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# Stochastic Modified MAJ Model for Measuring the Efficiency and Ranking of DMUs

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## Abstract

The present study focuses on the extention of modified MAJ model in stochastic-data presence. To this aim, a stochastic modified MAJ model in Stochastic Data Envelopment Analysis (SDEA) is proposed for measuring the stochastic $\alpha$- efficiency of Decision-Making Units (DMUs) with normal-distribution inputs and outputs. Furthermore, stochastic super efficiency based on modified MAJ model is proposed for ranking stochastic $\alpha$ - efficient DMUs in stochastic modified MAJ model. Finally, a numerical example is presented to demonstrate the proposed idea.

**Keywords:** Modified MAJ Model, Stochastic Data Envelopment Analysis, Stochastic Modified MAJ Model, Stochastic Super Efficiency Model

## 1. Introduction

Data Envelopment Analysis (DEA) proposed by Charnes et al.[1] and developed by Banker et al.[2] is an approach for evaluating the efficiencies of Decision Making Units (DMUs) with similar quantitative characteristics. This is reflected by the assumption that each DMU uses the same set of inputs to produce the same set of outputs, but the inputs are consumed and outputs are produced in varying amounts. Additional DEA approaches and applications can be found in, but are not limited to, Kordrostami et al.[3], Kordrostami et al.[4], Shamsi et al.[5].

In cases for which several DMUs have the same efficiency score of one, a standard DEA approach is not able to discriminate amongst this DMUs. The related literature provide several approaches to rank efficient DMUs in DEA. Adler et al.[6], for example, classify and present these approaches into six streams: 1) Cross-efficiency ranking methods; 2) Benchmark ranking method; 3) Ranking with multivariate statistics in the DEA context; 4) Ranking inefficient DMUs; 5) DEA and Multi-Criteria Decision-Making (MCDM) Methods and 6) Super-efficiency ranking techniques. The sixth stream is super-efficiency ranking techniques proposed by Anderson et al.[7] They rank efficiency DMUs by measuring the distance from an efficiency DMU to a frontier, based on a set of observations, excluding the efficiency DMU in question. Therefore, the most efficient DMU is the one that can proportionally reduce outputs relative to the most efficient one without becoming inefficient.

Olesen et al.[8] developed a chance-constrained DEA model which utilizes the piecewise linear envelopments of confidence regions for use with stochastic multiple inputs and multiple outputs. Huang et al.[9] utilized this joint chance-constrained concept to discuss general dominance structures in the stochastic situations. Cooper et al.[(10-12)] have introduced the chance-constrained models to deal with the technical inefficiencies and congestion in the stochastic situation. Dibachi et al.[13] utilized stochastic multiplicative DEA model for measuring the efficiency and ranking of DMUs under VRS technology.

In this paper, we propose the stochastic modified MAJ model and stochastic super efficiency based on modified MAJ model for measuring the efficiency and ranking of DMUs, respectively. We consider this DMUs with the inputs and outputs having normal distributions. The

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paper unfolds as follows:

Some basic concepts in statistics and deterministic modified MAJ model will be introduced in the next section. Section 3 addresses the proposed method for introducing the stochastic modified MAJ model. Section 4 addresses the stochastic super efficiency based on modified MAJ model proposed for ranking of DMUs. A numerical example is given in section 5. Conclusions will appear in section 6.

## 2. Preliminaries

In this section, we recall some basic concepts and results which will be used throughout the paper.

### 2.1 Normal Distribution

The normal distribution is a family of probability density functions that is frequently used in practical situations. In quantitative economics and finance, the normal distribution is ubiquitous and it arises, among other things, in connection with Brownian motion, the standard model for the price dynamics of securities in mathematical finance.

**Definition 2.1** *A random variable X is said to have normal distribution if its probability density function is given as follows*

$$f_X(x)=\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{1}{2\sigma^2}(x-\mu)^2} \quad ; \quad x\in R \tag{1}$$

We will use the notation $X\sim N(\mu,\sigma^2)$ to denote a random variable X following a normal distribution with parameters $\sigma>0$ and $\mu\in R$. The corresponding cumulative distribution function has the following form:

$$F_X(x)=\int_{-\infty}^{x}\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{1}{2\sigma^2}(t-\mu)^2}dt \tag{2}$$

Note that if $Z\sim N(0,1)$ then $f_z(z)$ is called standard normal distribution and $F_z(z)$ is denoted by $\Phi(z)$ and $\Phi^{-1}$, its inverse, is the so-called fractile function. Specially, $\Phi^{-1}(0.5)=0$, $\Phi^{-1}(0.1)=-1.28$ and $\Phi^{-1}(0.67)=0.44$, .

### 2.2 Modified MAJ Model

One of the basic DEA models for measuring the efficiency of DMUs is the modified MAJ model introduced by Saati et al.[14]. The modified MAJ-efficiency of a specific $DMU_0$ is obtained by solving the following model

$$\begin{aligned}
& w_o^*=\min && \{1+w_o\} \\
& s.t. && \\
& \sum_{j=1}^{n}\lambda_j x_{ij}+s_i^-=x_{io}+w_o, && i=1,\ldots,m, \\
& \sum_{j=1}^{n}\lambda_j y_{rj}-s_r^+=y_{ro}-w_o, && r=1,\ldots,s, \\
& \sum_{j=1}^{n}\lambda_j=1, && \\
& s_r^+\geq 0, && r=1,\ldots,s, \\
& s_i^-\geq 0, && i=1,\ldots,m \quad and \\
& \lambda_j\geq 0, && j=1,\ldots,n,
\end{aligned} \tag{3}$$

$X_j=(x_{1j},x_{2j},\ldots x_{mj})$ and $Y_j=(y_{1j},y_{2j},\ldots y_{sj})$ are the input and output vectors of $DMU_j$, respectively where $x_{ij}\geq 0$ and $y_{rj}\geq 0$ for each $r\in\{1,2,\ldots,s\}$, $j\in\{1,2,\ldots,n\}$, and $i\in\{1,2,\ldots,m\}$.

**Definition 2.2** *(Modified MAJ-efficient) $DMU_0$ is said to be modified MAJ-efficient if and only if the following two conditions are both satisfied:*

i. $w_o^*=1$

ii. All slack variables are zero in the alternative optimal solution.

### 2.3 Super Efficiency based on Modified MAJ Model

Perhaps super-efficiency is the most well known, most widely applied and researched ranking method in DEA which was developed by Andersen et al.[7]. To solve the important difficulties of super-efficiency model, Mehrabian et al.[15] proposed another model for ranking efficient units. Their proposed model is

$$
\begin{aligned}
& w_o^* = min && \{1+w_o\} \\
& s.t. && \\
& \sum_{\substack{j=1 \\ j\neq o}}^{n} \lambda_j x_{ij} \le w_o + x_{io}, && i=1,\dots,m, \\
& \sum_{\substack{j=1 \\ j\neq o}}^{n} \lambda_j y_{rj} \ge y_{ro}, && r=1,\dots,s, \\
& \sum_{\substack{j=1 \\ j\neq o}}^{n} \lambda_j = 1 && and \\
& \lambda_j \ge 0, && j=1,\dots,n,
\end{aligned}
\tag{4}
$$

The MAJ model may be infeasible in some cases. To solve this problem, Saati et al.[14] proposed the following modified MAJ model

$$
\begin{aligned}
& w_o^* = min && \{1+w_o\} \\
& s.t. && \\
& \sum_{\substack{j=1 \\ j\neq o}}^{n} \lambda_j x_{ij} \le x_{io} + w_o, && i=1,\dots,m, \\
& \sum_{\substack{j=1 \\ j\neq o}}^{n} \lambda_j y_{rj} \ge y_{ro} - w_o, && r=1,\dots,s, \\
& \sum_{\substack{j=1 \\ j\neq o}}^{n} \lambda_j = 1 && and \\
& \lambda_j \ge 0, && j=1,\dots,n
\end{aligned}
\tag{5}
$$

## 3. Stochastic Modified MAJ Model

In this section, we present the stochastic modified MAJ model. For each $DMU_j$, let $X_j = (x_{1j}, x_{2j},...x_{mj})$ and $Y_j = (y_{1j}, y_{2j},...y_{sj})$ be the input and output random vectors of $DMU_j$, respectively. Moreover, suppose that $x_{ij} \ge 0$ and $y_{rj} \ge 0$. If $X_{ij} \sim N(\mu_{ij}, \sigma^2_{ij})$ and $Y_{rj} \sim N(\gamma_{rj}, \tau^2_{rj})$ then by using model (3) the stochastic modified MAJ model can be obtained as follows

$$
\begin{aligned}
& w_o^*(\alpha) = min && \{1+w_o\} \\
& s.t. && \\
& P(\sum_{j=1}^{n} \lambda_j X_{ij} \le X_{io} + w_o) \ge 1-\alpha && i=1,\dots,m, \\
& P(\sum_{j=1}^{n} \lambda_j Y_{rj} \ge Y_{ro} - w_o) \ge 1-\alpha, && r=1,\dots,s, \\
& \sum_{j=1}^{n} \lambda_j = 1 && and \\
& \lambda_j \ge 0, && j=1,\dots,n
\end{aligned}
\tag{6}
$$

where $\alpha$ is a predetermined number between 0 and 1 which specifies the significance level and P stands for probability.

### 3.1 Deterministic Equivalent

If $X_{ij} \sim N(\mu_{ij}, \sigma^2_{ij})$ and $Y_{rj} \sim N(\gamma_{rj}, \tau^2_{rj})$ then for all $r \in \{1,2,...,s\}$, $j \in \{1,2,...,n\}$, and $i \in \{1,2,...,m\}$ we have that

$$
\begin{aligned}
\sigma_i^2(\lambda) &= Var(\sum_{j=1}^{n} \lambda_j X_{ij} - X_{io} - w_o) \\
&= Var(\sum_{j=1}^{n} \lambda_j X_{ij} - X_{io}) = \sum_{k=1}^{n} \sum_{j=1}^{n} \lambda_j \lambda_k Cov(X_{ik}, X_{ij}) - 2\sum_{j=1}^{n} \lambda_j Cov(X_{ij}, X_{io}) + \sigma_{io}^2
\end{aligned}
\tag{7}
$$

Similarly,

$$
\begin{aligned}
\tau_r^2(\lambda) &= Var(\sum_{j=1}^{n} \lambda_j Y_{rj} - Y_{ro} + w_o) \\
&= Var(\sum_{j=1}^{n} \lambda_j Y_{rj} - Y_{ro}) = \sum_{k=1}^{n} \sum_{j=1}^{n} \lambda_j \lambda_k Cov(Y_{rk}, Y_{rj}) - 2\sum_{j=1}^{n} \lambda_j Cov(Y_{rj}, Y_{ro}) + \tau_{ro}^2
\end{aligned}
\tag{8}
$$

**Theorem 1** *The deterministic equivalent of stochastic modified MAJ model (6) is*

$$
\begin{aligned}
& w_o^*(\alpha) = min\ \{1+w_o\} \\
& s.t. \\
& \sum_{j=1}^{n} \lambda_j \mu_{ij} + s_i^- - \sigma_i(\lambda)\Phi^{-1}(\alpha) = \mu_{io} + w_o, \quad i=1,\dots,m, \\
& \sum_{j=1}^{n} \lambda_j \gamma_{rj} - s_r^+ + \tau_r(\lambda)\Phi^{-1}(\alpha) = \gamma_{ro} - w_o, \quad r=1,\dots,s, \\
& \sum_{j=1}^{n} \lambda_j = 1\ , \\
& s_i^- \ge 0, \quad i=1,\dots,m, \\
& s_r^+ \ge 0, \quad r=1,\dots,s \quad and \\
& \lambda_j \ge 0, \quad j=1,\dots,n
\end{aligned}
\tag{9}
$$

***Proof.*** From the first constraint in model (6) and statement (7) we have

$$
\begin{aligned}
&P\left(\sum_{j=1}^{n}\lambda_j X_{ij}\le X_{io}+w_o\right)\ge 1-\alpha \Leftrightarrow \\
&P\left(\frac{\sum_{j=1}^{n}\lambda_j X_{ij}-X_{io}-w_o-\left(\sum_{j=1}^{n}\lambda_j\mu_{ij}-\mu_{io}-w_o\right)}{\sigma_i(\lambda)}\le\frac{-\sum_{j=1}^{n}\lambda_j\mu_{ij}+\mu_{io}+w_o}{\sigma_i(\lambda)}\right)\ge 1-\alpha \Leftrightarrow \\
&P\left(Z\le\frac{-\sum_{j=1}^{n}\lambda_j\mu_{ij}+\mu_{io}+w_o}{\sigma_i(\lambda)}\right)\ge 1-\alpha \Leftrightarrow P\left(Z\le\frac{\sum_{j=1}^{n}\lambda_j\mu_{ij}-\mu_{io}-w_o}{\sigma_i(\lambda)}\right)\le\alpha \Leftrightarrow \\
&P\left(Z\le\frac{\sum_{j=1}^{n}\lambda_j\mu_{ij}-\mu_{io}-w_o+s_i^-}{\sigma_i(\lambda)}\right)=\Phi\left(\frac{\sum_{j=1}^{n}\lambda_j\mu_{ij}-\mu_{io}-w_o+s_i^-}{\sigma_i(\lambda)}\right)=\alpha \Leftrightarrow \\
&\Phi^{-1}(\alpha)=\frac{\sum_{j=1}^{n}\lambda_j\mu_{ij}-\mu_{io}-w_o+s_i^-}{\sigma_i(\lambda)} \Leftrightarrow \sum_{j=1}^{n}\lambda_j\mu_{ij}+s_i^- -\sigma_i(\lambda)\Phi^{-1}(\alpha)=\mu_{io}+w_o
\end{aligned}
\tag{10}
$$

Similarly, from the second constraint in model (6) and statement (8) we have that

$$
\begin{aligned}
&P\left(\sum_{j=1}^{n}\lambda_j Y_{rj}\ge Y_{ro}-w_o\right)\ge 1-\alpha \Leftrightarrow \\
&P\left(\frac{\sum_{j=1}^{n}\lambda_j Y_{rj}-Y_{ro}+w_o-\left(\sum_{j=1}^{n}\lambda_j\gamma_{rj}-\gamma_{ro}+w_o\right)}{\tau_r(\lambda)}\ge\frac{-\sum_{j=1}^{n}\lambda_j\gamma_{rj}+\gamma_{ro}-w_o}{\tau_r(\lambda)}\right)\ge 1-\alpha \Leftrightarrow \\
&P\left(Z\ge\frac{-\sum_{j=1}^{n}\lambda_j\gamma_{rj}+\gamma_{ro}-w_o}{\tau_r(\lambda)}\right)\ge 1-\alpha \Leftrightarrow P\left(Z\le\frac{-\sum_{j=1}^{n}\lambda_j\gamma_{rj}+\gamma_{ro}-w_o}{\tau_r(\lambda)}\right)\le\alpha \Leftrightarrow \\
&P\left(Z\le\frac{-\sum_{j=1}^{n}\lambda_j\gamma_{rj}+\gamma_{ro}-w_o+s_r^+}{\tau_r(\lambda)}\right)=\Phi\left(\frac{-\sum_{j=1}^{n}\lambda_j\gamma_{rj}+\gamma_{ro}-w_o+s_r^+}{\tau_r(\lambda)}\right)=\alpha \Leftrightarrow \\
&\Phi^{-1}(\alpha)=\frac{-\sum_{j=1}^{n}\lambda_j\gamma_{rj}+\gamma_{ro}-w_o+s_r^+}{\tau_r(\lambda)} \Leftrightarrow \sum_{j=1}^{n}\lambda_j\gamma_{rj}-s_r^+ +\tau_r(\lambda)\Phi^{-1}(\alpha)=\gamma_{ro}-w_o
\end{aligned}
\tag{11}
$$

Thus, by (11) and (10), the deterministic model is completely specified where $w_o^*(\alpha)$, $s_i^{-*}$, and $s_i^{+*}$ can be determined by solving model (9).

## 4. Stochastic Super Efficiency based on Modified MAJ Model

In this section we propose stochastic super efficiency based on modified MAJ model as follows

$$
\begin{aligned}
w_o^*(\alpha)=min\quad & \{1+w_o\} \\
s.t.\quad & \\
& P\Big(\sum_{\substack{j=1\\ j\ne o}}^{n}\lambda_j X_{ij}\le X_{io}+w_o\Big)\ge 1-\alpha, \quad i=1,\dots,m, \\
& P\Big(\sum_{\substack{j=1\\ j\ne o}}^{n}\lambda_j Y_{rj}\ge Y_{ro}-w_o\Big)\ge 1-\alpha, \quad r=1,\dots,s, \\
& \sum_{\substack{j=1\\ j\ne o}}^{n}\lambda_j=1 \quad and \\
& \lambda_j\ge 0, \quad j=1,\dots,n,
\end{aligned}
\tag{12}
$$

In a similar manner to the proof of Theorem (1) the deterministic equivalent of stochastic super efficiency MAJ model (12) is specified as follows

$$
\begin{aligned}
w_o^*(\alpha)=min\ & \{1+w_o\} \\
s.t.\ & \\
& \sum_{\substack{j=1\\ j\ne o}}^{n}\lambda_j\mu_{ij}+s_i^- -\sigma_i(\lambda)\Phi^{-1}(\alpha)=\mu_{io}+w_o, \ i=1,\dots,m, \\
& \sum_{\substack{j=1\\ j\ne o}}^{n}\lambda_j\gamma_{rj}-s_r^+ +\tau_r(\lambda)\Phi^{-1}(\alpha)=\gamma_{ro}-w_o, \ r=1,\dots,s, \\
& \sum_{\substack{j=1\\ j\ne o}}^{n}\lambda_j=1 \qquad , \\
& s_i^-\ge 0, \quad i=1,\dots,m, \\
& s_r^+\ge 0, \quad r=1,\dots,s \ and \\
& \lambda_j\ge 0, \quad j=1,\dots,n
\end{aligned}
\qquad 13)
$$

Note that in the above model we have that

$$
\begin{aligned}
\sigma_i^2(\lambda) &= Var\Big(\sum_{\substack{j=1\\ j\ne o}}^{n}\lambda_j X_{ij}-X_{io}-w_o\Big) \\
&= Var\Big(\sum_{\substack{j=1\\ j\ne o}}^{n}\lambda_j X_{ij}-X_{io}\Big)=\sum_{\substack{k=1\\ k\ne o}}^{n}\sum_{\substack{j=1\\ j\ne o}}^{n}\lambda_j\lambda_k Cov(X_{ik},X_{ij})-2\sum_{\substack{j=1\\ j\ne o}}^{n}\lambda_j Cov(X_{ij},X_{io})+\sigma_{io}^2
\end{aligned}
\tag{14}
$$

and,

$$\tau_r^2(\lambda) = Var\Big(\sum_{\substack{j=1 \\ j\neq o}}^{n} \lambda_j Y_{rj} - Y_{ro} + w_o\Big)$$
$$= Var\Big(\sum_{\substack{j=1 \\ j\neq o}}^{n} \lambda_j Y_{rj} - Y_{ro}\Big) = \sum_{\substack{k=1 \\ k\neq o}}^{n} \sum_{\substack{j=1 \\ j\neq o}}^{n} \lambda_j \lambda_k Cov(Y_{rk}, Y_{rj}) - 2\sum_{\substack{j=1 \\ j\neq o}}^{n} \lambda_j Cov(Y_{rj}, Y_{ro}) + \tau_{ro}^2 \quad (15)$$

The rank of the DMU under evaluation is determined by solving (12).

In order to determine the stochastic *modified* MAJ $\alpha$-efficiency we consider the following theorem

**Theorem 2** *For $\alpha = 0.5$. The inefficiency vs. efficiency classification of $DMU_0$ in the modified MAJ model (3) is the same as in the stochastic modified MAJ model (6).*

***Proof.*** If $\alpha = 0.5$ then $\Phi^{-1}(0.5)=0$. Thus, the inefficiency vs. efficiency classification of $DMU_0$ in deterministic modified MAJ model (3) is the same as in stochastic modified MAJ model (6).

## 5. Numerical Example

In this section, a numerical example is presented to demonstrate the modeling idea and the effectiveness of the proposed method.

We apply the proposed stochastic modified MAJ model and stochastic super efficiency modified MAJ model for measuring the efficiency and ranking of 12 DMUs. Suppose that $Y_{rj}$, $r$=1,2 are the random outputs of $DMU_j$ where have the normal distribution with parameters $\tau^2_{rj}$ and $\gamma_{rj}$ which are denoted with $Y_{rj} \sim N(\gamma_{rj}, \tau^2_{rj})$. Moreover, suppose that $X_{ij}$, $i$=1,2 are the random inputs of $DMU_j$ where have the normal distribution with parameters $\sigma^2_{ij}$ and $\mu_{ij}$ which are denoted with $X_{ij} \sim N(\mu_{ij}, \sigma^2_{ij})$.

Thus, by solving models (9) and (13) can be obtain the stochastic $\alpha$-efficiency of DMUs and its rank. The data set for this example is shown in Table 1.

We run models (9) and (13) by means of GAMS software for all $\alpha \in \{0.1, 0.5, 0.67\}$ and the results are shown in Table 2.

There are a lot of number of the DMUs, which are $\alpha$-efficient and thus, the rank of the DMU under evaluation is determined by solving the stochastic super efficiency modified MAJ model . Table 2 expresses that for a set of 12 DMUs, if $0 \le \alpha < \alpha' < \frac{1}{2}$ then the number of DMUs $\alpha'$-efficient is less than or equal to the number of DMUs $\alpha$-efficient. If $\frac{1}{2} < \alpha < \alpha' \le 1$ then the number of DMUs $\alpha'$-efficient is greater than or equal to the number of DMUs $\alpha$-efficient. Moreover, $DMU_6$ is the worst DMUs for each $\alpha \in \{0.1, 0.5, 0.67\}$.

**Table 1.** The data set of the numerical example

| $DMU_j$ | Input 1 | Input 2 | Output 1 | Output 2 |
|---|---|---|---|---|
| $DMU_1$ | $X_{11} \sim N(20,25)$ | $X_{21} \sim N(25,16)$ | $Y_{11} \sim N(1000,100)$ | $Y_{21} \sim N(900,400)$ |
| $DMU_2$ | $X_{12} \sim N(15,4)$ | $X_{22} \sim N(23,18)$ | $Y_{12} \sim N(800,200)$ | $Y_{22} \sim N(950,300)$ |
| $DMU_3$ | $X_{13} \sim N(10,4)$ | $X_{23} \sim N(9,9)$ | $Y_{13} \sim N(950,400)$ | $Y_{23} \sim N(500,450)$ |
| $DMU_4$ | $X_{14} \sim N(18,8)$ | $X_{24} \sim N(10,8)$ | $Y_{14} \sim N(850,500)$ | $Y_{24} \sim N(550,430)$ |
| $DMU_5$ | $X_{15} \sim N(17,6)$ | $X_{25} \sim N(18,7)$ | $Y_{15} \sim N(980,550)$ | $Y_{25} \sim N(800,100)$ |
| $DMU_6$ | $X_{16} \sim N(16,4)$ | $X_{26} \sim N(19,15)$ | $Y_{16} \sim N(700,520)$ | $Y_{26} \sim N(600,250)$ |
| $DMU_7$ | $X_{17} \sim N(11,9)$ | $X_{27} \sim N(20,14)$ | $Y_{17} \sim N(750,700)$ | $Y_{27} \sim N(650,230)$ |
| $DMU_8$ | $X_{18} \sim N(19,20)$ | $X_{28} \sim N(17,4)$ | $Y_{18} \sim N(850,350)$ | $Y_{28} \sim N(830,450)$ |
| $DMU_9$ | $X_{19} \sim N(12,10)$ | $X_{29} \sim N(15,17)$ | $Y_{19} \sim N(600,150)$ | $Y_{29} \sim N(580,160)$ |
| $DMU_{10}$ | $X_{110} \sim N(13,5)$ | $X_{210} \sim N(10,12)$ | $Y_{110} \sim N(970,300)$ | $Y_{210} \sim N(560,400)$ |
| $DMU_{11}$ | $X_{111} \sim N(16,6)$ | $X_{211} \sim N(22,16)$ | $Y_{111} \sim N(780,110)$ | $Y_{211} \sim N(700,350)$ |
| $DMU_{12}$ | $X_{112} \sim N(9,4)$ | $X_{212} \sim N(8,3)$ | $Y_{112} \sim N(650,90)$ | $Y_{212} \sim N(860,310)$ |

**Table 2.** Results of measuring the efficiency and ranking of DMUs

| $DMU_j$ | Efficiency $\alpha$=0.5 | Super Efficiency $\alpha$=0.5 | Ranking $\alpha$=0.5 | Efficiency $\alpha$=0.67 | Super Efficiency $\alpha$=0.67 | Ranking $\alpha$=0.67 | Efficiency $\alpha$=0.1 | Super Efficiency $\alpha$=0.1 | Ranking $\alpha$=0.1 |
|---|---|---|---|---|---|---|---|---|---|
| $DMU_1$ | 1 | 64.6364 | 1 | -0.7820 | 59.6241 | 1 | 27.7153 | 79.4697 | 2 |
| $DMU_2$ | 1 | 51.0000 | 2 | -0.3325 | 40.0767 | 2 | 28.8062 | 82.7769 | 1 |
| $DMU_3$ | 1 | 3.7160 | 5 | 0.2329 | 3.0141 | 4 | 4.8433 | 6.0304 | 6 |
| $DMU_4$ | -0.3289 | -0.3289 | 7 | -0.5328 | -1.2727 | 7 | 2.0944 | 2.2202 | 7 |
| $DMU_5$ | 1 | 2.9339 | 6 | -0.0707 | 1.2110 | 6 | 5.6635 | 8.0954 | 5 |
| $DMU_6$ | -5.8106 | -5.8106 | 12 | -6.8016 | -6.8016 | 12 | -3.6277 | -3.6277 | 12 |
| $DMU_7$ | -0.6611 | -0.6611 | 8 | -1.5401 | -1.5401 | 8 | 1.3723 | 1.7217 | 9 |
| $DMU_8$ | -1.3085 | -1.3085 | 9 | -2.5069 | -2.5069 | 9 | 2.0589 | 2.0589 | 8 |
| $DMU_9$ | -2.0000 | -2.0000 | 10 | -3.1181 | -3.1181 | 10 | 0.3723 | 0.3723 | 10 |
| $DMU_{10}$ | 1 | 4.8462 | 4 | -0.1294 | 2.7005 | 5 | 7.3962 | 9.1336 | 4 |
| $DMU_{11}$ | -5.3428 | -5.3428 | 11 | -6.3447 | -6.3447 | 11 | -2.8452 | -2.8452 | 11 |
| $DMU_{12}$ | 1 | 11.0000 | 3 | -0.0700 | 9.7650 | 3 | 5.1911 | 14.7714 | 3 |

## 6. Conclusion

The purpose of the classic DEA model is to evaluate the performance of a set of DMUs using deterministic inputs and outputs. In real-world scenarios, stochastic models may be better suited for DEA model when there is uncertainty associated with the inputs and/or outputs of DMUs. The stochastic inputs and outputs in DEA model are represented with random variables. This paper attempted to extend the modified MAJ model and develop a new model with the stochastic inputs and outputs. Therefore, we proposed a stochastic modified MAJ model for measuring the stochastic $\alpha$-efficiency of DMUs which inputs and outputs following a normal distribution. Furthermore, we proposed a stochastic super efficiency modified MAJ model for ranking of DMUs by using stochastic modified MAJ model. Some basic concepts in statistics were stated and the concepts of stochastic $\alpha$-efficient, $\alpha$-inefficient are defined. Finally, a numerical example was used to demonstrate the capability of the proposed approach. This example was run in three cases of $\alpha$ and it was observed that the number of DMUs featured stochastic $\alpha$-inefficient increases when the value of $\alpha$ increases for $0 \le \alpha < \frac{1}{2}$ and also for $\frac{1}{2} < \alpha \le 1$ the number of DMUs featured stochastic $\alpha$-efficient increases when the value of $\alpha$ increases. The approach of this research may be extended to some other DEA models and other distributions the as well.

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