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Vibration analysis of composite laminated plates using higher-order shear deformation theory with zig-zag function

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Abstract

The ever increasing use of composite materials in advanced technology areas like aerospace, automotive, and sports equipments, have prompted to play a prominent role. A theory which can predict accurately the behavior of the composite structure and the failure mechanism is very much essential. In this work an analytical procedure is developed, to investigate the free vibration characteristics of different laminated composite plates based on higher order shear displacement model with zig-zag function. This function improves slope discontinuities at the interfaces of laminated composite plates. The related functions are obtained using the dynamic version of principle of virtual work or Hamilton's principle. The solutions are obtained using Navier's and numerical methods for anti-symmetric cross-ply and angle-ply laminates with a specific type of simply supported boundary conditions SS-1 and SS-2 respectively. In this paper the Numerical results are presented for free vibration of anti-symmetric cross-ply and angle-ply laminated plates. All the solutions presented are close agreement with the theory of elasticity and closed form solutions available in the literature.

Keywords: Vibrational analysis, composite material, zig-zag function.

Introduction

The plates are straight, plane surface structures whose thickness is slight compare to other dimensions geometrically, they are bound either by straight or curved lines. Statically plates have, simply supported and fixed boundary conditions, including elastic supports and elastic restraints or in some cases point supports. The static or dynamic loads are carried by plates are predominantly perpendicular to the plate surface. The accurate prediction of the response characteristics of laminated structures is a challenging task because of their intrinsic anisotropy, heterogeneity, and low ratio of the transverse shear modulus to the in-plane Young's modulus. Hence, it is necessary to analyze the free vibration characteristics of laminated composite plates. Several plate theories have been developed to analyze laminated composite plates. Anderson & Irons *et al.* (1968), calculated frequencies and buckling loads of plates using simple displacement functions for the flexure of triangular plate elements. They attained good accuracy in frequency calculations, and made good predictions for stability, using coarse element subdivisions. Reddy (1979), has generalized the Mindlin's plate theory for isotropic plates to laminated anisotropic plates and included shear deformation and rotary inertia effects. He has presented the finite element solutions for rectangular plates of anti-symmetric angle-ply laminates whose material properties are typical of a highly anisotropic composite material. Reddy & Kuppusamy (1984) described the three dimensional elasticity equations and the associated finite element model for natural vibrations of laminated rectangular plates. A number of cross-ply and angle-ply rectangular plates are analyzed for natural frequencies. Reddy & Phan (1985) used a higher-order shear deformation theory to determine the natural frequencies and buckling loads of elastic plates. The theory accounts for parabolic distribution of the transverse shear strains through the thickness of the plate and rotatory inertia. Marur and Kant (1996) proposed a higher-order displacement model for the free vibration analysis of sandwich and composite beam fabrications. These theories are modeled by taking cubic variation of axial strain and eliminate the need for shear correction coefficient. The classical laminate theory (CLT), the FSDT and the cubic third order theory (TOT) for laminated beams and plates under mechanical loading, have been dealt in detail (Reddy,1997). Ji Wang *et al*. (1999) derived a finite element formulation of the piezoelectric vibrations of quartz resonators based on Mindlin plate theory. Kant & Swaminathan (2001) presented solutions to the natural frequency analysis of simply supported composite and sandwich plates. Carrera E (2004), discussed the use of the Murakami's zig-zag function in the modeling of layered plates and shells. In this, ZZF modeling was discussed with various loads and compared with other theories. Tongan Wan *et al*. (2008) developed eight dynamic governing equations and the corresponding boundary conditions were derived through the application of Hamilton's principle. The extended formulation was applied to the free vibration analysis of soft-core and honeycomb-core sandwich plates with anti-symmetric and symmetric lay-ups. Cetkovic (2009) dealt with the local-global analysis of laminated composite and sandwich plates using a layerwise displacement model. The proposed model assumes piece-wise linear variation of in-plane displacement components and constant transverse displacement through thickness of the plate. Shariyat (2010) developed a generalized global-local theory that guarantees the continuity condition of all of the displacement and transverse stress components and considers the transverse flexibility under thermo-mechanical loads is introduced. Song Xiang *et al.* (2011) proposed a nth-order shear deformation theory to analyze the free vibration of laminated composite plates. The nth-order shear deformation theory satisfies the zero transverse shear stress boundary conditions on the top and bottom surface of the plate. In this paper, a Higher -order shear deformation theory with Zig Zag function is proposed to develop the analytical procedure and to analyze the free vibration of laminated composite plates which takes care of the sudden change of properties from lamina to lamina.

The higher-order shear deformation theory with zig-zag function

A rectangular plate of $0 \le x \le a$; $0 \le y \le b$ and $-\frac{h}{2} \le z \le \frac{h}{2}$ is considered and the higher-order shear deformation theory with zig-zag function is assumed to be

$$\left.\begin{aligned} u(x,y,z) &= u_o(x,y) + z\theta_x(x,y) + z^2 u_o^*(x,y) + z^3\theta_x^*(x,y) + \theta_k s_1(x,y) \\ v(x,y,z) &= v_o(x,y) + z\theta_y(x,y) + z^2 v_o^*(x,y) + z^3\theta_y^*(x,y) + \theta_k s_2(x,y) \\ w(x,y,z) &= w_o(x,y) \end{aligned}\right\} \quad (1)$$

Where

u_0, v_0 , w_0, s_1 and s_2 denote the displacements of a point (x, y) on the mid-plane.

θ_x, θ_y are rotations of the normal to the midplane about y and x-axes

u_0^*, v_0^*, θ_x^*, θ_y^* are the higher-order deformation terms defined at the mid-plane.

θ_k is the Zig-Zag function, defined as:

$$\theta_k = 2(-1)^k \frac{Z_k}{h_k}$$

Z_k is the local transverse coordinate with its origin at the center of the k^{th} layer.

h_k is the corresponding layer thickness.

The strain components are

$$\begin{aligned} \varepsilon_x &= \varepsilon_{xo} + zk_{sx} + z^2\varepsilon_{xo}^* + z^3 k_x^* \\ \varepsilon_y &= \varepsilon_{yo} + zk_{sy} + z^2\varepsilon_{yo}^* + z^3 k_y^* \\ \varepsilon_z &= 0 \\ \gamma_{xy} &= \varepsilon_{xyo} + zk_{sxy} + z^2\varepsilon_{xyo}^* + z^3 k_{xy}^* \\ \gamma_{yz} &= \phi_{sy} + z\varepsilon_{yzo} + z^2\phi_y^* \\ \gamma_{xz} &= \phi_{sx} + z\varepsilon_{xzo} + z^2\phi_x^* \end{aligned} \quad (2)$$

The stress-strain relationships in the global x-y-z coordinate system can be written as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix}^L = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} & 0 & 0 & 0 \\ Q_{12} & Q_{22} & Q_{23} & 0 & 0 & 0 \\ Q_{13} & Q_{23} & Q_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & Q_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & Q_{66} \end{bmatrix}^L \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix}^L$$

(3)

The governing equations of displacement model will be derived using the principle of virtual work as

$$\int_0^T (\delta U + \delta V - \delta K)\, dt = 0 \quad (4)$$

The virtual work statement shown in Eq. (4) , integrating through the thickness of laminate, the in-plane and transverse force and moment resultant relations in the form of matrix obtained as:

$$\begin{Bmatrix} N \\ N^* \\ -- \\ M \\ M^* \\ -- \\ Q \\ Q^* \end{Bmatrix} = \begin{bmatrix} A & | & B & | & 0 \\ \overline{B^t} & | & \overline{D_b} & | & \overline{0} \\ 0 & | & 0 & | & D_s \end{bmatrix} \begin{Bmatrix} \varepsilon_0 \\ \varepsilon_0^* \\ -- \\ K_s \\ K^* \\ -- \\ \phi \\ \phi^* \end{Bmatrix} \quad (5)$$

Equating the coefficients of each of virtual displacements δu_o δv_0, δw_0, $\delta\theta_x$, $\delta\theta_y$, δu_0^*, δv_0^*, $\delta\theta_x^*$, $\delta\theta_y^*$, δs_1, δs_2 to zero, the equations of motion obtained as:

$$\delta u_0 : \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = I_1\ddot{u}_0 + I_2(\ddot{\theta}_x + R\ddot{s}_1) + I_3\ddot{u}_0^* + I_4\ddot{\theta}_x^*$$

$$\delta\theta_y : \frac{\partial M_y}{\partial y} + \frac{\partial M_{xy}}{\partial x} - Q_y = I_2\ddot{v}_0 + I_3(\ddot{\theta}_y + R\ddot{s}_2) + I_4\ddot{v}_0^* + I_5\ddot{\theta}_y^*$$

$$\delta u_0^* : \frac{\partial N_x^*}{\partial x} + \frac{\partial N_{xy}^*}{\partial y} = I_3\ddot{u}_0 + I_4(\ddot{\theta}_x + R\ddot{s}_1) + I_5\ddot{u}_0^* + I_6\ddot{\theta}_x^*$$

$$\delta v_0^* : \frac{\partial N_y^*}{\partial y} + \frac{\partial N_{xy}^*}{\partial x} = I_3\ddot{v}_0 + I_4(\ddot{\theta}_y + R\ddot{s}_2) + I_5\ddot{v}_0^* + I_6\ddot{\theta}_y^*$$

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$$\delta\theta_x^*: \frac{\partial M_x^*}{\partial x} + \frac{\partial M_{xy}^*}{\partial y} - Q_x^* = I_4\ddot{u}_0 + I_5(\ddot{\theta}_x + R\ddot{s}_1) + I_6\ddot{u}_0^* + I_7\ddot{\theta}_x^*$$

$$\delta\theta_y^*: \frac{\partial M_y^*}{\partial y} + \frac{\partial M_{xy}^*}{\partial x} - Q_y^* = I_4\ddot{v}_0 + I_5(\ddot{\theta}_y + R\ddot{s}_2) + I_6\ddot{v}_0^* + I_7\ddot{\theta}_y^*$$

$$\delta S_1: \frac{\partial RM_x}{\partial x} + \frac{\partial RM_{xy}}{\partial y} - RQ_x = I_2R\ddot{u}_0 + I_3R(\ddot{\theta}_x + R\ddot{s}_1) + I_4R\ddot{u}_0^* + I_5R\ddot{\theta}_x^*$$

$$\delta S_2: \frac{\partial RM_y}{\partial y} + \frac{\partial RM_{xy}}{\partial x} - RQ_y = I_2R\ddot{v}_0 + I_3R(\ddot{\theta}_x + R\ddot{s}_1) + I_4R\ddot{v}_0^* + I_5R\ddot{\theta}_y^*$$

(6)

The Eq. (6) is expressed in terms of displacements u_0, v_0, w_0, θ_x, θ_y, u_0^*, v_0^*, θ_x^*, θ_y^*, s_1, s_2 by substituting for the force and moment resultants

The Navier's solutions of simply supported anti symmetric cross ply laminated plates :

The SS-1 boundary conditions for the anti symmetric cross ply laminated plates are:

At edges x = 0 and x = a

$v_0 = 0$, $w_0 = 0$, $\phi_y = 0$, $M_x = 0$, $v_0^* = 0$, $\phi_y^* = 0$, $M_x^* = 0$, $N_x = 0$, $N_x^* = 0$, $s_2 = 0$ 7(a)

At edges y = 0 and y = b

$u_0 = 0$, $w_0 = 0$, $\phi_x = 0$, $M_y = 0$, $u_0^* = 0$, $\phi_x^* = 0$, $M_y^* = 0$, $N_y = 0$, $N_y^* = 0$, $s_1 = 0$ 7(b)

The SS-2 boundary conditions for the anti symmetric angle ply laminated plates are:

At edges x = 0 and x = a

$u_0 = 0$, $w_0 = 0$, $\phi_y = 0$, $N_{xy} = 0$, $M_x = 0$, $u_0^* = 0$, $\phi_y^* = 0$, $M_x^* = 0$, $N_{xy}^* = 0$, $s_1 = 0$ 8(a)

At edges y = 0 and y = b

$v_0 = 0$, $w_0 = 0$, $\phi_x = 0$, $N_{xy} = 0$, $M_y = 0$, $v_0^* = 0$, $\phi_x^* = 0$, $M_y^* = 0$, $N_{xy}^* = 0$, $s_2 = 0$ 8(b)

The displacements at the mid plane will be defined to satisfy the boundary conditions in Eq.(7) & (8). These displacements will be substituted in governing equations to obtain the equations in terms of A,B,D parameters. The obtained equations will be reduced to the Eigen value problem. For a non-trivial solution the determinant of the coefficient matrix obtained should be zero, which yields the characteristic equation as :

$([S] - \lambda [M]) = 0$ (9)

Where $\lambda = \omega^2$ is the Eigen value.

7 (a)

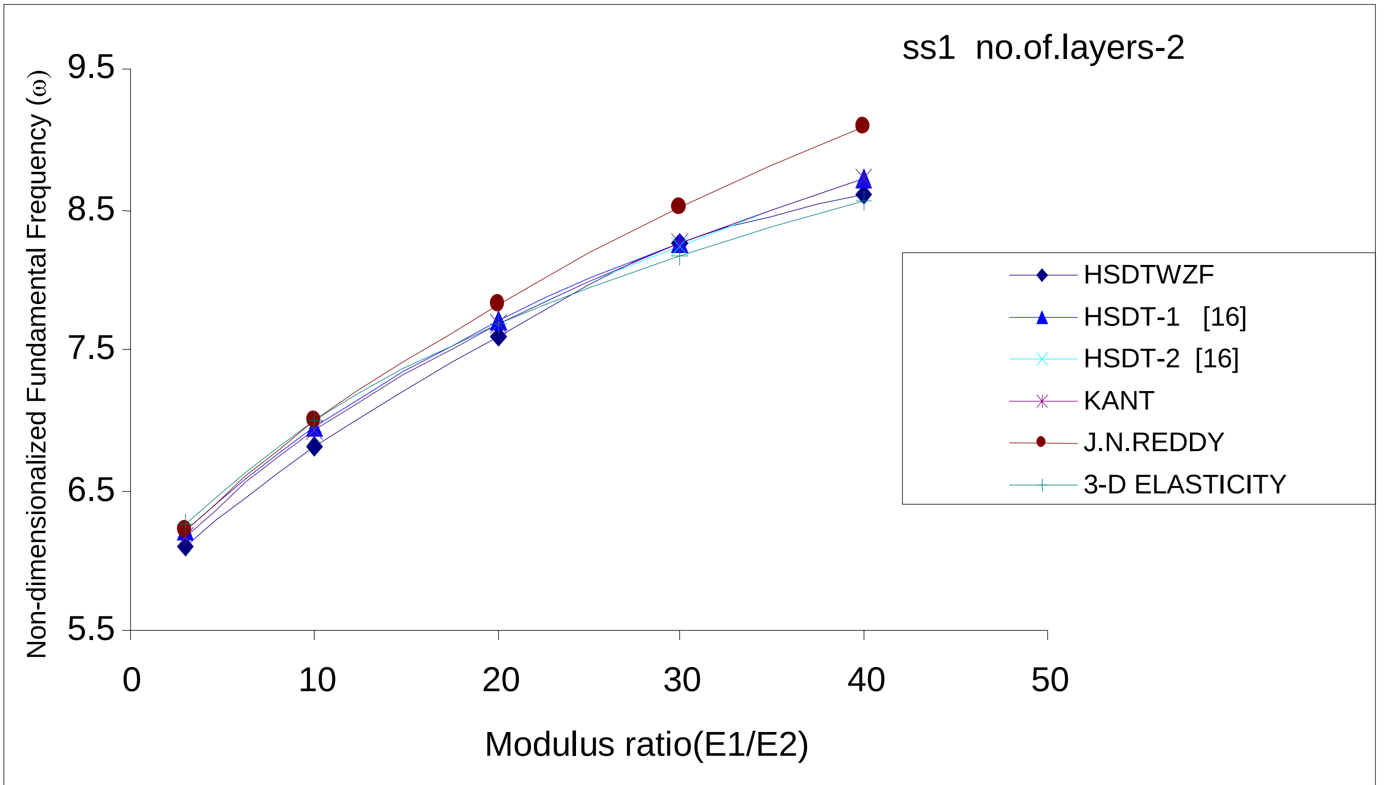

Fig 1. Non-dimensionalized fundamental frequency (w) Vs Modulus ratios (E1/E2) for simply supported anti-symmetric cross-ply laminated square plate

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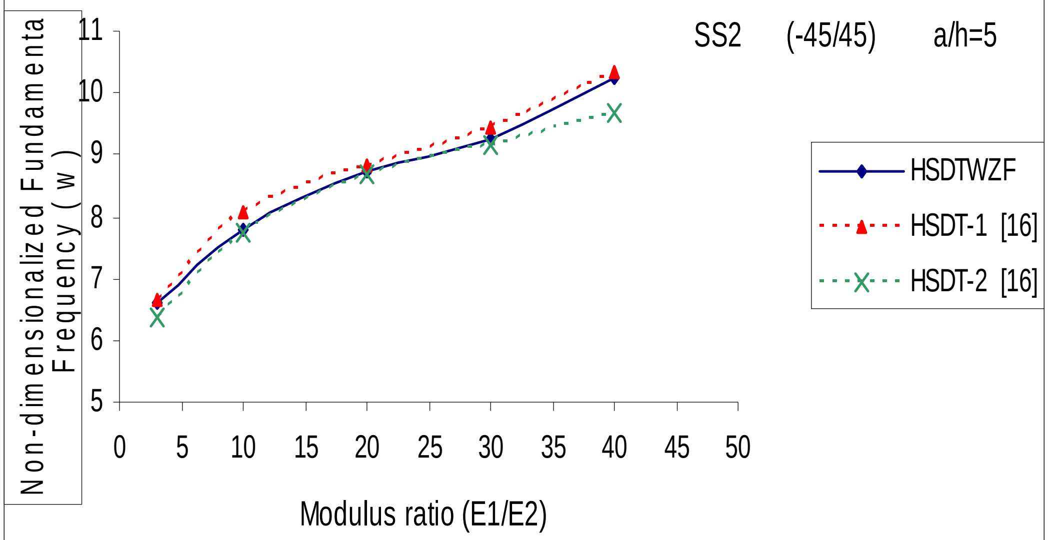

Fig. 2 Non-dimensionalized fundamental frequency (ω) Vs Modulus ratios (E1/E2) for simply supported anti-symmetric angle-ply laminated square plate

The real positive roots of the Eq.(9) gives the square of the natural frequency ω_{mn} associated with mode (m, n). The smallest of the equation is called the fundamental natural frequency.

Results and discussion

The simply supported boundary conditions (SS-1) shown in Eq. (7) are considered for solutions of anti-symmetric cross-ply laminates ,whereas Eq. (8) for solutions of anti-symmetric angle-ply laminates using a higher order shear deformation theory with zig- zag function. The material properties of graphite epoxy used for each lamina of the laminated composite plate are

Material-1

$E_1/E_2=3$, $G_{12}/E_2=0.6$, $G_{23}/E_2=0.5$, $E_2=E_3=10^6$, $G_{12}=G_{13}=0.5$ and $\mu_{12} = \mu_{23} = \mu_{13} = 0.25$.

Material-2

$E_1/E_2=40$, $G_{12}/E_2=0.5$, $G_{23}/E_2=0.2$, $E_2=E_3=10^6$, $G_{12}=G_{13}=0.5$ and $\mu_{12}=\mu_{23}=\mu_{13}=0.25$.

The natural frequencies of general rectangular composites are presented here in non-dimensional form using the following multiplier

$$\overline{\omega} = (\omega\, a^2/h)\sqrt{(\rho/E_2)}\,.$$

Fig 1 shows the variation of non dimentionalised fundamental natural frequency as a function of modulus ratio. The results clearly indicate that even at higher modes of vibration, the natural frequency values obtained using the present theory are in good agreement with the 3D Elasticity. Fig 2 and 3 shows the non-dimensionalized natural frequency as a function of modulus ratio for anti-symmetric cross-ply and angle-ply laminated composite plates. From the figures it is observed that with the increase in modulus ratio the effect of shear deformation increases on the fundamental frequencies. The non-dimensionalized natural frequencies of the anti-symmetric cross-ply and angle-ply laminates are shown in Fig 4 and 5 as a function of side to thickness ratio and number of layers respectively. From the plots it is noticed that the shear deformation effect is increasing significantly on the vibration of plates with decrement in bending-stretching coupling. The effect of shear deformation decreases with increasing values of a/h. This decrease is slower for 4, 6 and 8 layered plates. Fig.6 and 7 show the plots of fundamental frequencies as a function of aspect ratio (a/b) for anti-symmetric cross-ply and angle-ply laminated composite plates. As the number of layers increase without changing the total thickness, decreases the extensional coupling effect and thus increase the fundamental frequencies. This effect of the number of

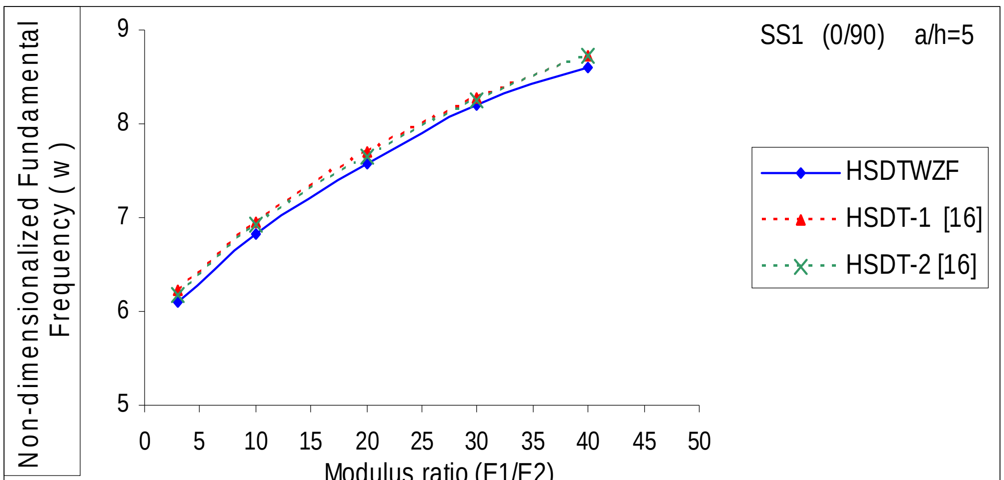

Fig.3 Non-dimensionalized fundamental frequency (ω) Vs Modulus ratios (E1/E2) for simply supported anti-symmetric cross-ply laminated square plate

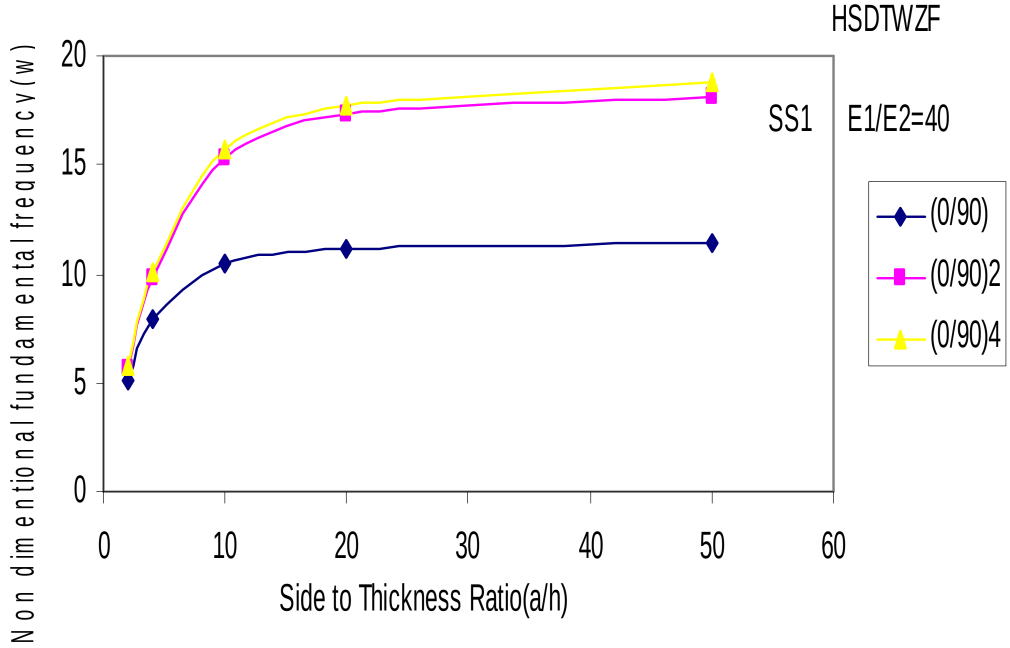

Fig-4 Non-Dimentional fundamental frequency(w) vs side to thickness ratio(a/h) for simply supported anti symmetric cross ply laminated square plate

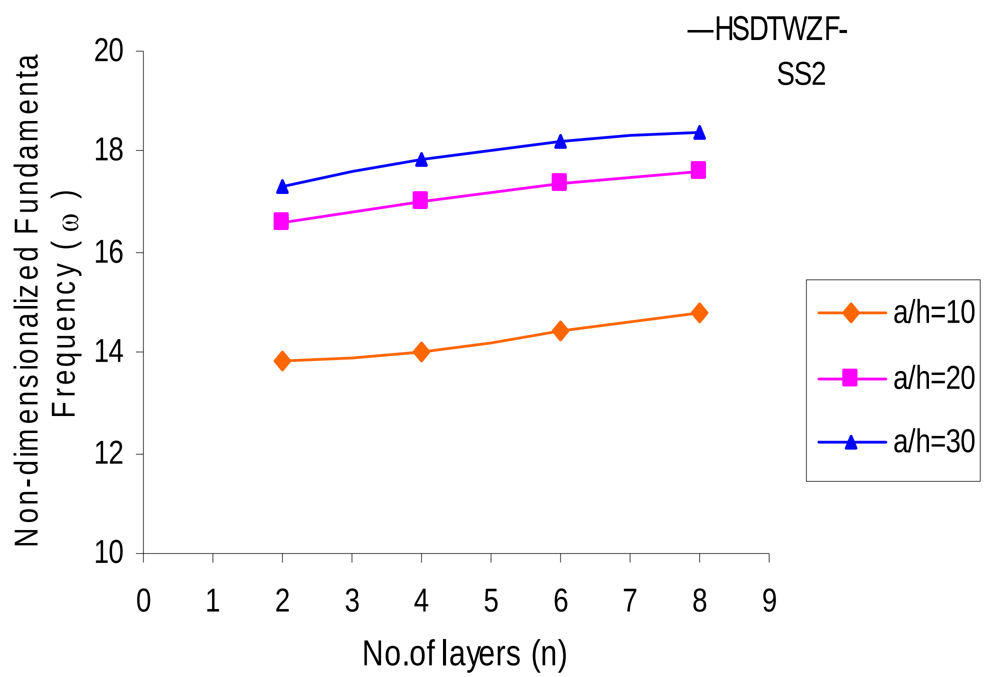

Fig.5 Non-dimensionalized fundamental frequency (ω) Vs no.of layers (n) for simply supported anti-symmetric angle-ply (-45/45) laminated square plate

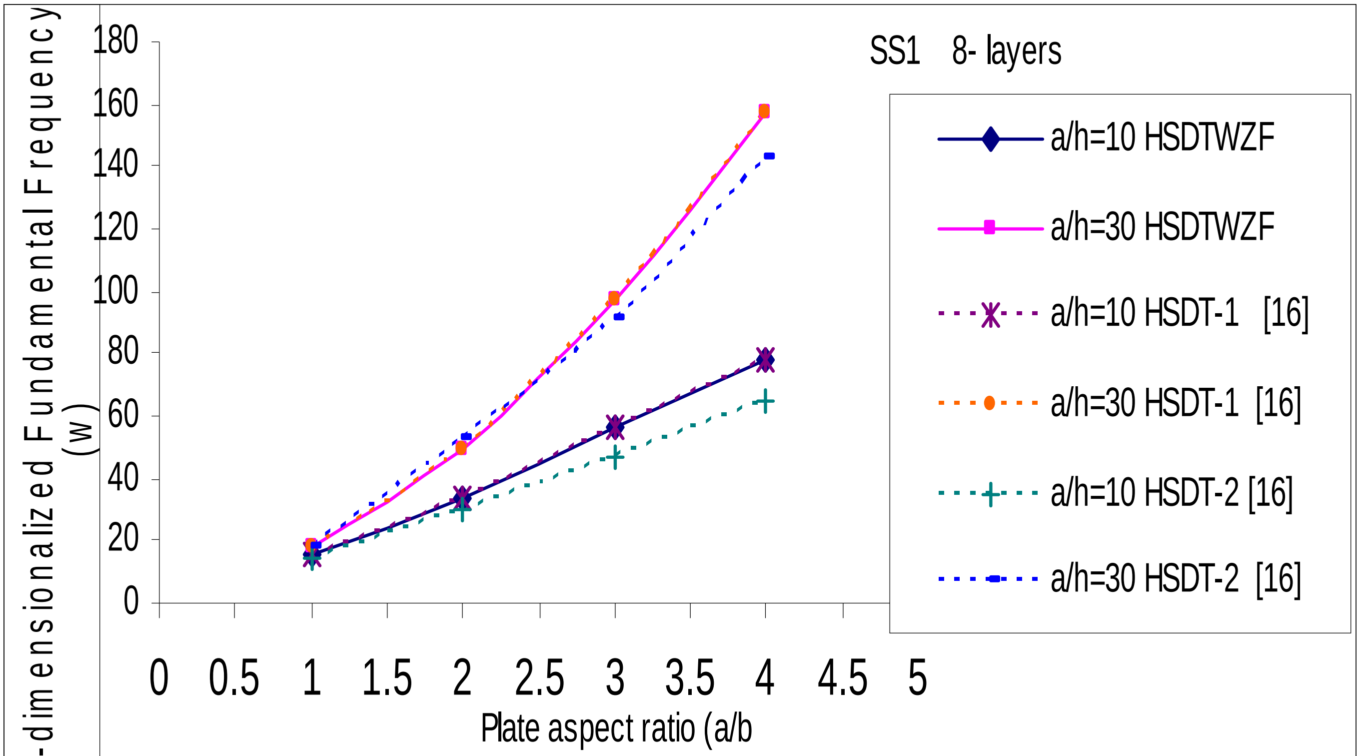

Fig. 6 Non-dimensionalized fundamental frequency (ω) Vs plate aspect ratio (a/b) for simply supported anti-symmetric cross-ply laminated plate

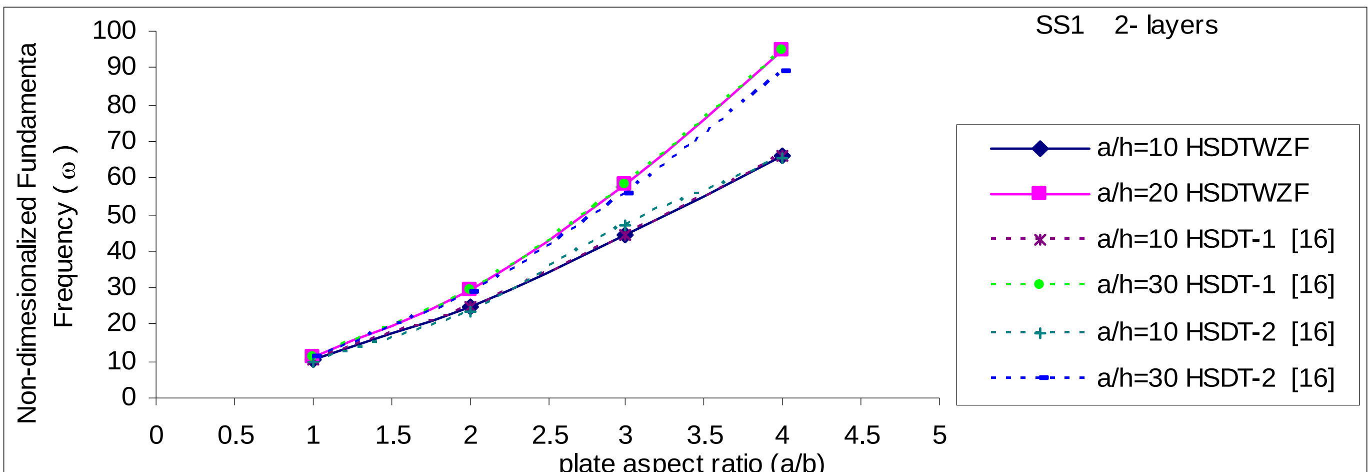

Fig.7 Non-dimensionalized fundamental frequency (ω) Vs plate aspect ratio (a/b) for simply supported anti-symmetric cross-ply laminated plate

layers is most pronounced for angle-ply laminated plates. It is seen that the coupling between bending and stretching had a significant effect on the behavior of anti-symmetric laminates with few Lamina. In all the cases the natural frequency values obtained using the present theory are found to effective when compared with other published work.

Conclusions

In this paper a powerful analytical method is developed to find the vibrational characteristics of laminated composite plates using higher order theory. For accurate free vibration analysis, the displacement model with zig-zag function is used for laminated composite anti-symmetric cross-ply and angle-ply plates. The inclusion of the zig -zag function in a displacement model has resulted to be more effective than the introduction of higher order polynomials. It is observed that multi layered plate theories are improved using zig -zag function. In anti-symmetric laminates, the higher order terms u_0^*, v_0^*, w_0^* and ϕ_x^*, ϕ_y^*, ϕ_z^* lead to reduce the natural frequencies.

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