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Simulation of Elasto-plastic Deformations in Composites by Flow Rules

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Abstract

In this paper, a novel model is proposed to predict stress behavior in the fibrous polymer composites under an applied tensile axial loading. Tangential stress is obtained by simulating between polymeric matrix flow (polyester) and fluid flow. In the other words, creep behavior of the petrochemical material or polymeric resin (matrix) is modeled using fluid flow theory. Furthermore, behavior of the shear stress is predicted by suitable mathematical functions. Finally, important relation is extracted by combining the solid and fluid mechanics formulations.

Keywords: Composite, Creep, Fluid Mechanics, Solid Mechanics

1. Introduction

Numerous researchers have studied the stationary creep behavior by analytical, experimental, and numerical approaches. A small number of studies have been performed for determining the shear (tangential) stress in the creeping matrix by the tangential stress definitions and models employing fluid mechanics methods and theories analytically. For instance, shear stress has been obtained by shear-lag model or other formulations by previous researchers in the creeping matrix and elastic fiber.

Previously, widespread investigations have been conducted to obtain the creep properties of the short fiber composites[1–3]. Viscous flow of aligned composites under tensile creep in the direction of the fibers has been analyzed[4]. An analysis has been carried out for predicting the stress distribution in unidirectional discontinuous fiber composites, based on the shear-lag theory, and the load transfer at fiber ends[5].

Other kinds of deformations have also been studied about SMA's, PZT's, rails, and so on, using theoretical method and FEM[6,7]. Herschel-Bulkley model is also known as the yield extended power law form which is a hybrid of Bingham plastic and the power law models[8]. The power law model does not consider the yield point, while the Herschel-Bulkley model takes the yield point into account[9-11].

Additionally, creep life prediction of thermally exposed Rene 80 super alloy and creep constitutive model, and component lifetime estimation (the case of niobium-modified 9Cr-1Mo steel weldments) have been investigated respectively[12,13]. Also, analytical solutions for determining creep stresses in thick-walled spherical pressure vessels and cylinders have been done[14,15]. Recently, theoretical analyses and different methods have been analytically proposed with the purpose of determining proper solutions and algorithms for analyzing nonlinear differential and ordinary equations. Moreover, some analytical

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models and such methods were presented to solve and analyze the nonlinear partial equations theoretically[16–18].

A number of studies have been carried out regarding engineering and mathematical problems for investigating such engineering and mathematical problems, which can help investigators[19–24].

In this research paper, a novel concept is proposed; in which viscosity of the creeping polymer matrix, such as polyester is used for determining the shear stress by mechanics of fluid theory. So, shear stress (tangential) in the creeping matrix is obtained by the tangential stress using fluid mechanics models. Therefore, tangential stress of the crept resin (matrix) is determined utilizing the suitable and correct tangential (shear) strain rates and viscosity of the crept polymeric resin (matrix) in the steady state creep like the Newtonian fluid. Shear strain rates are determined by the assumed and satisfied axial and radial displacement rates with considering suitable boundary and incompressibility conditions. Ultimately, shear stress in the crept matrix is obtained by substitution of the axial and radial displacement rates into the shear stress formulation utilizing fluid mechanics theory. Moreover, these new axial and radial displacement rates (velocities) are mathematically obtained. Also, they satisfy all the boundary and incompressibility conditions.

2. Simulation of the Creeping Matrix

Figure 1 schematically shows a simulation between the shear stress in the fluid, and creeping polymer matrix in the steady state creep in the state of small and large deformations. That is, most of the solids at high temperature are similar to the high viscous fluids.

For presentation of the new work, models and relations must be introduced initially. It is used from a micro mechanical model to reduce calculations, nevertheless preserving the physical conditions of the composite structure. In the mentioned form, it is supposed that fibers are parallel, regular, and unidirectional generally.

Figure 2 schematically shows a simple model of the fibrous composite. This simple model is used for more understanding the proper boundary conditions.

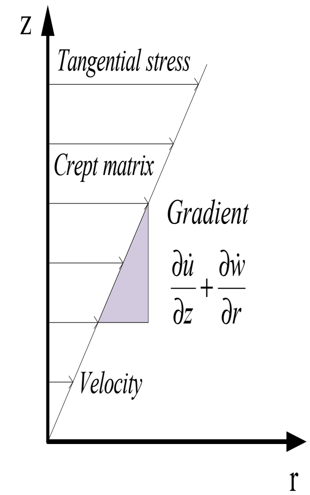

Figure 1. Simulation between the shear stress in the fluids and crept (creeping) polymeric matrix.

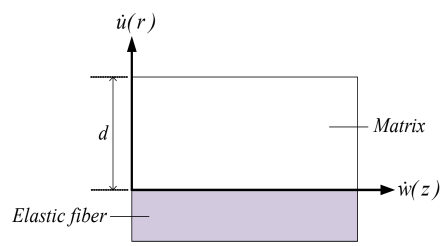

Figure 2. Simple presentation of composite model.

3. Boundary Conditions

For determining the correct and suitable tangential stress, tangential strain, axial and radial displacement rates of the fibrous composite, it is necessary to employ appropriate boundary conditions to the mathematical and analytical model (such as $\dot{u}(d)=\dot{u}, \dot{u}(0)=\dot{w}(0)=0, \tau_{r=0}=\tau_0, \tau_{r=d}=0$ *and so on*). Clearly, the iso-displacement condition is proper for the present problem. It should be mentioned that accurate and suitable boundary conditions led to acceptable behavior of the unknowns and parameters.

4. Formulation

The present approach is based on two displacement rates and functions $\dot{u}$ and $\dot{w}$ which are radial and axial displacement rates respectively. After determining $\dot{u}$, the coupled displacement rate $\dot{w}$ is obtained by incompressibility conditions. So we have:

$$\dot{u} = analytical\ \ function\ \psi = c_0 + c_1 r + c_2 r^2 + c_3 r^3 \quad (1)$$

Axial displacement rate $\dot{w}$ is determined using incompressibility condition ($\Delta V = 0, \quad \dot{u}_r + \frac{\dot{u}}{r} + \dot{w}_z = 0$), It then follows that:

$$\dot{w} = -\left[\frac{c_0}{r} + 2c_1 + 3c_2 r + 4c_3 r^2\right] z \quad (2)$$

That is, Eqs. (1) and (2) should satisfy the below relation:

$$\frac{\dot{u}}{r} + \frac{\partial \dot{u}}{\partial r} - \frac{\partial}{\partial z}\left[\int \frac{\dot{u}}{r} + \frac{\partial \dot{u}}{\partial r} dz\right] = 0 \quad (3)$$

Also, strain rate components are obtained by the following formulations:

$$\dot{\varepsilon}_r = \frac{\partial \dot{u}}{\partial r} \quad (4)$$

$$\dot{\varepsilon}_\theta = \frac{\dot{u}}{r} \quad (5)$$

$$\dot{\varepsilon}_z = \frac{\partial \dot{w}}{\partial z} \quad (6)$$

And for shear strains we have:

$$\dot{\gamma}_{rz} = \frac{\partial \dot{u}}{\partial z} + \frac{\partial \dot{w}}{\partial r} \quad (7)$$

$$\dot{\varepsilon}_{rz} = 0.5\dot{\gamma}_{rz} = 0.5\left(\frac{\partial \dot{u}}{\partial z} + \frac{\partial \dot{w}}{\partial r}\right) \quad (8)$$

Also shear stress in the creeping matrix is obtained in the below forms:

$$\tau_{rz} = \mu \dot{\gamma}_{rz} \quad (9)$$

$$\tau_{rz} = \tau_y + \mu_p \dot{\gamma}_{rz} \quad (10)$$

$$\tau_{rz} = k\dot{\gamma}^m \quad (11)$$

$$\tau_{rz} = k\dot{\gamma}^m + \tau_y \quad (12)$$

where μ_p, μ, $\dot{\gamma}$, k, τ_y and m are respectively plastic viscosity, dynamic viscosity, shear strain rates, consistency index, yield stress and exponent of power law formulation. Thermosetting materials are like the polymeric materials. Dynamic and active thermally viscous flow in the amorphous materials such as glasses and polymers is similar to the following comprehensive formulation,

$$\mu = A.e^{\frac{Q}{RT}} \quad (13)$$

In where, μ is dynamic viscosity, Q is activation energy, T is temperature, R is the molar gas constant and A is a constant value. The viscosity of the amorphous materials is generally explained by below two exponential equations,

$$\mu = T.A_1.\left[1 + C.e^{\frac{B_2}{RT}}\right] \times \left[1 + A_2.e^{\frac{B_1}{RT}}\right]. \quad (14)$$

In the above formulation, the constants A_1, A_2, B_1, C and B_2 are related to the thermo-dynamic parameters of the joining bonds of the amorphous materials. In various materials at normal temperatures, time-dependent plastic deformation is negligible because the stress is lesser than the yield stress. A straightforward and important model for explaining the mentioned effect is the Bingham model as the following,

$$\dot{\varepsilon}_e = \dot{\varepsilon}^i = \begin{cases} 0 & |\sigma| < \sigma_y \\ \left(1 - \frac{\sigma_y}{|\sigma|}\right)\frac{\sigma}{\mu} & |\sigma| \geq \sigma_y \end{cases} \quad (15)$$

At which parameter μ is the dynamic viscosity and the yield stress σ_y may depends on the strain and strain rates. The simplest model of the visco-plasticity is the Bingham model. Then, tangential stress behavior is analytically obtained by substituting radial displacement rate $\dot{\mu}$ and axial displacement rate $\dot{w}$ into the tangential stress formulation using fluid mechanics theory.

5. Results and Discussions

The viscosity of the crept polyester resin (matrix) with respect to the different temperatures is in the forms of the polynomial functions approximately as the following (Figure 3):

$$\mu[cP] \cong \sum_{i=0}^{6} T_i \;\; for\; 30 < T < 160\; celsius \quad (16)$$

Consequently, the exponential form in mentioned temperatures is:

$$\mu[cP] \cong e^{8-0.04T} \;\; for\; 30 < T < 160\; celsius \quad (17)$$

Finally, shear stress in the creeping polymeric polyester matrix is predicted by substitution of Equations (7) and (8) into Equations (9–15) with considering Equations (1–6). Therefore, these new models can be employed to obtain shear stress in the creeping polymer matrix. The recent explained method is a link and relationship between the solid mechanics (physics, solid state) and fluid mechanics generally.

Moreover, the results of the polynomial and exponential formulations for estimating the viscosity of the amorphous materials are acceptable generally. Specific and important model of the tangential stress is presented by the following formulations:

$$\tau_{rz}\Big|_{creeping\; polyester\; matrix} = [\mu[cP]] \times \left[\frac{\partial \dot{u}}{\partial z} + \frac{\partial \dot{w}}{\partial r}\right] + \tau_y\Big|_{polyester\; matrix} \quad (18)$$

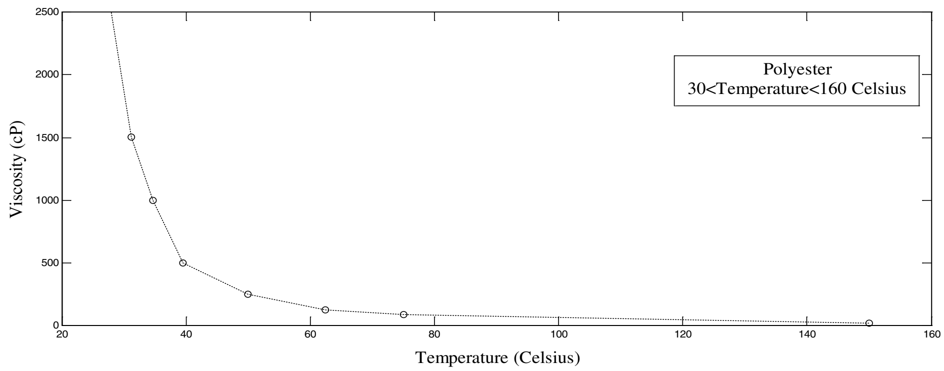

Figure 3. Nonlinear behavior of viscosity of polyester at various temperatures (30 < T < 160).

For instance, one of the recent explained formulations is given by the following:

$$\tau_{rz}\Big|_{creeping\ polyester\ matrix} = -\left[z \times e^{8-0.04T}\right] \times \left[\frac{-c_0}{r^2} + 3c_2 + 8c_3 r\right] + \tau_y\Big|_{polyester} \qquad (19)$$

In which, the constants $c_i' s$ are analytically obtained by proper boundary conditions. It should be mentioned that polynomial function has been assumed for predicting radial and axial displacement rates $\dot{u}$ and $\dot{w}$. Equation (19) is derived from the mentioned functions.

Above figure shows the results unambiguously.

6. Conclusions

In this article, behavior of the shear (tangential) stress in the creeping polyester matrix was predicted using viscosity of the polyester and assumed axial and radial displacement rates, employing mechanics of fluid theory. Also, novel axial and radial displacement rates, velocities, were achieved in the creeping matrix; in which the boundary and incompressibility conditions were satisfied by these displacement rates. In short, presented formulations may be employed to obtain the shear stress and displacement rates in the creeping matrix in the fibrous composites.

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