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Effect of Core Geometry on Shear Moduli in Cellular Core Sandwich Structures

Arash Shokrgozar Navi[1*], Amirhosein Ehsan[2] and Amir Ghafouri Sayyad[3]

[1]Department of Mechanics, Talesh Branch, Islamic Azad University, Talesh, Iran; Aarash.navii@gmail.com
[2]Department of Mechanics, Roudehen Branch, Islamic Azad University, Roudehen, Iran; Amirhosien_ehsani@yahoo.com
[3]Department of Mechanics, Ardebil Branch, Payamnur University, Ardebil, Iran; Amir.ghafouriii@gmail.com

Abstract

Cellular structures are lightweight structures with appropriate stiffness properties. Usually these cells are in the form of a honeycomb but various cell geometries can be considered as well, for special applications. Because of the repetitive nature of the cellular array the core can be considered as a homogeneous but anisotropic medium. In present study by changing in the number and positions of nodes, the number of cell walls and changing in size of RVE (Representative Volume Element), and also using homogenization method, the shear components of the elasticity tensor are obtained for heterogeneous geometries by assuming unconstraint core. The results show that the effect of these changes in geometry, increase the quality of shear stiffness and weight reduction for some core geometries of sandwich panels that have not been evaluated yet. Soar effect of rearrangement of walls and their located for transverse shear applications is the main characteristic of present study that results obtained have confirmed these claims. Advantage of the present study is providing new pattern applied for sandwich panels core those shear properties that have higher priority..

Keywords: RVE (Representative Volume Element), Homogeneous, Stiffness, Sandwich Panels

1. Introduction

Nowadays grid and cellular structures are used in a variety of engineering applications including aircraft, navy ship, constructions and transportations where strong, stiff and light structures are required. The typical cellular structure is structural sandwich panel that consists of three layers. Two high density face sheets are adhesively bonded to a low density core with high thickness that has to carry the transverse shear loads (Figure 1). Advantages sandwich structure is that large bending stiffness value in conjunction with very low specific weight are obtained. Because of the complexity of the core geometry and for reasons of numerical efficiency, analysis of sandwich structure is performed by considering of effective properties rather than by consideration of the real grid cellular structure which replaced by a quasi-homogeneous effective medium. At macroscopic level, equal mechanical behavior from quasi-homogeneous structure and cellular structure are the requisite for calculating the properties of the effective medium[1–4].

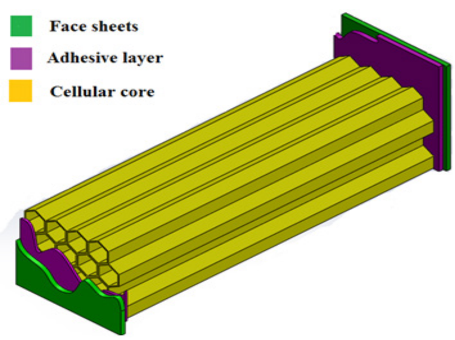

Figure 1. Structural Sandwich Panel.

Initial research on the two-dimensional cellular sandwich core is done by Kelsey et al.[5] also Chang and Ebcioglu[6] at (1961) who is brought up the transverse shear properties for regular hexagonal core. Because of the honeycomb geometry has some appropriate properties

** Author for correspondence*

like lightness, carry the normal, shear, and compound load, and easy produce and application, the most of studies were have be on hexagonal geometry, like Gibson et al.[7], Warren and Kraynik[8], also. The effective elastic properties of regular isotropic triangular cell structures and square cell structures have worked by Gibson and Ashby[9], also a study on the 3 x 3 unit cell has performed by Torquato et al.[10] and Huybrechts and Tsai[11]. In continuation of previous research Hohe and Becker[1,12] worked on the grid structure based on homogenization for calculating the strain energy in the real triangular microstructure, also their researchers Continued on the hexagonal and quadrilateral grid with unconstraint generals geometry core for two-dimensional with straight and curved walls[2,3], as well as they could obtain a method for general geometry with constraint core[4,13]. Hohe and Becker[12] worked on hyper plastic honeycombs[14,15]. As regards, the most of studies were on the regular geometries like honeycomb, triangular and square, in the present study with changes in the number of nodes and their position and arrangements on the boundary of Representative Volume Elements (RVE) the new stronger geometries for shear components of elasticity tensor is acquired. Also by comparing previous researches with changes in the size of RVE suitable results are obtained. Shear stiffness and weight are two factors have been considered in this study. In commercial applications regular core geometries is common but in this study is tried, the effect of changes for the shear stiffness indicated with the combined geometries. The method for calculating components of elasticity tensor is strain energy that based on homogenization[2–4]. Undoubtedly the acquisition of lighter sandwich structures can be considered as an advantage. Depending on the application of these structures, high elasticity tensor for a certain amount of force with less weight can be suitable. Certainly if the sandwich structures were put under the directional forces, conditions will require to high stiffness in the same directions for their greater longevity. Obviously, variations in the core geometry of sandwich structure can lead to the production of structures with high quality.

2. Basic Concept

2.1 Strain Energy based Homogenization

Because calculating the strain energy for the network structures has been complex, the studies on the strain energy of sandwich structures using homogenization have been carried out. In this method consider a body Ω consisting of a periodic cellular material which is limited by an external boundary $\Gamma = \Gamma_t \cup \Gamma_u$ which either strain T_i^0 or displacements u_i^0 are assigned (Figure 2). The body Ω has to be substituted by a similar body Ω^* with the same shape and subjected to the same boundary condition T_i^0 and u_i^0. The body Ω^* is supposed to consist of the homogeneous effective medium with yet unknown properties. For the calculation of the effective properties, a Representative Volume Element (RVE) for the microstructure Ω and a similar volume element consisting of the effective medium are assumed[3].

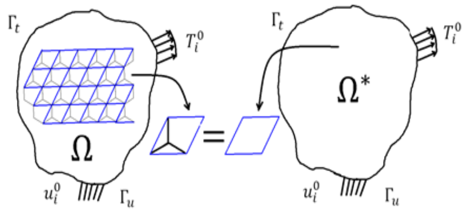

Figure 2. The concept of homogenization & Representative Volume Element (RVE).

The properties of the effective medium must be determined in such a way that the mechanical behavior of both volume elements to be equivalent on the macroscopic level. In present study equivalence on the macroscopic level is assumed, accordingly if the strain energies w and w^* in both volume elements are equal.

$$\frac{1}{V_{RVE}}\int_{V_{RVE}} w(\varepsilon_{ij})dV = \frac{1}{V_{RVE}}\int_{V^*_{RVE}} w^*(\varepsilon_{ij}^{\ *})dV \qquad (1)$$

Both volume elements are subjected to macroscopically equivalent strain states ε_{ij} and ε_{ij^*} respectively. Equivalence of the strain states in both volume elements is assumed, if the volume average of the infinitesimal strain tensors is equal[16].

$$\overline{\varepsilon ij} = \frac{1}{V_{RVE}}\int_{V_{RVE}} \varepsilon_{ij} dV = \frac{1}{V_{RVE}}\int_{V^*_{RVE}} \varepsilon ij^* dV = \overline{\varepsilon ij^*} \qquad 2)$$

The effective properties now have to be determined in such a way that Equations (1) and (2) are satisfy all strain states. A linear elastic constitutive equation in a local formulation is posited on the effective level.

$$\sigma_{ij} = C_{ijkl^{\varepsilon}ij}, \qquad (3)$$

Thus, the components C_{ijkl} of the effective elasticity tensor are specified by the second partial derivatives of the strain energy density with respect to the strains[3]. Therefore, equation is presented in this way;

c↓ijkl = {(2/Vw↑ * (ε↓ij↑-)1/(ε↓ij↑(1 2)) i = j,k = l,i = k@@1/2Vw↑ * (ε↓ij↑1)1/(ε↓ij↑(1 2)) (4)

In Equation (4) indicates the average strain energy density in their preventative volume element is affected by a homogeneous strain state, where all components of the macroscopic strain tensor except $\overline{\varepsilon_{ij}}$ and $\overline{\varepsilon_{kl}}$ are equal to zero. Thus, the representative volume element is thought to be deformed by a number of reference strain states for which the strain energy has to be evaluated. Afterwards, all components of the effective elasticity tensor can be calculated by means of Equation (4).The strain energy can be assessed either analytically or numerically.

2.2 The Calculations of the Strain Energy of RVE

Two-dimensional cellular RVE is presented in Figure 3, also this parallelogram-shape shown by a, b, c, where cell walls connected together by some nodes (unit cell) inside or on the boundary. For calculating the strain energy of RVE, unit cell must be decomposed into individual cell wall elements, thus the strain energy of entire volume elements can be determined by the sum of strain energies of each cell wall, also displacement field can be implication in terms of the deflections of the cell walls end.

There is a method for determination the strain energy for unconstraint core that it is presented by Hohe and Becker[4]. The stress field conform strain field by Hook's law. Due to this, the strain energy density can be calculated as half of the product of the stress and strain fields. Union of the strain energy density according to the volume of the cell wall element finally obtains the strain energy of the cell wall element in terms of the nodal deflections[3].

W = {E/(2(1 - ν↑2))htl{((ν¯↓(1)1/Ll@ν¯↓(2)1/l@ε¯↓33))↑ T((1 & -1 & -ϑ@ - 1 & 1 & ϑ@ - ϑ & ϑ & 1))ν¯↓(1)1/l@ν¯↓ (2)1/l@ε¯↓33))↑ - β((ν¯↓(1)2/l@(Δφ)¯↓((1))@ν¯↓(2)2/l@(Δφ)˜↓((2))))↑T((1

Also, $\overline{C_{ijkl}} = \frac{C_{ijkl}}{E} x100$ (6)

Where E is *Young's modulus* and υ is the *Poisson's ratio* and l, h and t are the length, height and thickness of the cell wall respectively. Rotation is $\Delta_{\tilde{\varphi}(i)}$ with respect to the transverse axis at node i and $\tilde{\upsilon}_{(i)j}$ is the displacement of node i in direction j. It is obvious that a complete linear system of equations must satisfy some constraints and conditions. For calculation the components of c_{ijkl}, 2^{n+1} equation should be created for n node.ForcesF_i, torques M_i, deflection ϑ_i , and rotations $\Delta\varphi_i$ can be obtained by some condition and constraint that list below;

1. No rigid body motions of the whole RVE are allowed. Four equations are given by this condition[1,3]
2. Each node in the Periodic boundary of RVE must constitute at least a triple force with another neighbor RVE.
3. The effective strain field is assumed homogeneous therefore the relationship between rotations and deflection of nodes for whole of RVE can be obtained[4].
4. $\overline{\varepsilon}_{ij}$ Can be calculated for whole of RVE using Green's theorem in terms of displacement field u_i. Five equations are given by this condition[4].
5. All of the inside nodes in RVE should have balance of force and torque[1,4].
6. Length wall should be less than 0.2 h so that Timoshenko beam condition can be true.
7. Minimum number of nodes must be four.

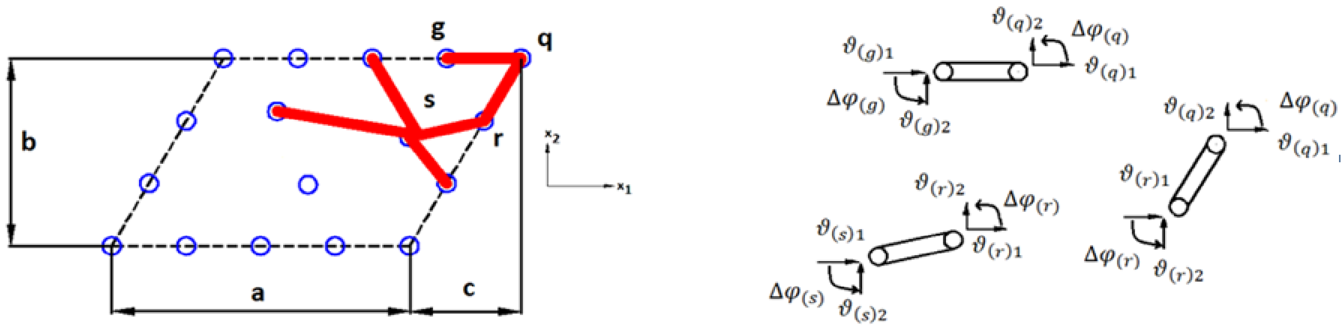

Figure 3. RVE for General Sandwich Coregeometry - Decomposition of the RVE.

2.3 Variables, Constraints, Parameters

In present study M, Z, Wn, NP, W2 are variables in representative volume elements. M is the number of nodes on parallelogram shape's boundary (RVE) that are limited to $4 \le M \le 12$. Z is the number of nodes inside of the parallelogram shape that are limited to $0 \le Z \le 4$. Wn is the number of walls in the parallelogram shape must not cross over each other's lines. Np is nodes position in the parallelogram shape that depends on the size of the parallelogram and finely Wa is wall arrangements that depends on the others variables. On the other hand there are some parameters like, La, Lb, Lc which are the length of parallelogram shape in a,b,c direction respectively. $\bar{\rho}$ which is density that depends on by the volume of cell walls (VCW) and volume of RVE (VRVE) that define by $\bar{\rho} = \frac{V_{CW}}{V_{RVE}}$ and h is the height of the core. With this interpretation, thickness depends on other variables like RVE and cell walls and the condition of Timoshenko beam. Also the weight of core depends on thickness of wall, wall lengths and core's height.

$$thickness = \frac{\bar{\rho} \mathrm{x} (Area)_{RVE}}{\sum_{k=1}^{Wn} (\text{wall lengths})_k} \tag{7}$$

$$Weight = 100 \mathrm{x}\, h \mathrm{x}\, thickness\, \mathrm{x} \sum_{k=1}^{Wn} (\text{wall lengths})_k \tag{8}$$

3. Result and Discussion

3.1 The Effect of Variables on $\overline{C_{1212}}$

In present study, the effect of displacement of nodes on the same RVE with sameWn, M, Z is indicated; also the important location for the unit cell is visible in the Figure 4. Two unit cells with the combination geometries in a same shape (RVE) with

$$\frac{b}{a} = 4, c = 0, M = 10, z = 0, W_n = 8, \bar{\rho} = 0.02,$$

which are similar in every respect with just a difference in arrangement are indicated. Applying an infinitesimal transverse shear strain in direction 1 to 2 ($\overline{\varepsilon_{12}}$) on the geometries of Figure 4. The amount of transverse shear components of elasticity tensor is acquired by Equation (4) and weight by Equation (8), that they are $\overline{C_{1212}} = 2.77, weight = 10.89\ ah$ (Figure 4a), and $\overline{C_{1212}} = 1.9, weight = 11.05\ ah$ (Figure 4b). It's clear that W_1, W_2 could be more resistant than W_3, W_4 in direction 1 to 2. Also for W_1, W_2 their displacement field provides better conditions that cause to high strain energy, that help to increase the shear properties. Rearranging increase $\overline{C_{1212}}$ amounted to 45.7% and reduces the rate of weight to 1.5%. It seems that increasing the size of RVE and its changes from parallelogram to rectangular improve the transverse shear properties. However, long wall and proximity of the walls is specified the thickness and length of walls together.

Also same shape as pervious example, the effect of changing wall arrangement is indicated in (Figure 5). It is observed that the change in the arrangement of the walls can produce inappropriate amount for shear component of elasticity tensor that $\overline{C_{1212}} = 1.072, weight = 12.74\, ah$ for Figure 5a and $\overline{C_{1212}} = 0.1, weight = 13.2\ ah$ for Figure 5b. Two equal RVE (Figure 5) with difference in $\overline{C_{1212}}, weight\ ghit$ is shown the importance of the node position and the number of walls. Node number 2 is the same node number 7 in repetitive shape that five walls leading to them, but in Figure 5b node number 12 and 17 have four walls. The node number 2 is a very strong support for Figure 5a in appropriate location. Also nodes number 5, 9 are same in repetitive shape, that the total wall leading into them are four walls. In contrast the nodes number 15, 19 have three walls in the general case. Such this kind of conditions, hence nodes with high number of walls in direction 1 to 2 improve shear properties, though there is some increasing in weight.

In previous research by Hohe and Becker[3,12] for a hexagonal core with a single cell wall is composed of three walls (Figure 6a) that $\overline{C_{1212}}, = 0.042, weight = 1.74\ ah$ is obtained while $\frac{b}{a} = 0.5, \quad c = 0.4, M = 3, z = 1, W_n = 3, \bar{\rho} = 0.02 \text{ or } t = 0.014a$.

Amount of $\overline{C_{1212}}$ is very less than hybrid geometries in Figure 4, though a small proportion of RVE size must be considered. Accordingly, the ratio the RVE's size to each other is $\lambda = 5.71$, indeed, if the λ ratio considered for weight and stiffness of hexagonal core hybrid core (Figure 4a) can be an appropriate geometry used in industries yet. Displacement field and rotations and subsequent shear deformation in nodes of honeycomb geometry are more than nodes of in hybrid geometries. Although in some previous studies[3, 4] for increasing the stiffness, the thickness of the horizontal cell walls is considered twice as large as the thickness of the inclined cell walls, but in

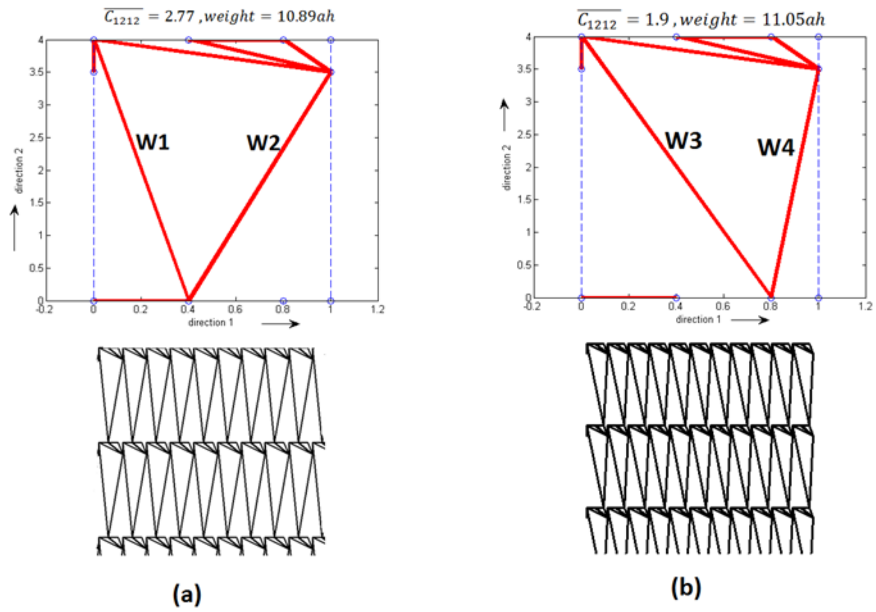

Figure 4. Rectangular RVE for combination geometries (Positive Effect on $\overline{C_{1212}}$).

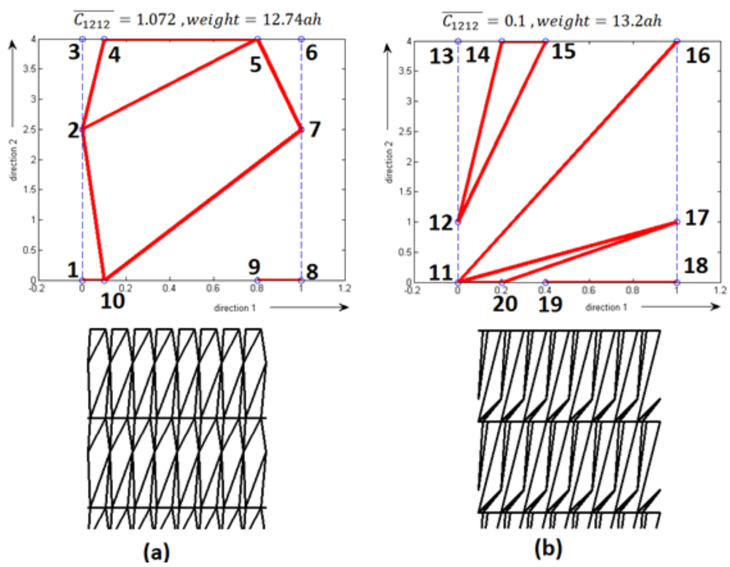

Figure 5. Rectangular RVE for Combination Geometries (Positive-Negative Effect on $\overline{C_{1212}}$).

present study rather than the change in thickness of wall, is used from changes in arrangement. The node number 25 is inside node that has suitable properties like other nodes in parallelogram shape (RVE) in Figure 6a.

Also some researches have been on triangular geometry that similar results have been found (Figure 6b) while it has been shown that with increasing $\frac{b}{a}$ for triangular geometry, the amount of stiffness will decrease1. Subsequently in present study for a triangular geometry $\overline{C_{1212}} = 0.01$, $weight = 14.1\ ah$ is obtained while RVE have $\frac{b}{a} = 4, c = 0, M = 4, z = 0, \bar{\rho} = 0.012$, $\mathbf{W_n} = 4$ Node numbers 28, 29 in general view have four walls but the location of them and the nature of triangular are caused low stiffness in compare with hybrids and hexagonal geometries.

3.2 The Effect of Variables on $\overline{C_{2323}}$

Following the $\overline{C_{1212}}$ the second component of elasticity tensor is $\overline{C_{2323}}$ that applying an infinitesimal transverse shear strain in direction 2 to 3 ($\overline{\varepsilon_{23}}$) with

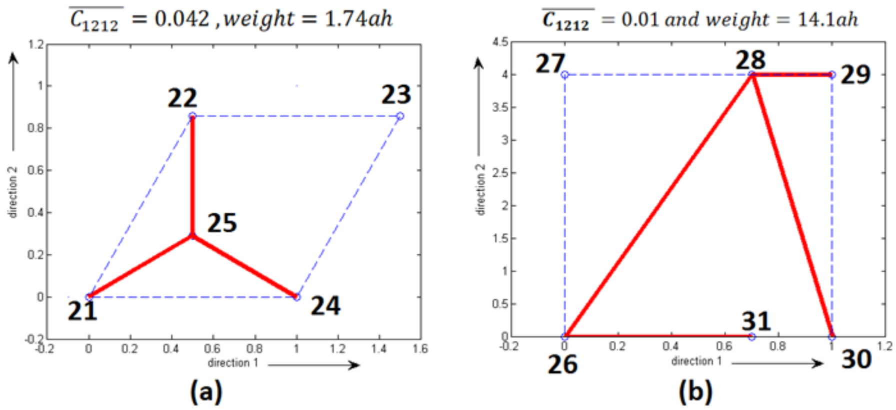

Figure 6. Rectangular RVE for Combination Geometries for $\overline{C_{1212}}$).

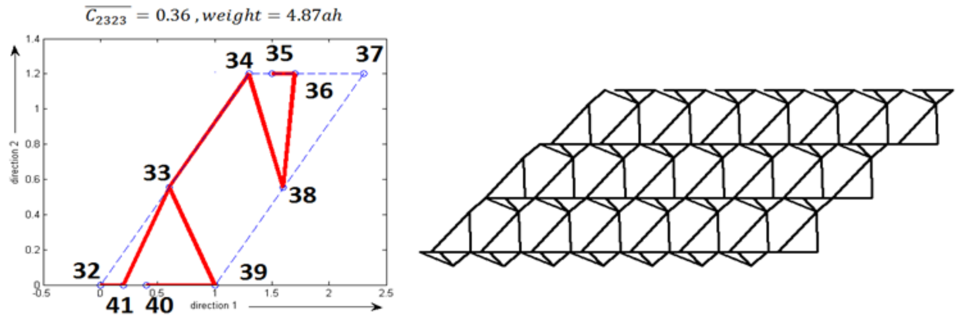

Figure 7. Rectangular RVE for Combination Geometries (Positive Effect on $\overline{C_{2323}}$).

$\frac{b}{a} = 1.2, \quad c = 1.3, M = 10, z = 0, W = 8, \bar{\rho} = 0.012$

is indicated for heterogeneous geometry in Figure 7. The effect of nodes and walls of RVE according to Figure 7 on the low density is shown, while proximity of walls has a large impact on the increasing amount of $\overline{C_{2323}}$ that cause the increasing of the shear properties in direction 2 to 3 (the height of core). With these conditions $\overline{C_{2323}} = 0.36$, $weight = 4.87\,ah$ is acquired the node number 33 and 38 also 32, 34, 39 have five walls in general form that wall 33-34 has a high strain energy. For a hexagonal geometry of approximately the same size RVE $\overline{C_{2323}} = 0.21$, and $weight = 3.25\ ah$ is obtained that the rotation of nodes in hexagonal increase the stiffness properties that shown by Hohe and Becker[3,4], nevertheless the effect of the proximity of walls, M, Z, W_n, P_n, and the size of RVE in this study with some example is checked. Although the weight of Figure 7 is heavy than hexagonal geometry but high stiffness is more concretely.

4. Conclusion

In the present study, the energetic homogenization method presented by the authors in previous papers on the analysis of triangular, hexagonal and quadrilateral core geometries is extended to combinatorial and heterogeneous geometry for sandwich panel cores. Indeed some new geometries with appropriate properties for transverse shear stiffness is acquired by changes in the number of nodes and walls, and also the size of RVE and the position of node. Soar effect of rearrangement of walls and their located for transverse shear applications is the main characteristic of present study that results obtained have confirmed these claims. Advantage of the present study is providing new pattern applied for sandwich panels core those shear properties that have higher priority.

5. References

1. Hohe J, Becker W. Effective elastic properties of triangular grid structures. Compos Struct. 1999; 45:131–45.
2. Hohe J, Beschorner C, Becker W, Effective elastic properties of hexagonal and quadrilateral grid structures. CompósStruct. 1999; 46:73–89.
3. Hohe J, Becker WA. Mechanical model for two-dimensional cellular sandwich cores with general geometry. Computational Materials Science. 2000; 19:108–15.
4. Hohe J, Becker W. A refined analysis of the effective elasticity tensor for general cellular sandwich cores. International Journal of Solids and Structures. 2001; 38:3673–87.
5. Kelsey S, Gellatley RA, Clark BW. The shear modulus of foil honeycomb cores. Aircraft Eng. 1958; 30:294–302.
6. Chang CC, Ebcioglu IK. Effect of cell geometry on the shear modulus and on density of sandwich panel cores. J Basic Eng. 1961; 83:513–8.
7. Gibson LJ, Ashby MF. Cellular solids. Oxford: Pergamon Press; 1988.
8. Warren WE, Kraynik AM. Foam mechanics: the linear elastic response of two-dimensional spatially periodic cellular materials. Mech Mater. 1987; 6:27–37.
9. Gibson LJ, Ashby MF, Schajer GS, Robertson CI. The mechanics of two-dimensional cellular materials. Proc Roy Soc A. 1982; 382:25–42.
10. Torquato S, Gibianski LV, Silva MJ, Gibson LJ. Effective mechanical and transport properties of cellular solids. Int J Mech Sci. 1998; 40:71–82.
11. Huybrechts S, Tsai SW. Analysis and behavior of grid structures. Compos Sci Technol. 1996; 56:1001–15.
12. Hohe J. A direct homogenization approach for determination of the stiffness matrix for micro heterogeneous plates with application to sandwich panels. Composites: Part B. 2003; 34; 615–26.
13. Hohe J, Becker W. An energetic homogenizations procedure for the elastic properties of general cellular sandwich cores. Composites B. 2001; 32:185–97.
14. Hohe J, Becker W. Effective mechanical behavior of hyper elastic honeycombs and two-dimensional model foams at finite strain. International Journal of Mechanical Sciences. 2003; 45:891–913.
15. Demiray S, Becker W, Hohe J. Analysis of two- and three dimensional hyper plastic model foams under complex loading conditions. Mechanics of Materials. 2006; 38:985–1000.
16. Bishop JF, Hill R. A theory of the plastic distortion of a polycrystalline aggregate under combined stress. PhilosMag. 1951; 42:414–27.