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Analytical Solution of Nonlinear MHD Flow Past A Porous Sheet with Prescribed Heat Flux, Viscous Dissipation, Heat Generation and Radiation Effects

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Abstract

Influence of viscous energy dissipation, heat generation and radiation on nonlinear MHD boundary layer 2-D flow of gray fluid past a linear stretching porous sheet with prescribed heat flux is studied. Exact solution of momentum equation and skin friction coefficients are obtained with the help of similarity transformations. Energy equation is solved using nonhomogeneous confluent hypergeometric function. Influence of Eckert number, magnetic field together with heat generation is to enhance the temperature distribution whereas the effect of porosity, Prandtl number and radiation is to suppress it. Such flow of fluids has abundant practical applications in polymer processing industry, lubrication, energy storage and recovery, insulation of buildings and equipments and also in many engineering areas such as nuclear engineering, mechanical engineering and civil engineering etc.

Keywords: Heat Flux, Magnetic Field, Radiation, Stretching Porous Sheet, Viscous Dissipation, Viscous Energy Dissipation, Heat Source

Nomenclature

B_0 applied magnetic field
$\vec{H}_0$ uniform magnetic field of strength
C_p specific heat of the fluid at constant pressure
C_f local skin friction co-efficient
a Positive constant called stretching rate
v_0 constant suction velocity
S suction parameter ($S > 0$),
M^2 magnetic interaction parameter
E_0 positive constant
K thermal conductivity
Q_0 volumetric rate of heat generation
T fluid temperature
T_∞ Temperature of the fluid far away from the surface
' prime denotes differentiation with respect to η
m heat flux parameter
S_h heat source parameter
q_r radiative heat flux
q_w rate of heat transfer at the wall
ψ stream function
ν kinematic viscosity
ρ fluid density
σ electrical conductivity of the fluid
σ^* Stefan-Boltzmann constant
α^* Rosseland mean absorption coefficient
Pr Prandtl number
R_d Radiation parameter
Ec Eckert number

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1. Introduction

Heat transfer plays a crucial role in industrial processes, electronic devices and food processing etc. In particularly, thermohydrodynamic analysis of fluid film lubrication of mechanical components, viz., bearing, gears and seals is basically a heat transfer analysis where in heat generated due to viscous sheer of the lubricant is dissipated through convection, conduction and to some extend by radiation is balanced. In the analysis heat transfer due to convection and conduction is considered to seek balance between the heat generated and heat dissipated to achieve an equilibrium condition. This helps in determining the temperature rise in the fluid film and the surrounding solids[1]. Not only in lubrication, but also in engineering and applied scientific areas, boundary layer MHD flow with radiative heat transfer past a stretching sheet in the presence of viscous energy dissipation, heat generation and radiation plays a vital role. In view of all these, this work is mainly focused on a problem of such kind.

Gebhart was the first to reveal that whenever natural convection region is extensive or the body force is extremely high then the influence of viscous dissipation is considered[2]. He gave the solutions for temperature distribution using perturbation method for pr various from 0.01 to 10000. The pioneer work of fluid flow in the presence of transverse magnetic field was developed by Pavlov[3, 4]. However, the solution given by him was approximate and he didn't find the analytical solution.

Vajravelu and Hadjinicolaou and Chaim both expressed the analytical solution for temperature[5, 6]. They investigated the influence of viscous energy dissipation and heat source on the flow field with suction/blowing. The temperature distribution for the flow problem was obtained analytically for the two different cases namely (i) when the stretching surface is subjected to variable temperature (PST Case) and (ii) when the rate of change of heat per unit area is prescribed on the surface (PHF Case). The influence of magnetic field is also considered in Chaims's work[6].

Vajravelu found a numerical procedure to solve momentum, energy and mass equations for finding velocity, temperature and concentration distribution using Runge Kutta numerical method[7]. Raptis et al., considered such type of flow field with the influence of both magnetic field and radiation. But he also got, only the numerical solution[8].

Therefore, Cortell developed the analytical solution for the flow field with viscous energy dissipation and radiation effect[9]. He used Kummer's method to find the temperature with the correct boundary condition for both PST and PHF case. Das et al., gave the analytical solution obtained by the implicit relations with the help of boundary conditions for the elastic-viscous oldroyd model liquid over a semi-infinite region[10]. Ahmed and Kalita considered the porous plate subject to periodic suction velocity in the normal direction using perturbation method solutions are obtained by neglecting the powers of ε more than one[11].

Recently, the influence of heat source and radiation on MHD flow past a stretching porous sheet with heat and mass transfer was discussed by Anjalidevi and Kayalvizhi[12]. She developed the analytical solution for both temperature and concentration distribution using Kummer's method for prescribed heat and mass flux.

But so far, no contribution has been developed to find the analytical solution of 2-D hydromagnetic nonlinear gray fluid flow past a stretching porous sheet with the influence of radiation, viscous energy dissipation and heat generation. In order to obtain more realistic solution, prescribed heat flux is considered in this work.

2. Formulation of the Problem

The boundary layer 2-D, viscous gray fluid flow past a porous sheet with prescribed heat flux is considered. The sheet is stretched linearly with the velocity $u = ax$ where the fluid flow is along the parallel x - direction of the stretching sheet. The laminar flow of an electrically conducting fluid in the presence of viscous dissipation, heat generation, radiation and transverse magnetic field is considered.

In addition, the analysis is based on the following assumptions:

- The fluid flow with constant physical properties in Cartesian coordinates.
- The magnetic Reynolds number is assumed as so small so that the induced magnetic field is considered to be negligible.
- In the energy equation, Joule's dissipation is considered to be negligible and $\frac{\partial q_r}{\partial x}$ is negligible compared with $\frac{\partial q_r}{\partial y}$.

Incompressible, steady flow at a large Reynolds number over a stretching sheet with zero pressure gradients with the above assumptions, the equation for the flow and energy equations are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2 u}{\rho} \tag{2}$$

$$\rho C_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=K\frac{\partial^2 T}{\partial y^2}+Q_0(T-T_\infty)+\mu\left(\frac{\partial u}{\partial y}\right)^2-\frac{\partial q_r}{\partial y} \tag{3}$$

The last term on the right hand side of equation (3) signifies the radiation effect. Rosseland approximation is assumed

[Raptis[8]] which leads to q_r and hence

$$q_r=\frac{-16\sigma^*}{3a^*}T_\infty^3\frac{\partial T}{\partial y} \tag{4}$$

Substituting eqn. (4) in (3), hence eqn. (3) becomes

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{K}{\rho C_p}\frac{\partial^2 T}{\partial y^2}+\frac{16\sigma^* T_\infty^3}{3\rho C_p \alpha^*}\frac{\partial^2 T}{\partial y^2}+\frac{Q_0}{\rho C_p}(T-T_\infty)+\frac{\mu}{\rho C_p}\left(\frac{\partial u}{\partial y}\right)^2 \tag{5}$$

3. Mathematical Analysis

3.1 Flow Analysis

Boundary conditions pertaining to velocity are

At $y = 0 : u = ax, v = -v_0$

As $y \to \infty : u = 0$ (6)

Where, $v_0 = S\sqrt{a\nu}$.

The following similarity transformations are introduced to transform the partial differential equation (2) with boundary condition (6) is into ordinary differential equation.

$$\left.\begin{aligned}\psi&=\sqrt{a\nu}\,x\,f(\eta)\\ \eta&=\sqrt{\frac{a}{\nu}}\,y\end{aligned}\right\} \tag{7}$$

A stream function ψ is defined by:

$$u=\frac{\partial\psi}{\partial y}\text{ and } v=-\frac{\partial\psi}{\partial x} \tag{8}$$

Equation (1) is satisfied automatically.

Employing the equations (7) and (8), the partial differential equation (2) with boundary condition (6) is reduced to the following ordinary differential equation

$$f'''+f f''-f'^2-M^2 f'=0 \tag{9}$$

with boundary conditions

$$f(\eta)=S,\ f'(\eta)=S \text{ at } \eta=0$$

$$f'(\eta)=0 \text{ as } \eta\to\infty \tag{10}$$

The exact solution of equation (9) subject to the boundary condition (10) is

$$f(\eta)=S+\frac{1}{\alpha}(1-e^{-\alpha\eta}),\ f'(\eta)=e^{-\alpha\eta}$$

where, $\alpha=\dfrac{S+\sqrt{S^2+4(1+M^2)}}{2}$ and $M^2=\dfrac{\sigma B_0^2}{\rho a}$ is the magnetic interaction parameter. (11)

3.2 Skin Friction

$\tau^*=\mu\left(\dfrac{\partial u}{\partial y}\right)_{y=0}$ gives the shear stress at the wall and the non-dimensional shear stress is $f''(0)=-\alpha$ which is known as skin friction co-efficient C_f.

3.3 Heat Transfer Analysis

Solution of equation (5) is obtained by using the following prescribed heat flux boundary condition

$$-K\frac{\partial T}{\partial y}=q_\omega=E_0x^m \quad \text{for } y=0$$

$$T\to T_\infty \text{ as } y\to\infty \tag{12}$$

For $m = 0$, represents the rate of change of heat per unit area is uniform. Using the similarity variable

$$T-T_\infty=\left(E_0/K\right)\sqrt{\nu/a}\,x^m\theta(\eta) \tag{13}$$

the equation (5) can be written as

$$\theta''+\beta f\theta'-\beta(mf'-S_h)\theta=-Ec\beta f'' \tag{14}$$

where, $\beta=\dfrac{3\Pr R_d}{3R_d+4}$, $\Pr=\dfrac{\nu\rho C_p}{K}$ is the Prandtl number, $R_d=\dfrac{Ka^*}{4\sigma^* T_\infty^3}$ is the Radiation parameter, $S_h=\dfrac{Q_0}{\rho C_p a}$ is the heat source parameter, $Ec=\dfrac{U_w^2}{C_p\left(\dfrac{E_0}{K}\sqrt{\dfrac{\nu}{a}}\right)x^m}$ is the Eckert number.

Using (13), Equation (12) reduces to

$$\theta'(\eta)=-1 \text{ at } \eta=0$$

$$\theta(\eta) \text{ as } \eta\to\infty \tag{15}$$

New variable ξ is introduced such that $\xi = \frac{-\beta}{\alpha^2} e^{-\alpha\eta}$. Now equation (14) and (15) reduces to

$$\xi \frac{d^2\theta}{d\xi^2} + (1-\gamma-\xi)\frac{d\theta}{d\xi} + \theta\left(m + \frac{\beta S_h}{\xi \alpha^2}\right) = \frac{-Ec\alpha^4}{\beta}\xi \quad (16)$$

$$\frac{d\theta}{d\xi} = \frac{-\alpha}{\beta} \text{ at } \xi = \frac{-\beta}{\alpha^2}$$

$$\theta(\xi) = 0 \text{ at } \xi = 0 \quad (17)$$

where, $\gamma = \frac{\beta}{\alpha^2}(1 + S\alpha)$ and assuming heat flux parameter $m = 2$.

The solutions of the equation (16), satisfying boundary condition (17) in terms of dependent variable η, are

$$\theta(\eta) = b_0 e^{-2\alpha\eta}\beta + b_3\left(\frac{-\beta e^{-\alpha\eta}}{\alpha^2}\right)^b 1F_1\left[b-2, A+1; \frac{-\beta e^{-\alpha\eta}}{\alpha^2}\right] \quad (18)$$

where,

$$A = \sqrt{\gamma^2 - 4\left(\frac{\beta}{\alpha^2}\right)S_h}, \quad b_0 = \frac{-Ec}{4 - 2\gamma + \left(\frac{S_h\beta}{\alpha^2}\right)}, \quad b = \frac{A+\gamma}{2},$$

$$b_1 = {}_1F_1\left[b-2, A+1; \frac{-\beta}{\alpha^2}\right],$$

$$b_2 = {}_1F_1\left[b-1, A+2; \frac{-\beta}{\alpha^2}\right] \text{ and}$$

$$b_3 = \frac{2b_0 -^2 - \left(\frac{\alpha}{\beta}\right)}{\left(-\beta/\alpha^2\right)^b\left(\left(\frac{--^2 b}{\beta}\right)b_1 + \left(\frac{b-2}{1+A}\right)b_2\right)}.$$

The non-dimensional temperature at the wall is obtained by using eqn. (18) as

$$\theta(0) = b_0\beta + b_3\left(\frac{-\beta}{\alpha^2}\right)^b b_1.$$

4. Discussion of the Results

Numerical computations of results are demonstrated through graphs for different non-dimensional parameter. Equation (18) gives the solution for the non-dimensional temperature distribution for prescribed wall heat flux. This dimensionless temperature distribution $\theta(\eta)$ for several values of the dimensionless parameters is depicted through Figures 1 to 6. Also the values of non-dimensional wall temperature $\theta(0)$ for various values of M^2, S, R_d, S_h, Pr and Ec are presented in Table 1.

Figure 1 depicts the distribution of non-dimensional temperature $\theta(\eta)$ versus η, by choosing various values of M^2. The effect of magnetic field in the presence of

Table 1. Wall temperature $\theta(0)$ for various values of physical parameters

R_d	S_h	M^2	S	Pr	Ec	$\theta(0)$
1	0.05	4	1.5	0.71	0.01	1.48982
2						1.07702
3						0.940852
4						0.872936
10^9						0.669378
3	0	4	1.5	0.71	0.01	0.919706
	0.05					0.940852
	0.1					0.964539
	0.15					0.991492
	0.2					1.02283
3	0.05	0	1.5	0.71	0.01	0.84481
		1				0.87789
		4				0.940852
		9				1.00166
		16				1.05355
3	0.05	4	0.5	0.71	0.01	1.497
			1			1.15734
			1.5			0.940852
			2			0.788629
			2.5			0.676185
3	0.05	4	1.5	0.71	0.01	0.940852
				1		0.684997
				1.75		0.414379
				2.3		0.326034
				7		0.125648
3	0.05	4	1.5	0.71	0	0.933957
					0.001	0.934647
					0.01	0.940852
					0.05	0.968432
					0.1	1.00291

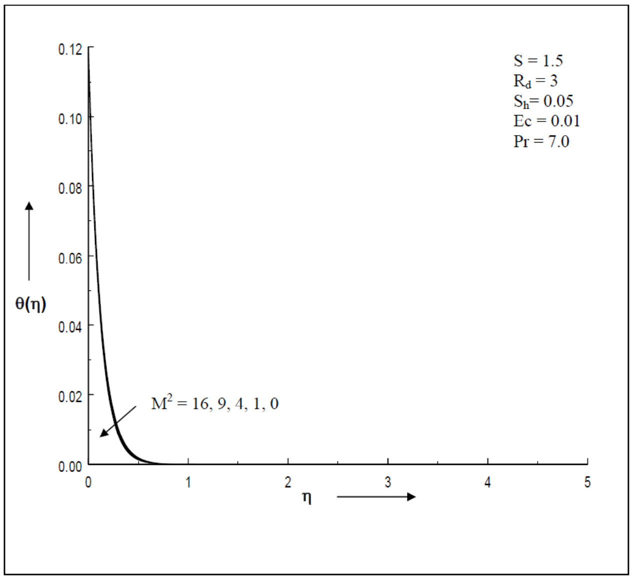

Figure 1. Effect of M^2 on temperature distribution.

porosity is to increase the temperature field. But it has less significant effect over temperature distribution.

Variation in dimensionless temperature due to the variation of suction parameter (S) is visualized using Figure 2. It is inferred that temperature decreases with an increase in S. The influence of radiation over non-dimensional temperature is demonstrated using Figure 3. As radiation increases, the temperature decreases.

Figure 4 reveals the fact that the influence of heat generation enhances the non-dimensional temperature

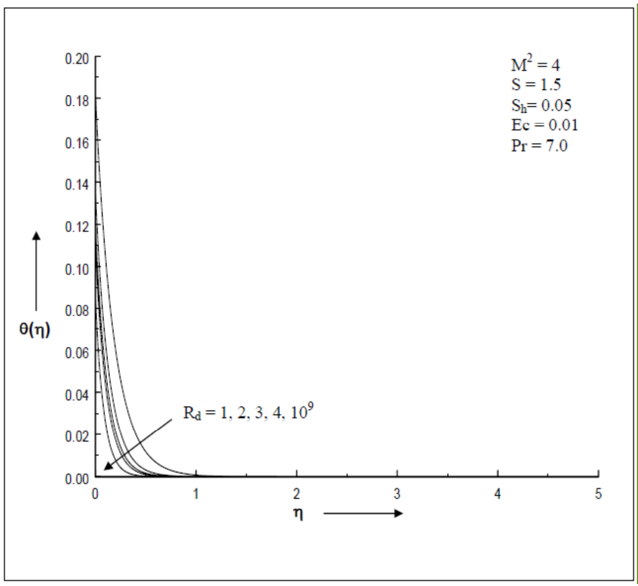

Figure 2. Effect of S on temperature distribution.

$\theta(\eta)$for both air and water. It is clearly noted that the influence of heat generation over non-dimensional temperature is less significant in water. However, the effect is clearly seen in the case of air.

Figure 5 clearly discloses influence of Eckert number (Ec) over the temperature field for both air and water. As Ec increases, the temperature also increases. It is clear from Figure 6 that the temperature decreases with increases of Prandtl number Pr.

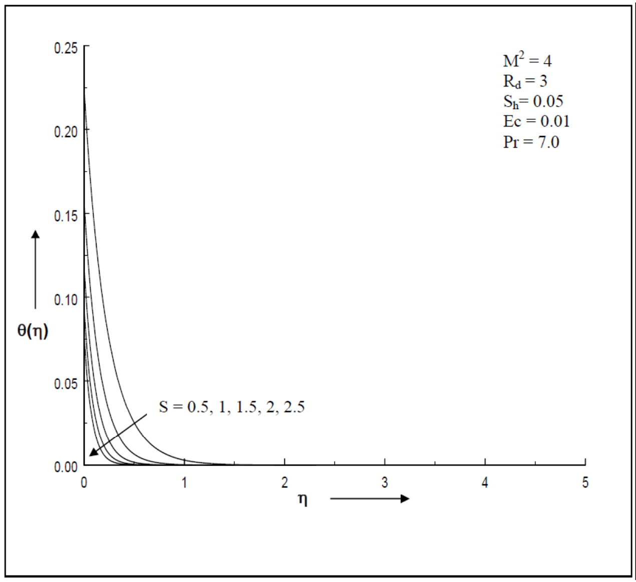

Figure 3. Effect of R_d on temperature distribution.

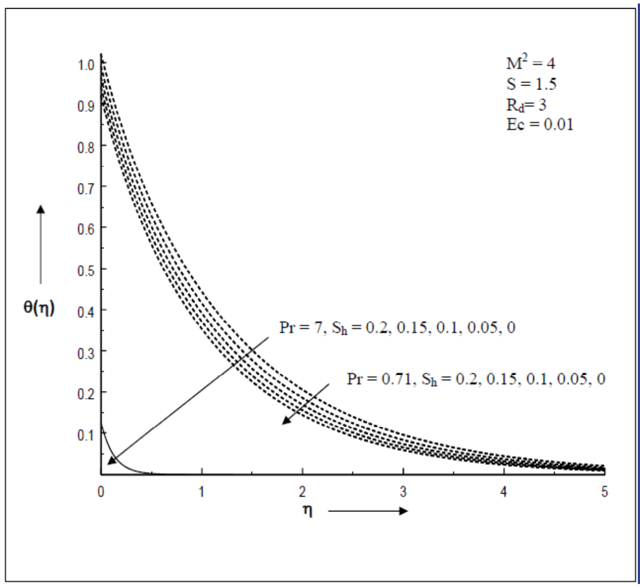

Figure 4. Influence of S_h over dimensionless temperature for air and water.

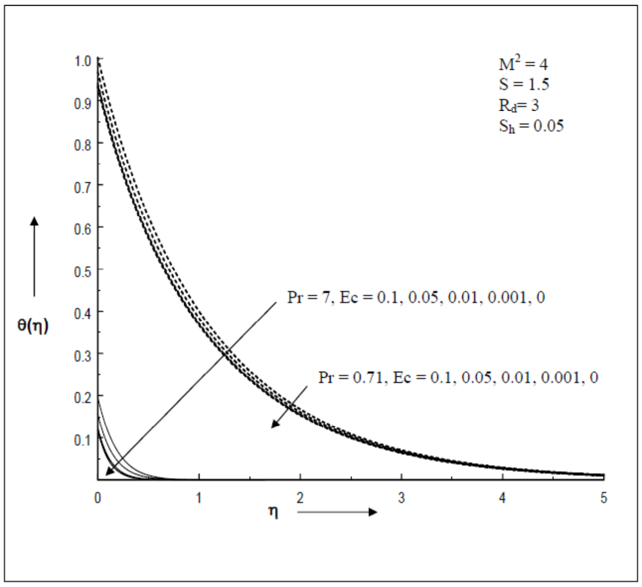

Figure 5. Dimensionless temperature for various *Ec*, for both air and water.

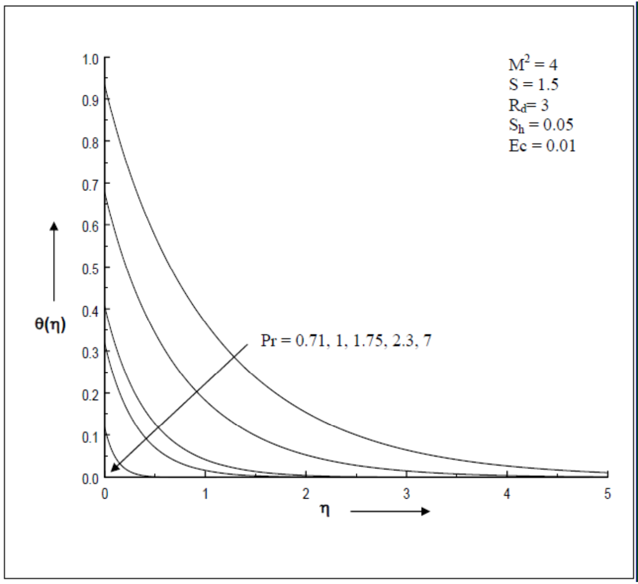

Figure 6. Non-dimensiona temperature for different *Pr*.

5. Conclusion

Generally, Influence of the physical parameters affects the velocity profile and temperature profile. When there is no radiation, the results are identical to that of Chaim for prescribed heat flux case[6]. When there is no magnetic field and in the absence of heat source, the analytical solution of momentum equation and numerical values obtained for temperature are identical to that of Cortell for prescribed heat flux case[9].

We concluded that the temperature distribution and the wall temperature enhance for increasing values of magnetic field, Eckert number and heat generation. But the influence of porosity, radiation and Prandtl number is to individually reduce the temperature distribution and wall temperature.

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