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Stochastic geometry and its application in calculating transmission capacity of wireless ad hoc networks

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Abstract

In this paper we analysed a special kind of wireless ad hoc networks. The elements of this network are distributed according to marked Poisson point processes in the plan so as to derive the characteristics of this network we use stochastic geometry. We assume that the channel between receivers and transmitters varies according to a gamma distribution and based on this assumption we model our network. The main topic of study in this issue is transmission capacity so we focus on it as well using some other related terms we try to find some bounds for it. Our work shows that different assumptions of channel variation in wireless ad hoc networks must be carried by precise calculation because of the complication of transmission capacity.

Keywords: Ad hoc networks, Outage probability, Stochastic geometry, Transmission capacity, Wireless networks.

Introduction

Due to magnificent work by Gupta and Kumar (2000), the study of wireless ad hoc networks in wireless communication has become one of the most interesting issues even more than before. They introduced transport capacity as the number of bit-meters pumped over a given time interval for a network of nods in a unit area of plan and showed it can be best achieved as O ($\sqrt{\lambda}$) in which λ is the density of transmitting nodes. This was a key for others like Webber *et al.* (2005, 2006, 2007 & 2010). They extended this idea to transmission capacity that is defined as the number of successful transmissions taking place a network per unit area subject to a constraint on the network outage probability as the definition shows it depends on outage probability that says whether a transmission is successful or not according to a signal to interference noise ratio (SINR) requirement.

A lot of important strides have been taken to analyze the wireless networks by this terminology. For an example, in the work of Baccelli *et al.* (2006), one can find more applications of mathematics tools especially stochastic geometry. Andrews *et al.* (2007) worked on a wireless network model in a way that is much closer to the reality of this networks. One of the reasons that make transmission capacity very important is that it is possible to calculate bounds on outage probability and by itself as well. In almost all work it has been tried to find the tightest upper and lower bounds on transmission capacity. Considering all aspects of wireless ad hoc networks is difficult because it depends on a lot of factors.

In a large network, all transmitters interferes each other and it makes difficult to find the exact expression for transmission capacity and thus finding bounds for transmission capacity becomes the main goal. However, in most works a few parameters are considered and physical layers are often ignored. There are different kinds of networks but experiences show that DS-CDMA and FH-CDMA are special and very good cases for study. Considering different approaches and comparing this two models are thus becomes the subject of many workers. In this paper we consider general wireless ad hoc networks and use a special model for this networks and using the famous inequalities to derive the bounds on transmission capacity.

Channel model

Consider a large ad hoc wireless communication network including transmitters and receivers in the plan that the location of transmitters shown by $\{X_i\}$ form a stationary poisson point processes. Each transmitter has a unique associated receiver that tries to send signal with satisfactory quality according to outage constraint.

Transmitters are distributed uniformly and independently in the area and all transmitters act independently. We look at the system in a time slot t. There are different channel models to work with for example fast fading shadowing and path loss attenuation. We consider a model in which both fading and path loss attenuation are indicated. Consider a marked Poisson point processes (MPPP) in a plan that is not homogenous with intensity λ denoted by $\Pi(\lambda)=\{X_i\ ,\ M_i\}$. In which M_i is a random no distance dependent channel effect.

We define the transmission capacity to be:

$$c(\varepsilon) = \lambda(\varepsilon)\, b\, (1-\varepsilon) \qquad \varepsilon\epsilon(0,1) \qquad (1)$$

In which $\lambda(\varepsilon)$ is the spatial intensity of attempted transmissions. In other word $\lambda(\varepsilon)$ is the maximum spatial density of nodes that can contend with the channel subject to constraint on the typical outage probability.

Using palm distribution (Stoyan *et al.,* 1996) we suppose that a receiver is in the origin and its intended transmitter is in d distance away from it and all other transmitters are interferers. We also suppose that all transmitters use the fix power ρ to transmit and we show the distance between transmitters i to origin by$|X_i|$. The received signal by a receiver i is:

$$\rho\, |X_i|^{-\alpha} H_i \qquad (2)$$

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That H_i is a random variable that is the random channel gain and α is a path loss exponent.
We suppose that H_i are identically, independently distributed (iid) and each H_i is a gamma random variable with $m\,,n$ parameters m, n are positive integers:

$$f_H(h) = \frac{h^{m-1}}{\Gamma(m)n^m}\, e^{\frac{-h}{n}}\, u(h) \qquad (3)$$

In which $\Gamma(m)$ is the gamma function and $u(h)$ is the unit step function.

Outage probability

An appropriate signal quality for our receiver in the origin must not be less than threshold β.
If we consider ε as the constraint on outage probability then the probability that the received signal at origin is not suitable is:

$$p\left(\frac{\rho\, d^{-\alpha} H}{\sum_{i\epsilon\Pi} \rho\, |X_i|^{-\alpha} H_i} \le \beta\right) \le \varepsilon \qquad (4)$$

In the above formula H is the random channel gain between our reference receiver in origin and its intended transmitter.
We show the outage probability by $P^o(\lambda)$.
If we summarize the above expression then we have:

$$p\left(\sum_{i\epsilon\Pi} |X_i|^{-\alpha}\, \frac{H_i}{H} \ge \frac{1}{\beta d^{-\alpha}}\right) \le \varepsilon \qquad (5)$$

According to our channel model H_i and H are gamma random variables with $m\,,n$ parameters so it is straightforward to prove that the pdf of random variable $\frac{H_i}{H} = Z$ is:

$$f_Z(z) = \frac{\Gamma(2m)}{\Gamma(m)^2}\, \frac{z^{m-1}}{(1+z)^m}\, u(z) \qquad (6)$$

To compute the outage probability upper and lower bounds we separate the interfering transmitters into two groups, far field and near field interferers. The first group is close enough to the origin that their interference is lonely enough to cause outage so we define:

$$\Pi_D(\lambda) = \left\{(X_i\,,H_i) : |X_i|^{-\alpha}\, \frac{H_i}{H} > k\right\}$$

In which $k = \frac{1}{\beta d^{-\alpha}}$. The first group is dominant interferers.
Using bounds in [5] we have two upper bounds for outage probability as:

$$P_1(\lambda) \le \frac{E(Z)}{k}\, \lambda \qquad (8)$$

and

$$P_2(\lambda) \le \frac{Var\,(Z)}{\left(k - \lambda\, Var(Z)\right)^2} \lambda \qquad (9)$$

The bounds have been derived from Markov and Chebyshev respectively. But in our model:

$$E(Z) = \frac{\Gamma(2m)}{\Gamma(\mathrm{m})^2} \int_0^\infty \frac{z^{m-1}}{(1+z)^{m-1}}\, z\, dz$$
$$= \frac{\Gamma(\mathrm{m}+1)\Gamma(\mathrm{m}-1)}{\Gamma(m)^2}$$
$$= \frac{m}{m-1} \qquad (10)$$

and:

$$E(Z^2) = \frac{\Gamma(2m)}{\Gamma(m)^2} \int_0^\infty \frac{z^{m-1}}{(1+z)^{2m}}\, z^2\, dz$$
$$= \frac{\Gamma(\mathrm{m}+2)\Gamma(\mathrm{m}-2)}{\Gamma(m)^2}$$
$$= \frac{m(m+1)}{(m-1)(m-2)} \qquad (11)$$

In above we suppose that $m > 2$. consequently:

$$Var\,(Z^2) = \frac{2m^2 - m}{(m-1)^2(m-2)} \qquad (12)$$

Replacing above calculations we the first upper bound as:

$$P_1(\lambda) \le \frac{m\lambda}{k(m-1)} \qquad (13)$$

Now solving the equation:

$$P_1(\lambda) = \varepsilon \qquad (14)$$

We get the upper bound on λ as:

$$\lambda = \frac{\varepsilon\, k\, (m-1)}{m} \qquad (15)$$

For the second bound:

$$P_2(\lambda) \le \frac{2m^2 - m}{\left(k - \frac{m}{m-1}\lambda\right)^2} \lambda \sim (2m^2 - m)\frac{\lambda}{k^2} \qquad (16)$$

and solving the equation

Fig. 1. Spatial intensity versus outage constraint

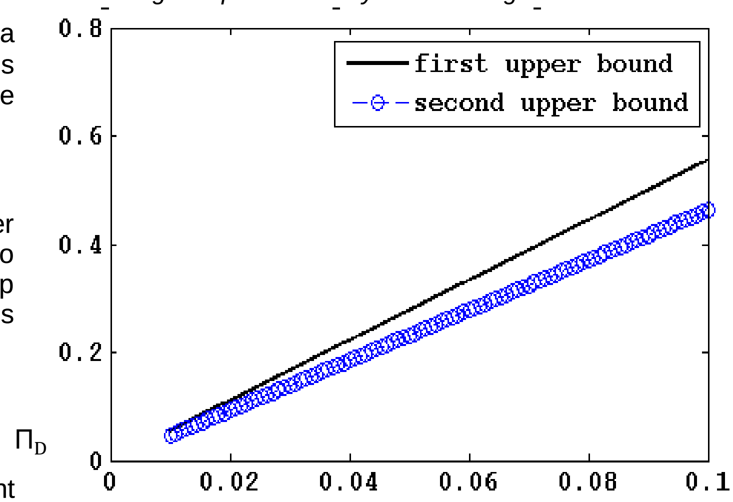
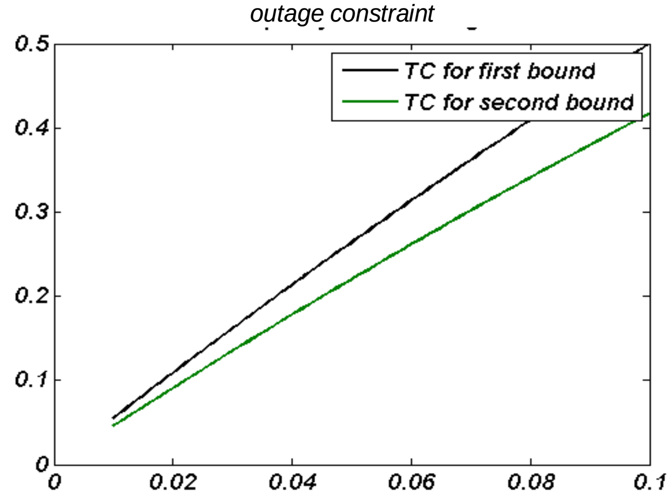


Fig.2. Transmission capacity versus outage constraint



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$P_2(\lambda) = \varepsilon$ (17)

We have:

$$\lambda = \frac{\varepsilon k^2}{2m^2 - m} \quad (18)$$

Simulation results

It is obvious from two derived upper bounds for k an m constant both bounds are linear for ε and it is not so difficult to prove that the second bound is much tighter than the first one. However in the simulation of these networks there are a lot of difficulty and restriction. There are many factors for a exact simulation that we have to ignore some of them. Computing the exact upper and lower bound is almost impossible because of the complexity of models and we are forced to use a lot of approximations as we used in the mathematic calculations. And even in considering of parameters we have to be so aware because for some values of parameters the conclusions are not true for example for α. Even though of being all this problems we see from the simulation results and diagram that two derived bounds are satisfactory and this shows that the model used in this paper is an acceptable model specially the Chebyshev bound that is always much closer to the outage probability. Simulate two upper bounds we suppose that d=5 and $\beta = 3$ and draw the bounds for $\alpha = 2$ and m=3.

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