . & . & c_{N-i-1} & . & . & . & 2c_0 & c_1 \\ 2c_N & c_{N-1} & . & . & . & c_{N-i} & . & . & . & c_1 & 2c_0 \end{pmatrix}, \quad (22)$$

where $i=[\frac{N}{2}]$.

Description of the method

In this section, we solve the nonlinear Volterra-Fredholm integral equations system by using the Chebyshev polynomials of the first kind.

With respect to the method of Section 2 for $i,j=0,1,2,...,n$ we have:

$$g_i(s)=T^T(s)G_i,$$

$$[g_j(s)]^m=T^T(s)G^*_{jm}, \quad for \quad m=p_j,q_j,$$

$$k'_{ij}(s,t)=T^T(s)K'_{ij}T(t),$$

$$k_{ij}(s,t)=T^T(s)K_{ij}T(t), \quad (23)$$

where $G^*_{jp_j}$ and $G^*_{jq_j}$ are operational vectors defined in Section 2 and G_i is a $(N+1)\times 1$ vector

$$G_i=[g_{i0},g_{i1},g_{i2},...,g_{iN}]^T. \quad (24)$$

Now, with substituting equation (23) in system (1) we'll have:

$$f_i(s)=T^T(s)G_i-(\sum_{j=0}^{n}\int_{-1}^{s}T^T(s)K_{ij}T(t)T^T(t)G^*_{jp_j}\,dt)$$

$$-(\sum_{j=0}^{n}\int_{-1}^{1}T^T(s)K'_{ij}T(t)T^T(t)G^*_{jq_j}\,dt),$$

$$=T^T(s)G_i-(\sum_{j=0}^{n}T^T(s)K_{ij}\int_{-1}^{s}T(t)T^T(t)G^*_{jp_j}\,dt)-(\sum_{j=0}^{n}T^T(s)K'_{ij}\int_{-1}^{1}T(t)T^T(t)G^*_{jq_j}\,dt),$$

$$=T^T(s)G_i-(\sum_{j=0}^{n}T^T(s)K_{ij}\tilde{G}^*_{jp_j}\int_{-1}^{s}T(t)dt)-(\sum_{j=0}^{n}T^T(s)K'_{ij}\tilde{G}^*_{jq_j}\int_{-1}^{1}T(t)dt).$$

(25)

$$f_i(s)=T^T(s)G_i-(\sum_{j=0}^{n}T^T(s)K_{ij}\tilde{G}^*_{jp_j}PT(s))$$

$$-(\sum_{j=0}^{n}T^T(s)K'_{ij}\tilde{G}^*_{jq_j}PT(1)), \; i=0,1,2,...,n. \quad (26)$$

Hence Equation (26) represent a system with $(n+1)$ equations and $(n+1)\times(N+1)$ unknowns, so we rewrite each equation of the system at the collocation points of $\{s_k\}_{k=0}^{\infty}$ in the interval [-1,1]. Then we'll have a system with $(n+1)\times(N+1)$ equations and $(n+1)\times(N+1)$ unknowns:

$$f_i(s_k)=T^T(s_k)G_i-(\sum_{j=0}^{N}T^T(s_k)K_{ij}\tilde{G}^*_{jp_j}PT(s_k))-(\sum_{j=0}^{N}T^T(s_k)K'_{ij}\tilde{G}^*_{jq_j}PT(1)),$$

$$for \;\; i=0,1,2,...,n \;\; and \;\; k=0,1,2,...,N. \quad (27)$$

Relation (27) leads to a linear or nonlinear system of equations such that the unknown coefficients can be found.

Convergence analysis

We can show the nonlinear terms in equation (1) by $F(g_j)=[g_j(t)]^{p_j}$ and $F'(g_j)=[g_j(t)]^{q_j}$. Let $(C[-1,1],\|.\|)$ be the Banach space of all continuous functions on interval [-1,1] with norm $\|f\|_\infty=\max_{\forall s\in[-1,1]}|f(s)|$. Suppose the nonlinear terms $F(u)$ and $F'(u)$ are satisfied in Lipschitz condition

$$|F(u)-F(v)|\le L_1|u-v|,$$

and

$$|F'(u)-F'(v)|\le L_2|u-v|.$$

We also assume for all $i,j=0,1,2,...,n,\; |k_{ij}(s,t)|\le M$ and $|k'_{ij}(s,t)|\le M'$. We show exact solutions of the nonlinear Volterra-Fredholm integral equations system by $g_j(s)$ and approximate solutions of the nonlinear Volterra-Fredholm integral equations system for N by

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$\bar{g}_{jN}(s)$. Moreover, we define $\alpha = ML_1(s+1)+2M^{'}L_2$. So, we are ready for presenting two theorems about convergence analysis.

Theorem 5.1 *For $n=0$ the solution of the nonlinear Volterra-Fredholm integral equation by using Chebyshev polynomials is convergent if $0<\alpha<1$.*

Proof.

$$\|g_0-\bar{g}_{0N}\|_\infty = \max_{\forall s\in[-1,1]} |g_0(s)-\bar{g}_{0N}(s)|$$

$$= \max_{\forall s\in[-1,1]} |(\sum_{j=0}^{n}\int_{-1}^{s}k_{00}(s,t)(F(g_0)-F(\bar{g}_{0N}))dt)$$

$$-(\sum_{j=0}^{n}\int_{-1}^{1}k_{00}^{'}(s,t)(F^{'}(g_0)-F^{'}(\bar{g}_{0N}))dt)|$$

$$\le ML_1(s+1)\|g_0-\bar{g}_{0N}\|_\infty + 2M^{'}L_2\|g_0-\bar{g}_{0N}\|_\infty$$

$$=\alpha\|g_0-\bar{g}_{0N}\|_\infty .$$

$$\Rightarrow \|g_0-\bar{g}_{0N}\|_\infty < \alpha\|g_0-\bar{g}_{0N}\|_\infty . \qquad (28)$$

By selection $0<\alpha<1$ we'll have:

$$N\to\infty,\quad \|g_0-\bar{g}_{0N}\|_\infty \to 0,$$

so the proof is completed.

Theorem 5.2 *For $n\ge 1$ the solution of the nonlinear Volterra-Fredholm integral equations system by using Chebyshev polynomials is convergent if*

$$0<\alpha<\frac{1}{1+n}.$$

Proof. Let us consider the following norm for the i th equation of system (1.1):

$$\|g_i-\bar{g}_{iN}\|_\infty = \max_{\forall s\in[-1,1]} |g_i(s)-\bar{g}_{iN}(s)|$$

$$= \max_{\forall s\in[-1,1]} |(\sum_{j=0}^{n}\int_{-1}^{s}k_{ij}(s,t)(F(g_j)-F(\bar{g}_{jN}))dt)$$

$$-(\sum_{j=0}^{n}\int_{-1}^{1}k_{ij}^{'}(s,t)(F^{'}(g_j)-F^{'}(\bar{g}_{jN}))dt)|$$

$$\le \max_{\forall s\in[-1,1]} (\sum_{j=0}^{n}\int_{-1}^{s}|k_{ij}(s,t)||(F(g_j)+F(\bar{g}_{jN})|dt$$

$$-\sum_{j=0}^{n}\int_{-1}^{1}|k_{ij}^{'}(s,t)||(F^{'}(g_j)-F^{'}(\bar{g}_{jN}))|dt)$$

$$\le \sum_{j=0}^{n}ML_1\int_{-1}^{s}\|g_j-\bar{g}_{jN}\|_\infty dt + \sum_{j=0}^{n}M^{'}L_2\int_{-1}^{1}\|g_j-\bar{g}_{jN}\|_\infty dt$$

$$=\sum_{j=0}^{n}(ML_1(s+1)+2M^{'}L_2)\|g_j-\bar{g}_{jN}\|_\infty ,$$

$$\Rightarrow \|g_j-\bar{g}_{jN}\|_\infty \le \sum_{j=0}^{n}\alpha\|g_j-\bar{g}_{jN}\|_\infty . \qquad (29)$$

If we rewrite relation (29) for $i=0,1,2,...,n$ and then add up the obtained unequal extremes, we'll have:

$$\Rightarrow \sum_{i=0}^{n}\|g_i-\bar{g}_{iN}\|_\infty \le \sum_{j=0}^{n}(n+1)\alpha\|g_j-\bar{g}_{jN}\|_\infty , \qquad (30)$$

so

$$\Rightarrow \sum_{i=0}^{n}(1-(1+n)\alpha)\|g_i-\bar{g}_{iN}\|_\infty < 0, \qquad (31)$$

according to relation (31) by selecting $0<\alpha<\dfrac{1}{1+n}$ we'll have:

$$N\to\infty,\quad \|g_i-\bar{g}_{iN}\|_\infty \to 0,$$

so the proof is completed.

Examples

In this section, the efficiency of the presented method is shown in following three examples. In examples 1 and 2, we use Newton's iterative method for solving the generated nonlinear system. Mathematica 5.2 software is applied in computing examples.

Example 1. As a first example we have the following system with 2 equations and 2 unknowns:

$$f_1(s) = g_1(s) - \int_{-1}^{1}st^2g_1(t)dt - \int_{-1}^{1}(st-1)g_2(t)dt,$$

$$f_2(s) = g_2(s) - \int_{-1}^{s}(s-t)g_1(t)dt - \int_{-1}^{s}2tg_2(t)dt, \qquad (32)$$

where $f_1(s)=2s+1$ and $f_2(s) = -\frac{2}{3}s^3+1$. The exact solutions of the above system are $g_1(s)=2s-1$ and $g_2(s)=s+1$. Table 1 illustrates the numerical results for $N=9$ and $N=11$.

Example 2. Consider the following nonlinear integral equations system:

$$f_1(s) = g_1(s) - \int_{-1}^{s}16st[g_1(t)]^2dt - \int_{-1}^{1}s^2t^3g_2(t)dt,$$

Table 1. The Numerical results of Example 6.1.

	Exact solution		Approximation solution with N=9		Approximation solution with N=11	
x_i	$g_1(s)$	$g_2(s)$	$g_1(s)$	$g_2(s)$	$g_1(s)$	$g_2(s)$
-1	-3	0	-2.99998	-0.00977	-3	0.00011
-0.75	-2.5	.25	-2.49998	0.24262	-2.5	0.25005
-0.5	-2	.5	-1.99998	0.49506	-2	0.50002
-0.25	-1.5	.75	-1.49998	0.74751	-1.5	0.75
0	-1	1	-0.99998	1	-1	1
0.25	-0.5	1.25	-0.49998	1.25	-0.5	1.25
0.5	0	1.5	0.00002	1.50506	0	1.49999
0.75	.5	1.75	0.50002	1.75763	0.5	1.74997
1	1	2	1.00002	2.01023	1	1.99999

$$f_2(s)=g_2(s)-\int_{-1}^{s}(3s^2-2t)[g_2(t)]^2\,dt-\int_{-1}^{1}(2t-4)g_1(t)\,dt,$$ (33)

where $f_1(s)=s^5-\frac{2}{5}s^2-\frac{s}{2}$, $f_2(s)=-s^5-\frac{5}{2}s^4-\frac{5}{3}s^5+s+\frac{1}{6}$, and the exact solutions $g_1(s)=\frac{s}{2}$ and $g_2(s)=s+1$. Table 2 illustrates the numerical results.

Table 2. The Numerical results of Example 5.2.

	Exact solution		Approximation solution with N=10		Approximation solution with N=12	
x_i	$g_1(s)$	$g_2(s)$	$g_1(s)$	$g_2(s)$	$g_1(s)$	$g_2(s)$
-1	-0.5	0	-0.48378	-0.03068	-0.49910	-0.00045
-0.75	-0.375	.25	-0.36599	0.22503	-0.37468	0.24956
-0.5	-0.25	.5	-0.24563	0.48074	-0.24999	0.49957
-0.25	-0.125	.75	-0.12360	0.73645	-0.12505	0.74959
0	0	1	0	0.99216	0	0.9996
0.25	0.125	1.25	0.12516	1.24788	0.12505	1.24961
0.5	0.25	1.5	0.25188	1.50359	0.24999	1.49963
0.75	0.375	1.75	0.380278	1.75930	0.37468	1.74964
1	0.5	2	0.51122	2.01501	0.49910	1.99965

Example 3. As a last example, we have the following nonlinear Volterra-Fredholm integral equations system:

$$f_1(s)=g_1(s)-\int_{-1}^{s}(t^2-s)g_1(t)\,dt-\int_{-1}^{1}st^2g_1(t)\,dt-\int_{-1}^{1}(t+1)s[g_2(t)]^2\,dt,$$

$$f_2(s)=g_2(s)-\int_{-1}^{s}2g_2(t)\,dt-\int_{-1}^{1}3s[g_1(t)]^2\,dt,$$ (34)

where $f_1(s)=-\frac{s^4}{4}+\frac{5s^3}{6}-s^2-\frac{s}{10}-\frac{5}{12}$ and $f_2(s)=-\frac{2}{3}s^3+2s^2-9s-\frac{5}{2}.$

The exact solution of above system is $g_1(s)=s-1$ and $g_2(s)=s^2-s$. Table 3 shows the numerical results for N=10 and N=12.

Table 3. The Numerical results of Example 5.3.

	Exact solution		Approximation solution with N=10		Approximation solution with N=12	
x_i	$g_1(s)$	$g_2(s)$	$g_1(s)$	$g_2(s)$	$g_1(s)$	$g_2(s)$
-1	-2	2	-2.01013	1.98970	-2.00010	2.00057
-0.75	-1.75	1.3125	-1.75755	1.30476	-1.75007	1.31291
-0.5	-1.5	0.75	-1.50501	0.74482	-1.50005	0.75025
-0.25	-1.25	0.3125	-1.25249	0.30989	-1.25002	0.31261
0	-1	0	-1	-0.00002	-1	-0.00001
0.25	-0.75	0.1875	-0.74754	-0.18493	-0.74998	-0.18762
0.5	-0.5	-0.25	-0.49511	-0.24483	-0.49996	-0.25022
0.75	-0.25	-0.1875	-0.24270	-0.17972	-0.24994	-0.18780
1	0	0	0.00967	0.01040	0.00008	-0.00039

Conclusion

In this paper, we solved a system of Volterra-Fredholm integral equations by using Chebyshev collocation method. The properties of Chebyshev polynomials are used to reduce the system of Volterra-Fredholm integral equations to a system of nonlinear algebraic equations. Computations are excuted using Mathematica 5.2 software. Three numerical examples demonstrate the validity and efficiency of proposed method.

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