A geometric method in data envelopment analysis to obtain the region of efficiency

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Abstract

In this paper we consider a geometric method in one of the issues in data envelopment analysis to obtain the region of efficiency (RE) for a super efficientdecision-making unit (DMU) especially when this region is concave. First, we identify production possibility set(PPS) and reduced production possibility set ($RPPS_O$). We determine region of efficiency and the region will be number of convex re-gions by applying this method. In this procedure, asuper-efficient DMU_O is connected to the most n adjacent DMUs on the efficient frontier of.Then we will use equations of faces of convex regions in the boundary of achieved region of efficiency. Finally examples are given for clarifying, this particular subject.

Keywords: data envelopment analysis; variable returns to scale(VRS); sensitivity analysis; linear programming(LP)

1. Introduction

Possible inputs and outputs changes in an extreme efficient-DMU always have been important to DEA researchers. Several researchers have studied data envelopment analysis and ways have been suggested to obtain input and output to allow changes in them. At first, this subject was studied by Charnes and Neralic (Charnes et al. , 1989) and S.Kordrostami, et al (S.Kordrostami et al. , 2007). In this way the optimal solution maintained its optimality after perturbation of optimal basis matrix. Other studies were done by maintaining Thompson et al (Thompson et al. , 1994) and Gonzales-Lima et al (Gonzalez-lima, 1996) and they discussed the optimal dual variables in a multiple model for evaluating desirable DMU. In subsequent studies the developed model is used so that is removed from the reference and variables have been defined by changing inputs and outputs. Besides we considered a definition for region of efficiency by maintaining efficiency of discussed DMU. The aim of this paper is introducing of geometric approach using any method (concave or convex) to find efficient region.

2. Production Possibility Set and Region of Efficiency

Suppose there are n DMUs and each DMU is determined by m inputs and outputs.

$$X_j = (x_{1j},...,x_{mj})^T$$

That DMU_j input's vectors, $j = 1,...,n$

$$Y_j = (y_{1j},...,y_{sj})^T \quad (1)$$

That DMU_j output's vectors, $j = 1,...,n$

Therefore $DMU_j = (X_j^T, Y_j^T)$ is a point in space of dimensional m + s assuming that inputs and outputs are greater than zero i.e. at least one component of inputs and outputs exist that is greater than zero. In this situationPPS in variable returns to scale is defined as bellow:

PPS =

$$\left\{ (X^T, Y^T) \middle| Y \le \sum_{j=1}^{n} \lambda_j Y_j ; X \ge \sum_{j=1}^{n} \lambda_j X_j ;\ \sum_{j=1}^{n} \lambda_j = 1; \lambda_j \ge 0, j = 1,...,n \right\}$$

3. Region of Efficiency

The efficient region of a supper efficient DMU is set of all possible values that gain to remain efficient. Of course will be divided in to two convex and concave regions. In an article, status of being convex of was considered by ValterBoljanicic (ValterBoljuncic, 2006). In this article identifying through concave region by offering desirable intervals of inputs and outputs values has been discussed. Figures 1 and2 respectively identify convex and concave regions and define them as bellow:

$$RE_o = \left\{ \begin{array}{l} (X_o^{*T}, Y_o^{*T})^T \,\middle|\, (X_o^*, Y_o^*) \in PPS \ and \ (X_o^{*T}, Y_o^{*T})^T \\ is\ efficient\ compared\ to\ remaining\ n-1\ DMUs \end{array} \right\}$$

4. Determining Convex Efficient Parts by Geometric Method

First we specify super efficient DMU, then by with eliminating it the reduced production will be created and finally efficient s on frontier will bedetermined by BCC linear programming model. Assume n = m + s and every DMU has m inputs and s outputs and k ≥ n and efficient DMUs must be on. By selecting at most n adjacent DMUs on and considering and selecting bound between inputs and outputs, simplex convex regions will be created. Note that each at most n-simplex region of is determiner of a convex hull. The following conditions are established.

$$\begin{aligned} &Min\left\{X_j^i \mid 1 \le j \le n+1\right\} = X_i' && i = 1,\dots,m, \\ &Min\left\{Y_j^i \mid 1 \le j \le n+1\right\} = Y_l' && l = 1,\dots,s, \\ &Max\left\{X_j^i \mid 1 \le j \le n+1\right\} = X_i'' && i = 1,\dots,m, \\ &Max\left\{Y_l^j \mid 1 \le j \le n+1\right\} = Y_i'' && l = 1,\dots,s. \end{aligned} \tag{4}$$

Then

$$\begin{aligned} &X_i' \le X_i^* \le X_i'' && i = 1,\dots,m, \\ &Y_l' \le Y_l^* \le Y_l'' && l = 1,\dots,s. \end{aligned} \tag{5}$$

If DMU_o be super efficient and it is out of $RPPS_o$'s frontier then can be formed by at most n adjacent DMUs on $RPPS_o$'s frontier and itself. If we change every DMU_o inputs and outputs until remain in RE_o then DMU_o preserve efficiency and with connecting mentioned DMUs will be a convex hull. By selecting n adjacent DMUs on frontier, a base including n-1 vectors defined as Adjacent DMUs can be determined by BCC optimal tables for each DMU_j on $RPPS_o$'s and by related table be considered sub region that table connected to each subset from RE_o are determined distinctive efficient regions. By continuing this action will be obtained the all of convex parts. It is noteworthy that each obtained region by represented method be included DMU_o and n adjacent DMUs on $RPPS_o$'s frontier that they are efficient and convex; Because they form a n-simplex that is convex hull from a set (at most, n+1 DMUs); at most n DMUs are on $RPPS_o$'s frontier in E^n.

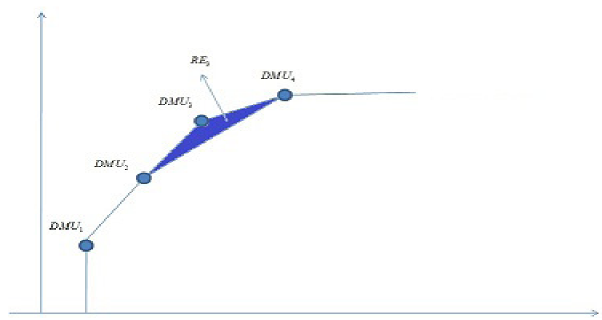
$$\left\{\begin{matrix} DMU_2 - DMU_1, DMU_3 - DMU_1 \\ ,\dots, DMU_n - DMU_1 \end{matrix}\right\}$$

In addition, this base can be extended out of $RPPS_o$'s frontier by DMU_o so that with n basic elements is formed a base as

$$\left\{\begin{matrix} DMU_2 - DMU_o, DMU_3 - DMU_o \\ ,\dots, DMU_n - DMU_o \end{matrix}\right\}$$

thus we will have a convex hull including n+1DMUs (that are not on a hyperplan) in E^n. In other case DMU_o (super efficient) is on passing hyperplan through n-1 adjacent DMUs $RPPS_o$'s frontier because DMU_o is on $RPPS_o$'s frontier. The value of DMU_o efficiency is equal to one and it is on the n-1 dimensional convex hull that is obtained by a base with n adjacent DMUs on built $RPPS_o$'s frontier. So this obtained convex hull will be n-1 dimensional. However the created regions will be efficient and convex.

Fig.1.*Convexregion of efficiency*



Numerical example 1

We find regions of efficiency between DMUs. Assume

$$DMU_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad DMU_2 = \begin{pmatrix} 3 \\ 6 \end{pmatrix} \quad DMU_3 = \begin{pmatrix} 9 \\ 8.5 \end{pmatrix}$$

$$DMU_4 = \begin{pmatrix} 2.5 \\ 4 \end{pmatrix} \quad DMU_6 = \begin{pmatrix} 6 \\ 7 \end{pmatrix}$$

At first, we consider super efficient DMU then we recognize that the efficient DMUs on the $RPPS_o$'s frontier is DMU_2 . Then we abandon DMU_2 and apply BCC model on each of the remaining DMUs. Optimized table of each DMU is shown below. Consider the following model to determine efficiency of DMU_2 on the boundary of $RPPS_2$;

Min Θ

$$\begin{aligned} \text{s.t} \quad & \lambda_1 + 9\lambda_3 + 2.5\lambda_4 + 6\lambda_5 \le \Theta \\ & 2\lambda_1 + 8.5\lambda_3 + 4\lambda_4 + 7\lambda_5 \ge 2 \\ & \lambda_1 + \lambda_3 + \lambda_4 + \lambda_5 \ge 1 \\ & \lambda_i \ge 0 \quad , \quad i = 1,3,4,5 \end{aligned} \tag{6}$$

Using Simplex method final table will be obtained as below:

Table1. *Optimalsimplex table for in BCC model*

	λ_1	λ_3	λ_4	λ_5	θ	S_1	S_2	R.H.S.
	0	-3.25	0	-1.25	0	-1	-0.75	1
λ_1	1	-2.25	0	-1.5	0	0	0.5	1
θ	0	-3.125	0	-1.25	1	-1	-0.75	1
λ_4	0	3.25	1	2.5	0	0	-0.5	0

From efficient table 1, It is clear that DMU_1 and DMU_4 are adjacent and they are on frontier of $RPPS_2$. Now according to geometric method specified minimum and maximum inputs and outputs and efficient region (E_2) is obtained by connecting DMU_2 to DMU_1 and DMU_4 (according to figure 2). In this case, region of efficiency is all of PPS points that are true in given interval as below:

$$1 \le X \le 3, \quad 2 \le y \le 6 \tag{7}$$

Besides we obtain the passed hyperplan on and . In this example because space has two dimensions, obtaining hyperplan is easily possible.

$$\begin{aligned} & DMU_4 - DMU_1 = \begin{pmatrix} 1.5 \\ 2 \end{pmatrix} \Rightarrow \\ & \frac{(x-x_o)}{a} = \frac{(y-y_O)}{b} \Rightarrow \frac{(x-1)}{1.5} = \frac{(y-2)}{2} \\ & \Rightarrow 1.5(y-2) = 2(x-1) \\ & \Rightarrow y = \frac{2}{1.5}x + \frac{1}{1.5} \end{aligned} \tag{8}$$

Now with putting DMU_2 coordinates in the above equation we see that equality is not established while $\frac{2}{1.5}(3) + \frac{1}{1.5} \le 6$ and it is showed that region of top of the passed hyperplan on DMU_1 and DMU_4 is acceptable. i.e.

$$y^* - \frac{2}{1.5}x^* - \frac{1}{1.5} \ge 0. \tag{9}$$

We obtain hyperplan of passing on DMU_2 , DMU_4 and DMU_1 DMU_2 respectively.

$$\begin{aligned} & 0.5x - 2 = -3, \quad 0.5y^* - 2x^* + 3 \ge 0, \\ & y = 2x, \quad y^* - 2x^* \le 0. \end{aligned} \tag{10}$$

This time we examine efficiency of DMU_1 on the boundary $RPPS_2$ of by BCC model. The optimal table is as

Fig.2. *Concave region of efficiency from example 1*

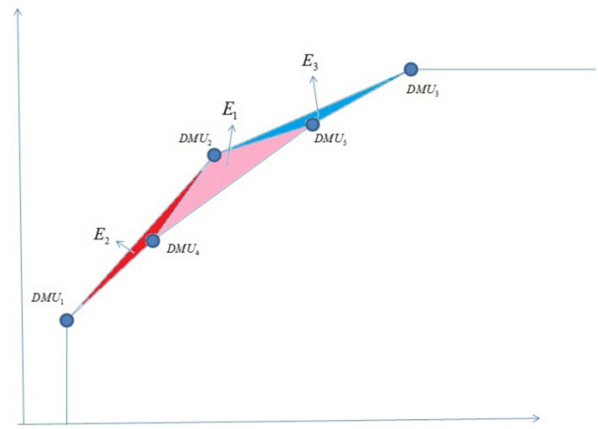


Table 2. *Optimal simplex table forDMU$_1$ in BCC model.*

	λ_1	λ_3	λ_4	λ_5	θ	S_1	S_2	R.H.S.
	-0.3333	-0.5	0	0	0	-0.4	-0.4667	1
λ_5	-0.6667	1.5	0	1	0	0	-0.3333	0
θ	-0.3333	-0.5	0	0	1	-0.4	-0.4667	1
λ_4	1.6667	-0.5	1	0	0	0	0.3333	1

θ value for λ_4 is equal to 1. So DMU_4 is on the boundary of $RPPS_2$. Besides it is clear from the table that it is adjacent with DMU_5 . Now we will have another region of efficient as E_1 and it is obtained by connecting DMU_2 , DMU_4 and DMU_5 and passed hyperplan of DMU_4 , DMU_5 , and DMU_2 , DMU_4 and DMU_2 , DMU_5. This hyperplans and their efficient regions will be calculated respectively.

$$\begin{aligned} & DMU_4 - DMU_5 = \begin{pmatrix} 3.5 \\ 3 \end{pmatrix} \Rightarrow \\ & \frac{(x-x_o)}{a} = \frac{(y-y_O)}{b} \Rightarrow \frac{(x-2.5)}{3.5} = \frac{(y-4)}{3} \\ & \Rightarrow 3.5y - 14 = 3x - 7.5 \\ & \Rightarrow y = \frac{3}{3.5}x + \frac{6.5}{3.5} \\ & \Rightarrow y^* - \frac{3}{3.5}x^* - \frac{6.5}{3.5} \ge 0 \end{aligned} \tag{11}$$

and

$$\begin{aligned} & DMU_2 - DMU_4 = \begin{pmatrix} 0.5 \\ 2 \end{pmatrix} \Rightarrow \\ & \frac{(x-3)}{0.5} = \frac{(y-6)}{2} \\ & \Rightarrow 0.5y - 2x + 3 = 0 \\ & \Rightarrow 0.5y^* - 2x^* + 3 \le 0 \end{aligned} \tag{12}$$

and

$$DMU_5 - DMU_2 = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \Rightarrow$$

$$\frac{(x-3)}{3} = \frac{(y-6)}{1} \tag{13}$$
$$\Rightarrow 3y - x - 15 = 0$$
$$\Rightarrow 3y^* - x^* - 15 \le 0.$$

At intervals

$$\begin{aligned} &2.5 \le x \le 6, \\ &4 \le y \le 7. \end{aligned} \tag{14}$$

All of PPS points are efficient. We consider efficiency of DMU_3 like previous methods.

Table 3. *Optimal simplex table for DMU_3 in BCC model*

	λ_1	λ_3	λ_4	λ_5	θ	S_1	S_2	R.H.S.
	-0.5556	0	-0.2778	0	0	-0.1111	-0.2222	1
λ_3	-3.3333	1	-2	0	0	0	-0.6667	1
θ	-0.5556	0	-0.2778	0	1	-0.1111	-0.2222	1
λ_5	4.3333	0	3	1	0	0	0.6667	0

According to table 3, are adjacent so we consider regions of efficiency that is including ,, and we determine each of active hyperplans of ,and , and , and that this region of efficiency ()is marked in the intervals

$$\begin{aligned} &3 \le x \le 9, \\ &6 \le y \le 7. \end{aligned} \tag{15}$$

That is a PPS part and it has been determined in figure 2. In this case we have divided concave region to 3 pieces (E_1,E_2,E_3).

Numerical Example 2

Showregion of efficiency between given DMUs from table 4.

Table 4. *Data for example 2*

	DMU_1	DMU_2	DMU_3	DMU_4	DMU_5	DMU_6
x-input	3.9	1.1	2.2	5.3	7.2	1
y-input	2.9	3.1	5.4	9.1	12.2	3.6
z- output	0	1	3	5.4	7.1	4.4

First, we determine super efficient DMU. $DMU_6$is taken as a super efficient DMU. Because the value of θ equal of DMU_6 BCC and CCR is one. Now with abandoning this DMU, we look for DMUs on boundary of $RPPS_6$. We review values θ for remaining DMUs i.e.DMU_1, DMU_2 ,DMU_3 ,DMU_4 ,DMU_5 by BCC model.Therefore,

$$\begin{aligned} &Min\,\theta \\ &s.t. \\ &\quad 3.9\lambda_1 + 1.1\lambda_2 + 2.2\lambda_3 + 5.3\lambda_4 + 7.2\lambda_5 \\ &\le 2.2\theta,\ 2.9\lambda_1 + 3.1\lambda_2 + 5.4\lambda_3 + 9.1\lambda_4 + 12.2\lambda_5 \le 5.4\theta, \\ &\quad \lambda_2 + 3\lambda_3 + 5.4\lambda_4 + 7.1\lambda_5 \ge 3, \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 \\ &= 1, \lambda_i \ge 0,\ i = 1,...,5. \end{aligned} \tag{16}$$

We will achieve optimal table by solving simplex table5.

Considering table 5, the value of efficiency of DMU_3 is equal to one so it is on the boundary of $RPPS_6$ also it can be seen that DMU_3 and DMU_2 are adjacent. Similarly, optimize tables related to efficiency of DMU_1, DMU_2 , DMU_4, DMU_5 have been determined in 6, 7 ,8 , 9 tables, respectively.

According to table 6 and table7, DMU_1 is adjacent only with DMU_2 on the $RPPS_6$ border i. e. we can not find adjacent DMUs on this border and a 3-simplex is built with its connection to DMU_6 but we can get a 2-simplex region by using DMU_1, DMU_2 and DMU_6 .

$$DMU_6 - DMU_1 = \begin{pmatrix} 1 \\ 3.6 \\ 4.4 \end{pmatrix} - \begin{pmatrix} 3.9 \\ 2.9 \\ 0 \end{pmatrix} = \begin{pmatrix} -2.9 \\ 0.7 \\ 4.4 \end{pmatrix} \tag{17}$$
$$DMU_2 - DMU_1 = \begin{pmatrix} 1.1 \\ 3.1 \\ 1 \end{pmatrix} - \begin{pmatrix} 3.9 \\ 2.9 \\ 0 \end{pmatrix} = \begin{pmatrix} -2.8 \\ 0.2 \\ 1 \end{pmatrix}$$

$$\begin{vmatrix} x-3.9 & y-2.9 & z \\ -2.9 & 0.7 & 4.4 \\ -2.8 & 0.2 & 1 \end{vmatrix} = 0 \Rightarrow -0.18x - y + 1.38z = -28.02$$

That

$$\begin{aligned} &1 \le x \le 3.9 \\ &2.9 \le y \le 3.6 \\ &0 \le z \le 4.4 \end{aligned} \tag{18}$$

This area is included in active hyperplans of DMU_1, DMU_2 , DMU_2 and DMU_6 and DMU_1 , DMU_6 that we obtain hyperplans and its efficient region respectively.

Table 5. *Optimal table for DMU_3 by BCC model*

	λ_1	λ_2	λ_3	λ_4	λ_5	θ	S_1	S_2	S_3	R.H.S.
	-1.5227	0	0	-0.8091	-1.2477	0	-0.4545	0	-0.25	1
λ_3	-0.5	0	1	2.2	3.05	0	0	0	-0.5	1
S_2	-7.2727	0	0	-3.4291	-4.6527	0	-2.4545	1	-0.2	0
θ	-1.5227	0	0	-0.8091	-1.2477	1	-0.4545	0	-0.25	1
λ_2	1.5	1	0	-1.2	-2.05	0	0	0	0.5	0

Table 6. *Optimal table for DMU_1 by BCC model*

	λ_1	λ_2	λ_3	λ_4	λ_5	θ	S_1	S_2	S_3	R.H.S.
	0	0	-0.7483	-1.982	-3	0	-0.0225	-0.3146	0	1
λ_1	1	0	-0.6494	-1.2607	-2	0	0.3258	-0.4382	0	1
S_3	0	0	-1.3506	-3.1393	-4.1	0	-0.3258	0.4382	1	0
λ_2	0	1	1.6494	2.2607	3	0	-0.3258	0.4382	0	0
θ	0	0	-0.7483	-1.982	-3	1	-0.0225	-0.3146	0	1

Table 7. *Optimal table for DMU_2 by BCC model*

	λ_1	λ_2	λ_3	λ_4	λ_5	θ	S_1	S_2	S_3	R.H.S.
	0	0	-0.7483	-1.982	-3	0	-0.0225	-0.3146	0	1
λ_2	0	1	-0.9011	0.2787	0	0	-0.3483	0.1236	0	1
S_3	0	0	-2.0989	-5.1213	-7.1	0	-0.3483	0.1236	1	0
θ	0	0	-0.7483	-1.982	-3	1	-0.0225	-0.3146	0	1
λ_1	1	0	0.0989	0.7213	1	0	0.3483	-0.1236	0	0

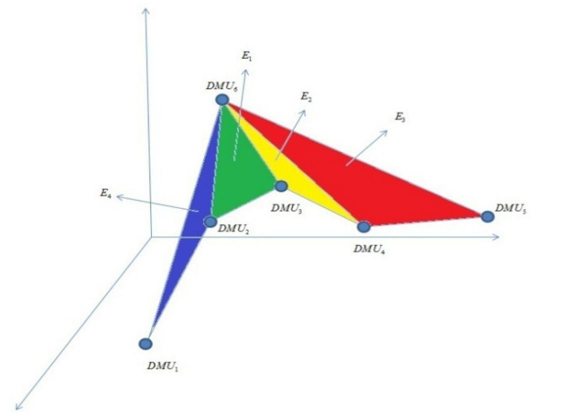
Table 8. *Optimal table for DMU_4 by BCC model*

	λ_1	λ_2	λ_3	λ_4	λ_5	θ	S_1	S_2	S_3	R.H.S.
	-0.2335	-0.0861	0	0	-0.0527	0	0	-0.1099	-0.1694	1
λ_3	2.25	1.8333	1	0	-0.7083	0	0	0	0.4167	0
S_1	4.3374	1.0271	0	0	-0.5749	0	1	-0.5824	0.3938	0
θ	-0.2335	-0.0861	0	0	-0.0527	1	0	-0.1099	-0.1694	1
λ_4	-1.25	-0.8333	0	1	1.7083	0	0	0	-0.4167	1

Table 9. *Optimal table for DMU_5 by BCC model*

	λ_1	λ_2	λ_3	λ_4	λ_5	θ	S_1	S_2	S_3	R.H.S.
	-0.2989	-0.1659	-0.0554	0	0	0	0	-0.082	-0.1495	1
λ_5	-3.1765	-2.5882	-1.4118	0	1	0	0	0	-0.5882	1
λ_4	4.1765	3.5882	2.4118	1	0	0	0	0	0.5882	0
θ	-0.2989	-0.1659	-0.0554	0	0	1	0	-0.082	-0.1495	1
S_1	2.4829	-0.4766	-0.8169	0	0	0	1	-0.5902	-0.0415	0

Figure 3. *Concave region of efficiency from example 2*



$$\begin{vmatrix} x-1.1 & y-3.1 & z-1 \\ 1.1 & 3.1 & 1 \\ 3.9 & 2.9 & 0 \end{vmatrix} = 0$$

$$\Rightarrow -2.9x + 3.9y - 8.9z = 0$$

$$\Rightarrow -2.9x^* + 3.9y^* - 8.9z^* \le 0$$

$$\begin{vmatrix} x-1.1 & y-3.1 & z-1 \\ 1.1 & 3.1 & 1 \\ 1 & 3.6 & 4.4 \end{vmatrix} = 0 \qquad (19)$$

$$\Rightarrow 10.04x - 3.84y + 0.86z = 0$$

$$\Rightarrow 10.04x^* - 3.84y^* + 0.86z^* \ge 0$$

$$\begin{vmatrix} x-3.9 & y-2.9 & z \\ 3.9 & 2.9 & 0 \\ 1 & 3.6 & 4.4 \end{vmatrix} = 0$$

$$\Rightarrow 12.76x - 17.76y + 11.14z = 0$$

$$\Rightarrow 12.76x^* - 17.76y^* + 11.14z^* \le 0$$

Region of efficiency is determined by E_4 in figure 3. According to table 5 DMU_2, DMU_3 according to table 8 are adjacent and DMU_3, DMU_4 are too, but because of DMU_2 and DMU_4 are not adjacent so a 3-simplex cannot be built by DMU_2, DMU_4, DMU_6. There for we consider a region that is built by DMU_2, DMU_3, DMU_6.

$$DMU_3 - DMU_2 = \begin{pmatrix} 1.1 \\ 2.3 \\ 2 \end{pmatrix} \qquad (20)$$

$$DMU_6 - DMU_2 = \begin{pmatrix} -0.1 \\ 0.5 \\ 3.4 \end{pmatrix}$$

$$\begin{vmatrix} x-1.1 & y-3.1 & z-1 \\ 1.1 & 2.3 & 2 \\ -0.1 & 0.5 & 3.4 \end{vmatrix} = 0$$

$$\Rightarrow 6.82x - 3.94y + 0.78z = -3.932$$

Also we calculate active hyperplans on DMU_2, DMU_3 and DMU_2, DMU_6 and DMU_3, DMU_6, respectively until we determine region of efficiency.

$$\begin{vmatrix} x-1.1 & y-3.1 & z-1 \\ 1.1 & 3.1 & 1 \\ 2.2 & 5.4 & 3 \end{vmatrix} = 0$$

$$\Rightarrow 3.9x - 1.1y - 0.88z = 0$$

$$\Rightarrow 3.9x^* - 1.1y^* - 0.88z^* \le 0$$

$$\begin{vmatrix} x-1.1 & y-3.1 & z-1 \\ 1.1 & 3.1 & 1 \\ 1 & 3.6 & 4.4 \end{vmatrix} = 0$$

$$\Rightarrow 10.04x - 3.84y + 0.86z = 0 \qquad (21)$$

$$\Rightarrow 10.04x^* - 3.84y^* + 0.86z^* \ge 0$$

$$\begin{vmatrix} x-1.1 & y-3.1 & z-1 \\ 2.2 & 5.4 & 3 \\ 1 & 3.6 & 4.4 \end{vmatrix} = 0$$

$$\Rightarrow 12.96x - 6.68y + 2.52z = 0$$

$$\Rightarrow 12.96x^* - 6.68y^* + 2.52z^* \le 0$$

At intervals

$$1 \le x \le 2.2$$
$$3.1 \le y \le 5.4 \qquad (22)$$
$$1 \le z \le 4.4$$

This region is shown in figure 3 with E_1.

According to table 8, DMU_3, and DMU_4, are adjacent; According to table 9, DMU_4, and DMU_5 are adjacent but there is no table showing that DMU_3, and DMU_5 are adjacent. So we cannot appoint a 2-simplex by DMU_3, DMU_4 and DMU_6. First we obtain this hyperplans equations and then we determine region of efficiency.

$$DMU_6 - DMU_3 = \begin{pmatrix} -1.2 \\ -1.8 \\ 1.4 \end{pmatrix}$$

$$DMU_4 - DMU_3 = \begin{pmatrix} 3.1 \\ 3.7 \\ 2.4 \end{pmatrix}$$

$$\begin{vmatrix} x-2.2 & y-5.3 & z-1 \\ -1.2 & -1.8 & 1.4 \\ 3.1 & 3.7 & 2.4 \end{vmatrix} = 0 \qquad (23)$$

$$\Rightarrow -9.5x + 7.22y + 1.14z = 18.506$$

$$(\textit{equation of hyperplan})$$

Now we search this effective region of this hyperplan, i.e. we get active hyperplans on DMU_3, DMU_4, and DMU_4, DMU_6 and DMU_3, DMU_6 and effective region respectively.

$$\begin{vmatrix} x-2.2 & y-5.4 & z-3 \\ 2.2 & 5.4 & 3 \\ 5.3 & 9.1 & 5.5 \end{vmatrix} = 0$$

$$\Rightarrow 1.86x + 4.02y - 8.6z = 0$$

$$\Rightarrow 1.86x^* + 4.02y^* - 8.6z^* \le 0$$

$$\begin{vmatrix} x-5.3 & y-9.1 & z-5.4 \\ 5.3 & 9.1 & 5.4 \\ 1 & 3.6 & 4.4 \end{vmatrix} = 0 \quad (24)$$

$$\Rightarrow 20.6x - 17.92y + 9.98z = 0$$

$$\Rightarrow 20.6x^* - 17.92y^* + 9.98z^* \le 0$$

$$\begin{vmatrix} x-2.2 & y-5.4 & z-3 \\ 2.2 & 5.4 & 3 \\ 1 & 3.6 & 4.4 \end{vmatrix} = 0$$

$$\Rightarrow 12.96x - 6.68y + 2.52z = 0$$

$$\Rightarrow 12.96x^* - 6.68y^* + 2.52z^* \ge 0$$

And at intervals

$$1 \le x \le 5.3$$
$$3.6 \le y \le 9.1 \quad (25)$$
$$3 \le z \le 5.4$$

Efficiency region of E_2 is determined. Table 9 shows that DMU_4 , DMU_5 are adjacent so first we calculate active hyperplans equations of DMU_4 , DMU_5, and DMU_6. So we will determine effective region of hyperplans of DMU_4 , DMU_5, DMU_4 , DMU_6, and DMU_5 , DMU_6, respectively. This region is shown in figure 3 with E_3.

$$DMU_6 - DMU_4 = \begin{pmatrix} -4.3 \\ -5.5 \\ -1 \end{pmatrix}$$

$$DMU_5 - DMU_4 = \begin{pmatrix} 1.9 \\ 3.1 \\ 1.7 \end{pmatrix}$$

$$\begin{vmatrix} x-5.3 & y-9.1 & z-5.4 \\ -4.3 & -5.5 & -1 \\ 1.9 & 3.1 & 1.7 \end{vmatrix} = 0$$

$$\Rightarrow -6.25x + 5.41y - 2.88z = 0.544$$
$$(equation\ of\ hyperplan)$$

$$\begin{vmatrix} x-5.3 & y-9.1 & z-5.4 \\ 5.3 & 9.1 & 5.4 \\ 7.2 & 12.2 & 7.1 \end{vmatrix} = 0$$

$$\Rightarrow -1.27x + 1.25y - 0.86z = 0 \quad (26)$$

$$\Rightarrow -1.27x^* + 1.25y^* - 0.86z^* \le 0$$

$$\begin{vmatrix} x-5.3 & y-9.1 & z-5.4 \\ 5.3 & 9.1 & 5.4 \\ 1 & 3.6 & 4.4 \end{vmatrix} = 0$$

$$\Rightarrow 20.6x - 17.92y + 9.98z = 0$$

$$\Rightarrow 20.6x^* - 17.92y^* + 9.98z^* \ge 0$$

$$\begin{vmatrix} x-7.2 & y-12.2 & z-7.1 \\ 7.2 & 12.2 & 7.1 \\ 1 & 3.6 & 4.4 \end{vmatrix} = 0$$

$$\Rightarrow 28.12x - 24.5y + 13.72z = 0$$

$$\Rightarrow 28.12x^* - 24.5y^* + 13.72z^* \ge 0$$

At intervals

$$1 \le x \le 7.2$$
$$3.6 \le y \le 12.2 \quad (27)$$
$$4.4 \le z \le 7.1$$

5. Conclusion

This paper has attempted to obtain region of efficiency. If region of efficiency is concave then it will separate to convex regions and efficient that each sub region is a s-simplex (s ≤n) in E^n .

Each convex region is obtained from connection super efficient DMU_O and at most, n adjacent DMUs on $RPPS_o$'s border. By this method has considered completed RE_o

6. References

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