

# Studying the Effect of Indentation on Flow Parameters and Slow Heat Transfer of Water-Silver Nano-Fluid with Varying Volume Fraction in a Rectangular Two-Dimensional Micro Channel

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## Abstract

The purpose of this study is to investigate the influence of indentations on the parameters of fluid flow and heat of water-silver nanofluid in a rectangular two-dimensional micro channel. It includes heat transfer water silver nano fluid in an indented micro channel under the constant temperature. The system is numerically modeled, by Finite Volume Method. After solving the governing equations for  $U$ ,  $V$  and  $\theta$ , other useful quantities such as Nusselt number and friction factor can be determined. The hot fluid inlet exits after cooling by the cold walls of the micro channel. Calculations are done for the two ranges of Reynolds number ( $Re$ ). It was observed that at times the fluid has more indentations; it has a greater temperature drop that is at the output cross section of the micro channel. With increasing Reynolds number ( $Re$ ), number of the indentations and the increasing volume fraction of the nanoparticles, greater temperature drop occurs. The presence of indentation in the micro channel increases the speed and the dimensionless temperature at the center line. Finally, the results are provided in the form of the contour of flow and isothermal lines, the coefficient of friction, Nusselt number, temperature and velocity profiles in different micro channel sections. The results of the numerical simulation indicate that the heat transfer rate is significantly affected by the solid volume fraction and Reynolds number.

**Keywords:** Heat Transfer, Local Friction Factor, Nanofluid, Nusselt Number, Rib Micro Channel

## 1. Introduction

Improved heat transfer using new methods makes significant savings in energy costs and protecting the environment. Methods such as using suspensions instead of fluid factor in heat transfer equipment and the use of finned surfaces to enhance heat transfer surfaces and the use of sub-surfaces to disrupt laminar layer in the boundary layer of turbulent flow and to create secondary flows in order to better incorporate the flow factor in a heat transfer environment are the new techniques. The purpose of

this study is to investigate the influence of indentations on the parameters of fluid flow and heat transfer of nano-fluid in a rectangular two-dimensional micro channel. However, creating artificial teeth and ribbed surfaces in the direction of the fluid passing through the channel can significantly improve the heat transfer performance. But the added tooth will have unpleasant issues such as further pressure drop, creating areas with lower heat transfer on the back of the teeth, and higher pumping power to move fluid in the channel, in comparison with the smooth channel.

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Many researchers investigated the flow behavior and heat transfer in channels and micro channels and Enclosure at different cross sections<sup>1-8</sup>. Much of this research focuses on examining the impact of parameters such as the Reynolds number, the channel cross-section, geometrical characteristics, arrangement of teeth, and attack angle of teeth and Type nanofluid<sup>9-13</sup>. Jung et al<sup>14</sup> experimentally investigated heat transfer in mandatory displacement of nano fluids in micro channels with water nano fluids - aluminum oxide and found that the coefficient of nano fluids displacement with volume ratio of 1/8% nano particles, more than 32 percent is higher than the rate of pure water displacement coefficient. In micro channels with smaller sizes, the heat transfer coefficient at comparable Reynolds numbers or larger than the coefficient of heat transfer in larger micro channels with higher Reynolds numbers, showing the micro channel heat transfer properties. Harris and colleagues<sup>15</sup> in an Experimental investigation of Water nano fluids - aluminum oxide concluded that the heat transfer coefficient increased turnover by 40 percent in comparison to pure water, while the thermal conductivity is increased up to 15 percent. Manka et al<sup>16</sup> studied the free displacement of heat transfer of nano fluids in a channel with teeth under constant heat flux. They concluded that the increasing Reynolds number and particle concentration both increase the average Nusselt number and the pressure drop. Han and colleagues<sup>17</sup> investigated the effects of tooth shape, angle of attack, and the ratio of step height on heat transfer. They concluded that with the same friction, the teeth with an angle of attack of 45° have a higher heat transfer performance than a tooth with 90°. Han and Park<sup>18</sup> studied the heat transfer in channels with different aspect ratios, and concluded that the channel with square cross section and a tooth with the angle of attack between 30° to 45° have the highest heat transfer performance. Jones and Emsit<sup>19</sup> examined the effect of height and spaces between teeth on heat transfer coefficient and concluded that the gap between the teeth is the most important geometrical parameter which can be considered as the characteristic length. Leo and Huang<sup>20</sup> examined the heat transfer performance of the fully developed flow in a channel with three square, semi-circular, and triangular teeth. They concluded that the above teeth have a comparable heat transfer performance and behind the square tooth compared to other areas, lower heat transfer areas are observed. Park et al<sup>21</sup> conducted

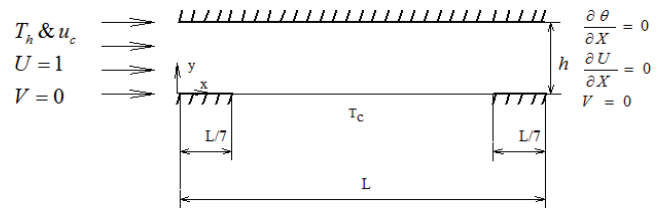
empirical investigations in order to explain the friction in the turbulent flow and heat transfer behavior of nano fluids composed of Al<sub>2</sub>O<sub>3</sub> and TiO<sub>2</sub> particles suspended in water in a circular pipe. They concluded that the increase in the volume concentration of nano particles increases the Nusselt number and there is a good relationship between Nusselt number and Reynolds number during the fully developed turbulent flow. Hove and colleagues<sup>22</sup> experimentally studied the effect of Tooth height on heat transfer in rectangular channels and found that increasing the blockage ratio (ratio of channel height to channel hydraulic diameter) improves the heat transfer. Leo and Huang<sup>23</sup> studied the numerical and experimental analysis of heat transfer and fluid flow behavior in rectangular channels with alternate arrangement of the teeth in the flow direction on one side of the main wall. They concluded that the speed and intensity of the turbulence are the important factors affecting the heat transfer coefficient. Wang and Sanden<sup>24</sup> in their empirical research to calculate the amount of local heat transfer in a square channel with teeth concluded that the local heat transfer rate is strongly dependent on the shape of the teeth. Rowe and colleagues<sup>25</sup> in their empirical research achieved the optimal step height and concluded that the increase in the rate of heat transfer not only depends on the aspect ratio of the teeth, but also depends on the shape of the teeth. Zhang and colleagues<sup>26</sup> in the study of heat transfer ratio in rectangular channels with grooved teeth found that creating grooves between the square teeth will not only increase the heat transfer rate but also decreases the pressure drop along the path. Han and colleagues<sup>27</sup> examined the transmission rate in a square channel for seven different angles (v-shaped) and concluded that the Chevron tooth with angle of 60° has heat transfer 4/5 times of the smooth channel and a better performance than the consistent teeth. Shen et al<sup>28</sup> experimentally investigated the flow and heat transfer in micro channel with rough walls with water fluid and concluded that the natural roughness has a very significant effect on the micro channel heat transfer in laminar flow and the coefficient of friction and the Nusselt number are significantly away from the classical theory. It is probably due to the roughness and the increased inflow temperature and power supply will promote the flow performance of heat.

## 2. Problem Statement

Figure 1 show the geometry used in this study that is a two-dimensional micro channel. Micro channel length is  $L = 2.5$  mm and the hydraulic diameter is  $2h = 50$ . The upper wall with the Length  $L$  and the bottom wall from the micro channel input and output sides are  $L/7$ . The central area of Micro channel is  $5L/7$  affected by temperature  $T_c = 293$ . The Water inlet temperature of water-silver nano fluids is  $T_h = 303^\circ$  K, and the Water and Silver Nano particles are in thermal equilibrium.

Silver nano particles are spherical and uniform in shape and the micro channel flow is assumed slow, Newtonian, and incompressible under non-slip boundary condition at the walls and the Radiation effects are negligible. Thermo properties of pure water (base fluid) and silver nano particles at  $298^\circ$  K are shown in Table 1 and the water Prandtl number is  $Pr = 6.2$ . In this study, the study of fluids parameters and heat transfer have been performed with Reynolds numbers  $Re = 10$  and the volume percentage of solid nano particles of 0, 0.02, 0.04 =  $\phi$ . Table 1 shows Thermo physical properties of water, silver (Ag) and nanofluid<sup>29</sup>.

At the present problem, in the micro channel Figure 1 we examine the effects of fluids parameters and heat transfer in the four states of D, C, B, A. In case (A) All calculations and analysis for micro channel of Figure 1 are performed and in case (B) the above parameters for micro channels of Figure 1 with respect to a tooth on the cold wall (below) with the length  $7L$  at the area  $7L < X < 4L/7$  is investigated from the length of the input micro channel. In case (C), the calculation of parameters associated with each tooth length  $L/7$  in the area of the micro channel (1) in the zone between  $7L < X < 3 \cdot 7L/2$  and  $7L < X < 5 \cdot 7L/4$  has been studied. In case (C) calculate the relevant parameters with two teeth along each  $L/7$  in the area of micro form (1) regions between  $7L < X < 3 \cdot 7L/2$  and  $7L < X < 5 \cdot 7L/4$  has been investigated. In case (D) parameters calculated for the three teeth during each  $7L/7$  in the micro channel bottom wall (1) in the zone  $5 \cdot 2L < X < 7L/7$  and  $7L < X < 5 \cdot 7L/4$  and  $7L/6 < X < 7L/5$  is investigated. For best results, all states in terms of form Figure 2 is shown. For all cases, D, C, B, A Tooth height is fixed and  $\alpha H$  values of  $\alpha = 0.1$ .



**Figure 1.** Schematic overview of the micro channel project areas A, B, C, D, is defined in.

## 3. Formulation

Dimensionless equations of governing equations include continuity, momentum and energy for steady-state in Cartesian coordinates<sup>30</sup>.

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{1}$$

$$U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{\mu_{nf}}{\rho_{nf} \nu_f} \frac{1}{Re} \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) \tag{2}$$

$$U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{\mu_{nf}}{\rho_{nf} \nu_f} \frac{1}{Re} \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) \tag{3}$$

$$U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{\alpha_{nf}}{\alpha_f} \frac{1}{Re Pr} \left( \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right) \tag{4}$$

In the above equations, the following dimensionless parameters are used,

$$\begin{aligned} X &= \frac{x}{h} & Y &= \frac{y}{h} & U &= \frac{u}{u_c} & V &= \frac{v}{u_c} & P &= \frac{\bar{P}}{\rho_{nf} u_c^2} \\ \theta &= \frac{T - T_c}{T_h - T_c} & Re &= \frac{u_c h}{\nu_f} & Pr &= \frac{\nu_f}{\alpha_f} \end{aligned} \tag{5}$$

The following formula is used to calculate the density of nanofluids<sup>30</sup>,

$$\rho_{nf} = (1 - \phi) \rho_f + \phi \rho_s \tag{6}$$

**Table 1.** Thermo physical properties of water, silver (Ag) and nanofluid

Units	Water	Ag	Nanofluid $\phi=0.02$	Nanofluid $\phi=0.04$
$c_p$ (J/kgK)	4179	235	3481.3	2976.2
$\rho$ (kg/m <sup>3</sup> )	997	10,500	1187.2	1377.2
$k$ (W/mK)	0.613	429	1.0052	1.4137
$\mu$ (Pa s)	$8.91 \times 10^{-4}$	-	$9.37 \times 10^{-4}$	$9.86 \times 10^{-4}$

Brinkman equation is used for calculating the effective dynamic viscosity of nanofluids<sup>31</sup>,

$$\mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}} \quad (7)$$

Nanofluids effective thermal diffusivity is calculated using the following formula<sup>30</sup>,

$$\alpha_{nf} = \frac{k_{eff}}{(\rho C_p)_{nf}} \quad (8)$$

Nanofluid specific heat capacity is calculated with the fol-

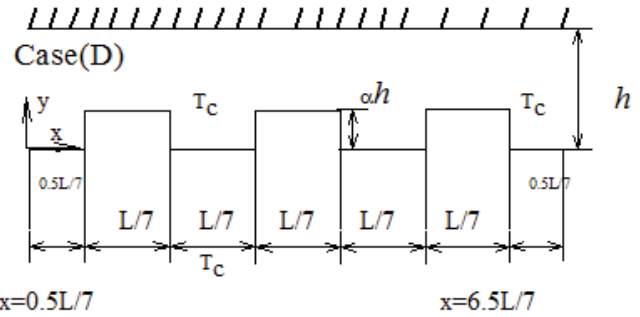
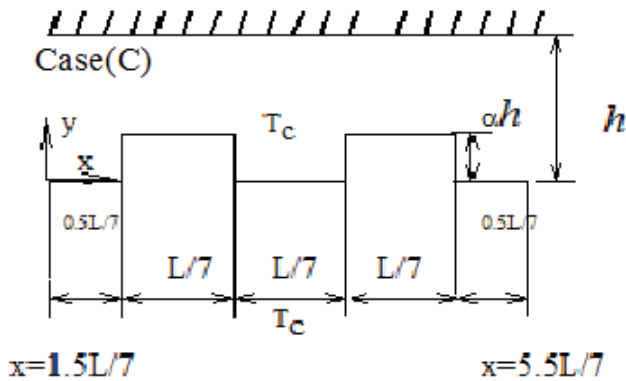
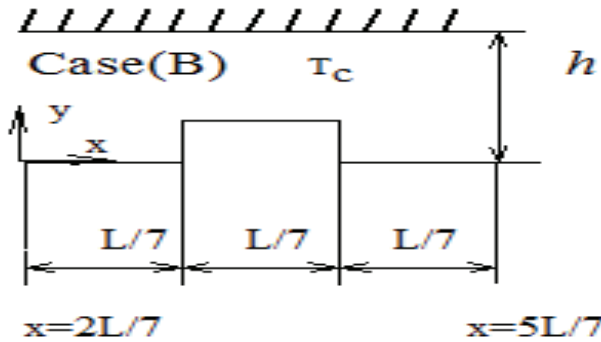
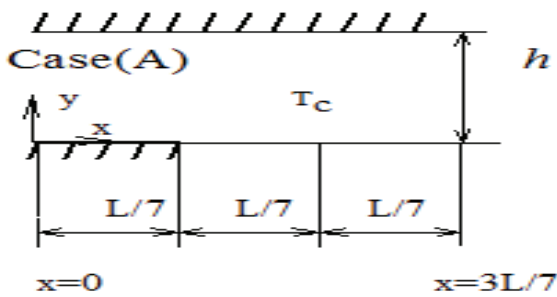


Figure 2. Schematic of Figures defined by A, B, C, D, show.

lowing formula<sup>30</sup>,

$$(\rho C_p)_{nf} = (1-\phi)(\rho C_p)_f + \phi(\rho C_p)_s \quad (9)$$

To calculate the effective thermal conductivity of nanofluids for suspension containing spherical particles Patel et al<sup>32</sup> equation is used:

$$k_{eff} = k_f \left[ 1 + \frac{k_s A_s}{k_f A_f} + c k_s Pe \frac{A_s}{k_f A_f} \right] \quad (10)$$

Empirical constant is  $c = 36,000$

$$\frac{A_s}{A_f} = \frac{d_f}{d_s} \frac{\phi}{1-\phi} \quad (11)$$

$$Pe = \frac{u_s d_s}{\alpha_f} \quad (12)$$

In equations (11 and 12) the diameter of the water molecule is  $d_f = 2\text{\AA}$  and the diameter of aluminum nano particles is equal to  $d_s = 50\text{nm}$ . The speed  $u_s$  Brownian motion of nano particles is calculated using the following formula:

$$u_s = \frac{2\kappa_b T}{\pi \mu_f d_s^2} \quad (13)$$

In equation (13), the value of  $\kappa_b = 1.3807 \times 10^{-23} \text{J/K}$  is Boltzmann's constant. To calculate the local Nusselt number along the bottom wall, we use the following equation,

$$Nu(X) = \frac{k_{eff}}{k_f} \left( \frac{\partial \theta}{\partial Y} \right)_{Y=0} \quad (14)$$

To calculate the local Nusselt number within the teeth width, we use the following equation:

$$Nu(Y) = \frac{k_{eff}}{k_f} \left( \frac{\partial \theta}{\partial X} \right)_{X=0} \quad (15)$$

To calculate the average Nusselt number along the horizontal portion of the lower wall, we use the following equation,

$$Nu_m |_{X=0} = \frac{1}{(L/7)} \int_{L/7}^{2L/7} Nu(X) dX \quad (16)$$

To calculate the average Nusselt number within any of the teeth we use the following equation,

$$Nu_m |_{Y=0} = \frac{1}{(\alpha_n H)} \int_0^{\alpha_n H} Nu(Y) dY; n=1,2 \quad (17)$$

The total Nusselt number on the surface of each tooth is equal to:

$$Nu_{m_{total}} = Nu_m |_{X=0} + Nu_m |_{Y=0} \quad (18)$$

$$C_f = \frac{\tau_w}{\frac{1}{2} \rho u_{ave}^2}; \mu = \mu_f \ \& \ \rho = \rho_f \ \& \ u_{ave} = u_c \quad (19)$$

By replacing the Dimensionless parameters of part (5) in equation (19), (20) and (21) equations can be achieved

$$C_f(X) = \frac{2}{Re} \left( \frac{\partial U}{\partial Y} \right)_{Y=0} \quad (20)$$

$$C_f(Y) = \frac{2}{Re} \left( \frac{\partial V}{\partial X} \right)_{X=0} \quad (21)$$

To calculate the average friction coefficient along the horizontal portion of the lower wall, we use the following equation,

$$C_{f_m} |_{X=0} = \frac{1}{(L/7)} \int_0^{L/7} C_f(X) dX \quad (22)$$

Flow lines have more changes with increasing Reynolds number in the area before and after the teeth. With the

increase in the number of teeth, changes of flow lines are greater, because of the loss of fluid momentum due to the collision of the teeth. In lower Reynolds numbers these changes are less (i.e. for Reynolds  $Re = 10$  flow line shifts towards Reynolds  $Re = 100$  is less). For temperature lines after entering fluid at high temperature the following equation is used to calculate the average friction coefficient within the width of each tooth:

$$C_{f_m} |_{Y=0} = \frac{1}{(\alpha_n H)} \int_0^{\alpha_n H} C_f(Y) dY; n=1,2 \quad (23)$$

The coefficient of local friction is equal to:

$$C_{f_{m_{total}}} = C_{f_m} |_{X=0} + C_{f_m} |_{Y=0} \quad (24)$$

## 4. Results and Discussion

### 4.1 Examining the Reynolds Number and Nanoparticle Volume Fraction

This section examines the effects of Reynolds number  $Re = 10$  and  $Re = 100$  and nano particle volume fraction ( $\phi = 0.04$ ) on flow lines and isothermal lines for all cases of A, B, C and D. Flow lines in Figures 3 to 6 show that, flow is developed after the arrival of flow to the micro channel. The development exists to the before hitting the teeth. After the encounter with the tooth, the flow is gone out of the development case. Discontinuities in the lower Reynolds and higher volume fraction of nano particles from heat transfer and the hot inlet increase to the cold inlet wall. Since the hot inlet fluid uses more time to deal with the mid-level at cold temperature, it can be seen that the fluid temperature drops more; it means that at the outlet micro channel with increasing Reynolds number, the number of teeth and the increasing volume fraction of nano particles, temperature drops more.

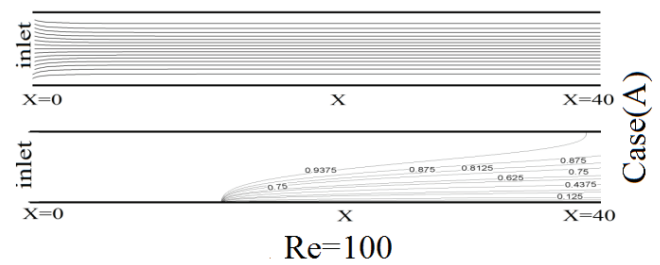
In Figure 7, the flow a is developed after going through the entrance area. This development will continue to hit the fluid to the teeth of the micro channel. With hitting

the teeth of micro channel, the speed increases and it is due to the shrinkage of micro channel for having no teeth and these changes are nearly constant with increasing the number of teeth. More changes in the dimensionless speed occur with increasing Reynolds number. Figure 8 shows the dimensionless temperature at the center of the micro channel for the cases A, B, C and D and the dimensionless temperature is closer to zero with increasing the number of teeth and reducing the Reynolds number in the output section. It means that more heat transfer occurs along the micro channel.

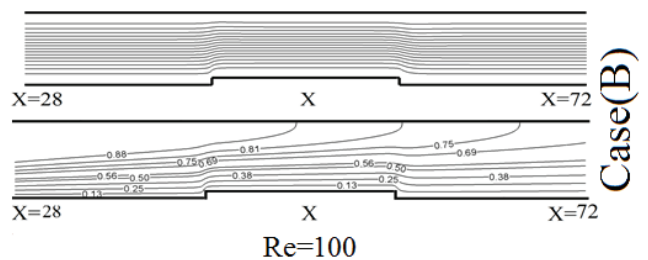
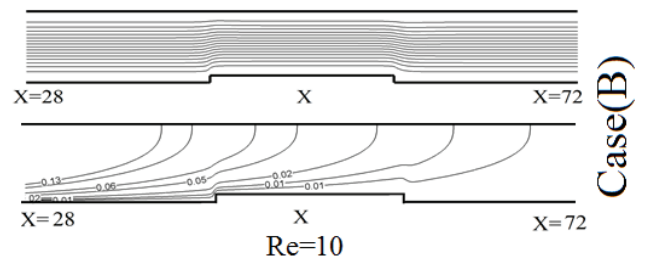
Figure 9 shows the graph for local Nusselt number along the dimensionless micro channel for cases A, B, C and D. It is observed that after hitting the fluid to the tooth due to the better mixing and more uniform temperatures in different layers of fluid, the Nusselt number increases suddenly over the micro channel. In the first teeth, the highest Nusselt number variations may occur. Nusselt number increases with the number of teeth, the volume fraction of nano particles and increasing Reynolds number.

Figure 10 shows the average Nusselt number in states A, B, C and D for the volume fraction of 0 to 0/4. It can be seen that the average Nusselt number at lower Reynolds with two teeth and different volume fraction has less changes. With more teeth and the volume fraction of nano particles, the Nusselt number increases. The effect of the presence of nano particles in higher Reynolds and higher volume fraction of nano particles is more pronounced at lower number of teeth. In  $Re = 100$  it is observed that with increasing number of teeth and volume fraction of nano particles, average Nusselt number increases. However, we compare the average friction coefficient along the x and y for scenarios A, B, C and D with Reynolds number  $Re = 100$ ,  $Re = 10$  and  $\phi = 0, 0$ .

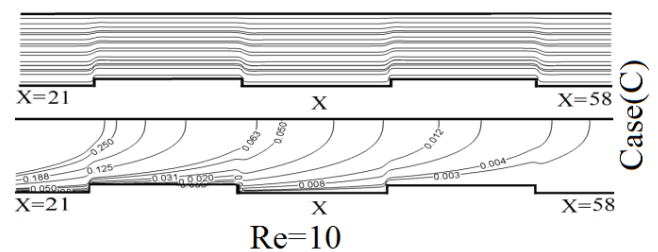
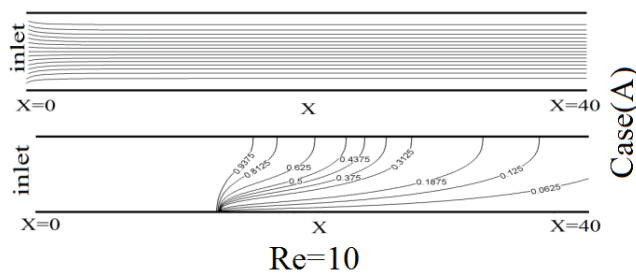
Figure 11 shows the comparison of average Cf along



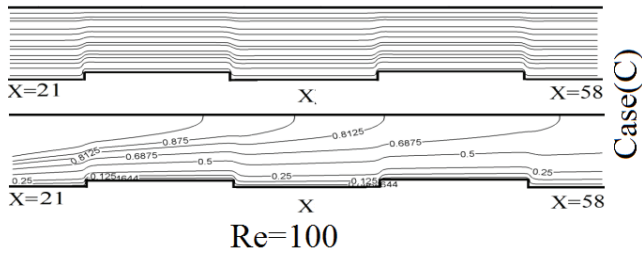
**Figure 3.** Graphs of flow lines and isotherms at  $Re = 10$  and  $Re = 100$  Reynolds range of nanoparticle volume fraction ( $\phi = 0.04$ ) for the case (A).



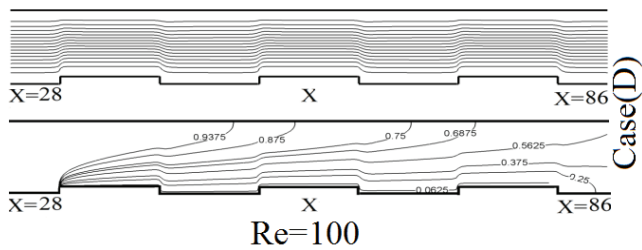
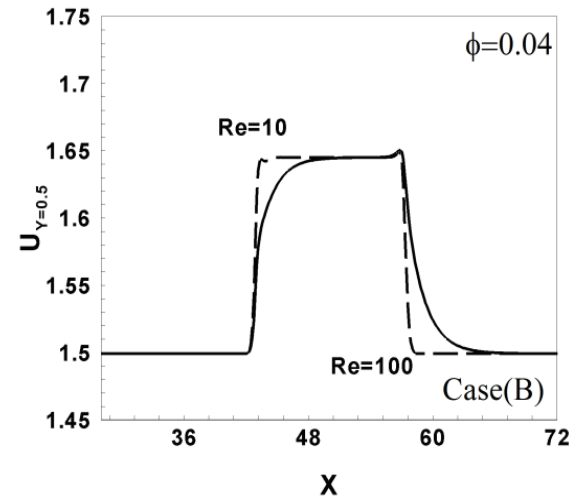
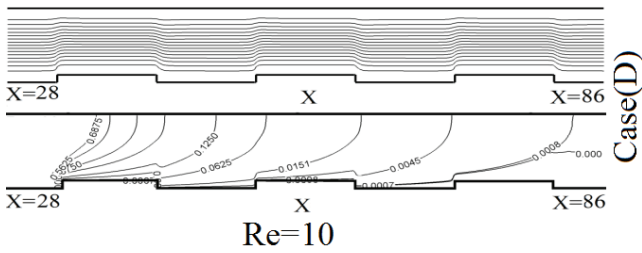
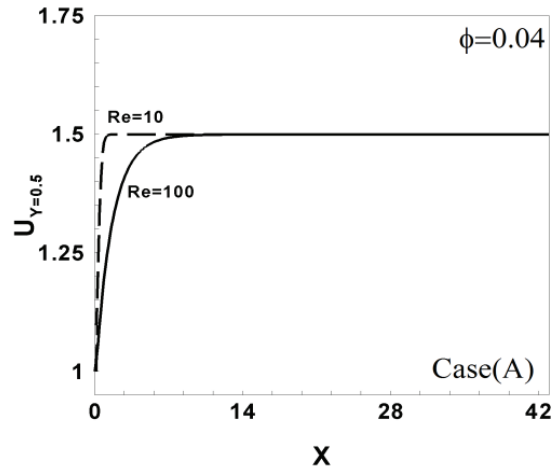
**Figure 4.** Graphs of flow lines and isotherms at  $Re = 10$  and  $Re = 100$  Reynolds range of nanoparticle volume fraction ( $\phi = 0.04$ ) for the case (B).



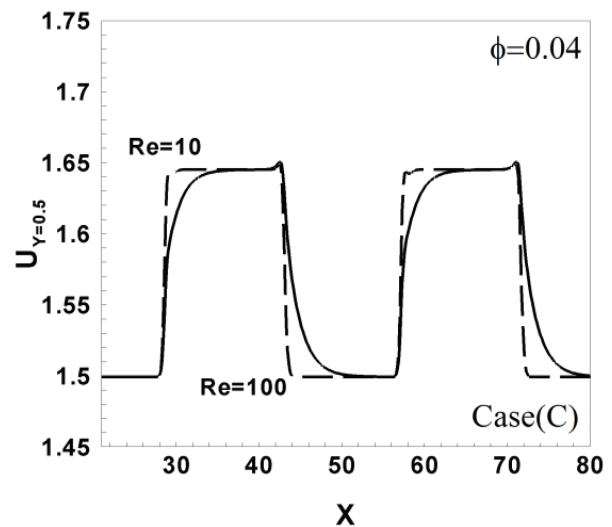




**Figure 5.** Graphs of flow lines and isotherms at  $Re = 10$  and  $Re = 100$  and Reynolds range of nanoparticle volume fraction ( $\phi = 0.04$ ) for the case (C).



**Figure 6.** Graphs of flow lines and isotherms at  $Re = 10$  and  $Re = 100$  and Reynolds range of nanoparticle volume fraction ( $\phi = 0.04$ ) for the case (D).



the  $x$  and  $y$  for scenarios A, B, C and D. In high Reynolds the above average friction coefficient values along the  $x$  and  $y$  decreases, that is due to the collision to the teeth and decreased flow speed near the tooth. The friction coefficient increases with the number of teeth both along  $x$  and along  $y$ . increased number of teeth will cause more turbulence in the flow and more contact with the surface. In this section, we compare the friction coefficient  $C_f$  for Cases of D, B and Reynolds numbers  $Re = 10$  and  $Re = 100$  and  $\phi = 0$ . In graphs of Figure 12 there is a sudden increase in the coefficient of friction such as the Nusselt number values for the coefficient of friction in areas with teeth. With decreasing Reynolds number, the fluid feels teeth more and the effects is transferred to the upper

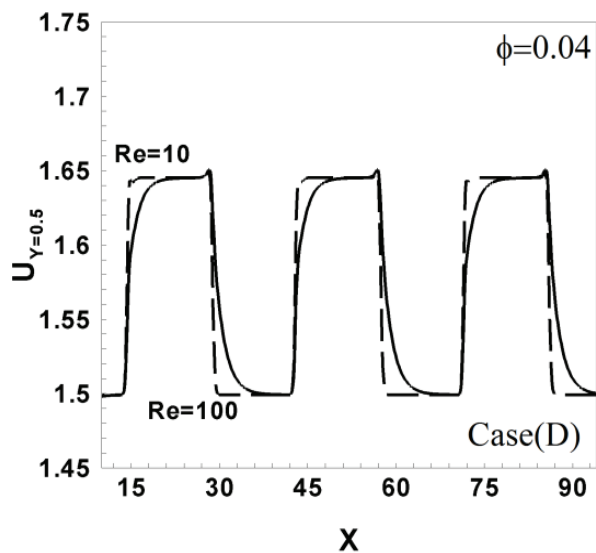


Figure 7. speeds at some point over the next micro channel four cases A, B, C, D.

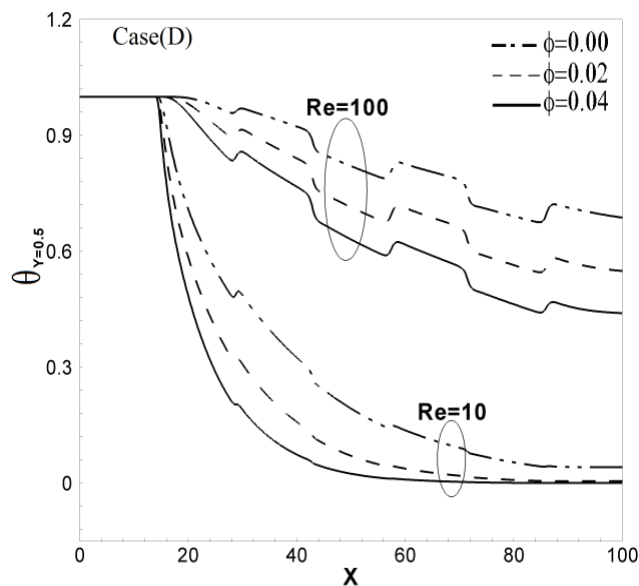
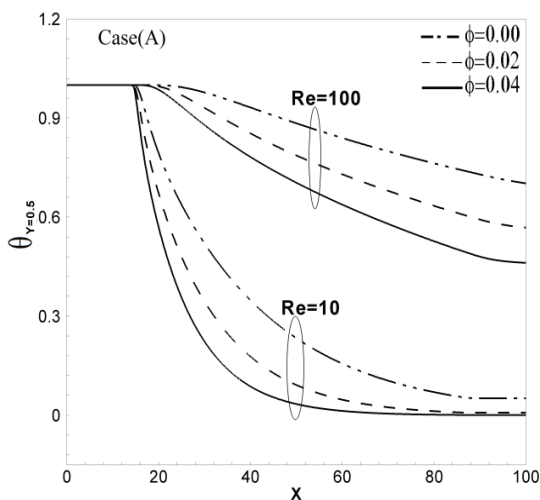
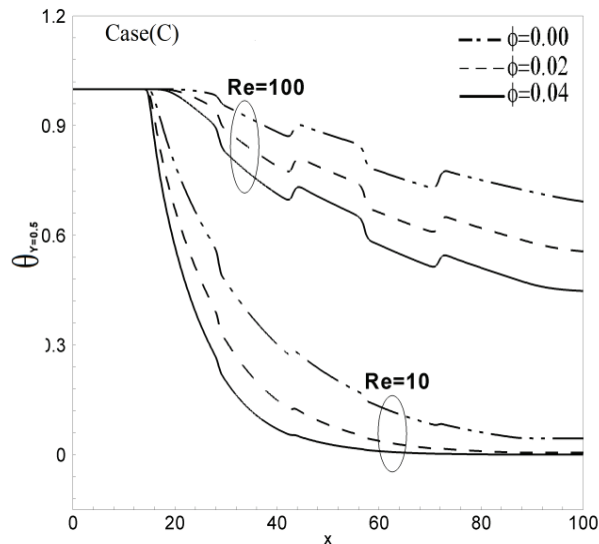
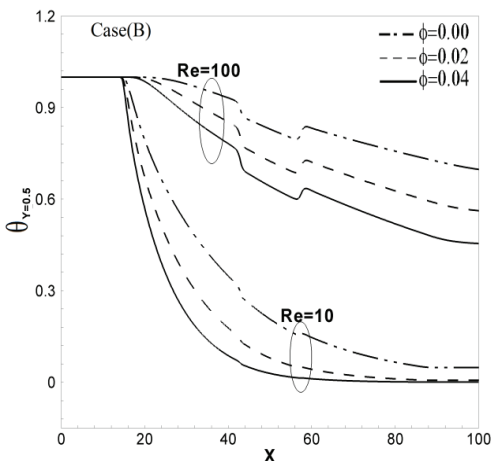


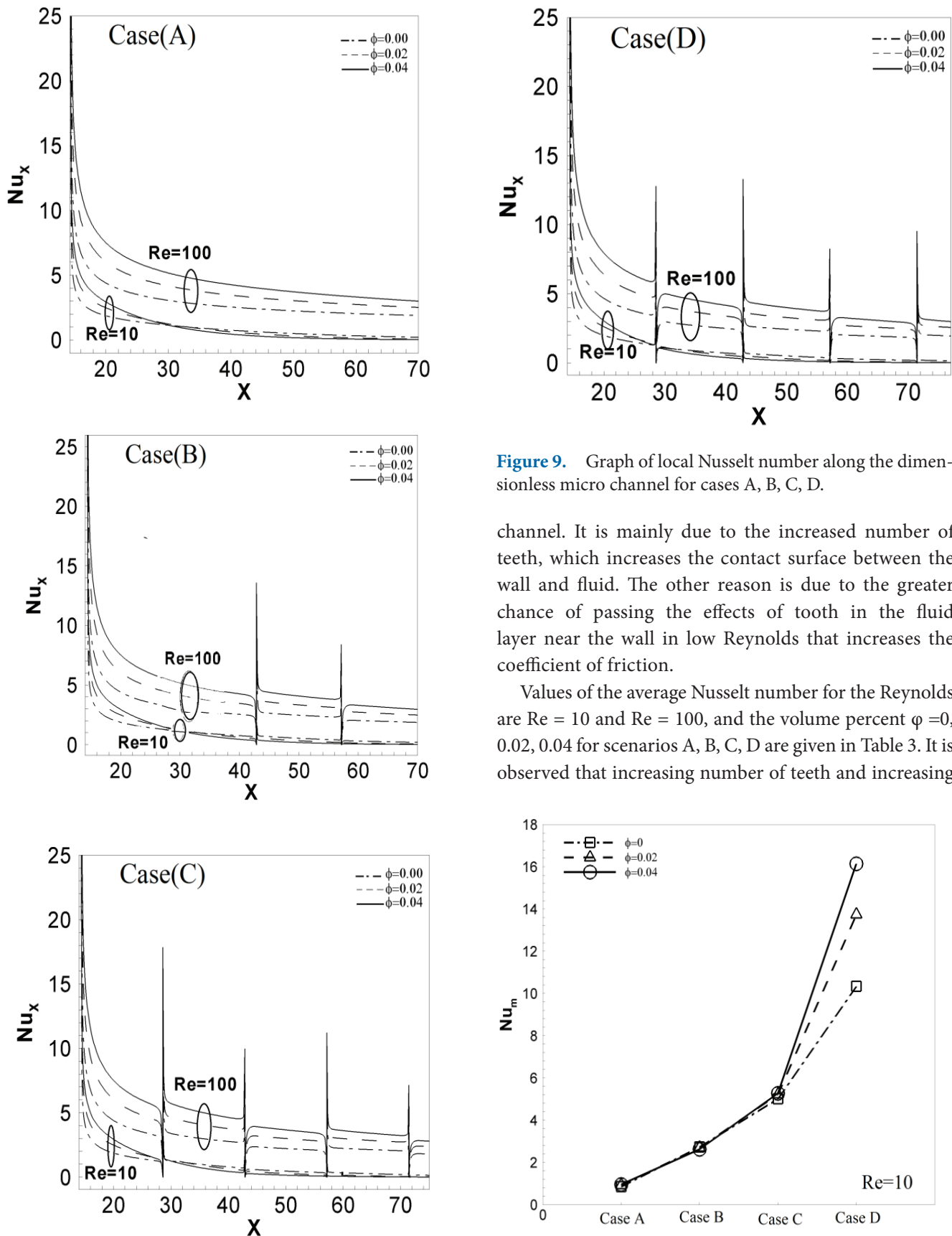
Figure 8. Temperature dimensionless micro channel Centre for case A, B, C, D.



layers of the fluid which increases the surface friction coefficient. The impact of the drop of transmission fluid velocity in the upper layer at  $Re = 100 \rightarrow$  less. Values of friction coefficient averaged along the axis (x) and (y) the Reynolds  $Re = 10$  and  $Re = 100$  and  $\% v 0 = \phi$  for scenarios A, B, C, D are precisely the values are given in Table 2.

The average friction coefficient increases with the increase of the number of teeth and reduced Reynolds number and it happens in horizontal walls and in line with the height of tooth in micro indentation occurs

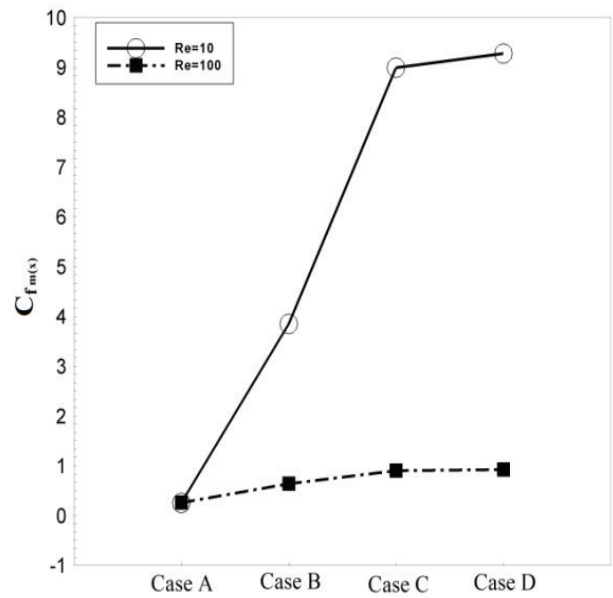
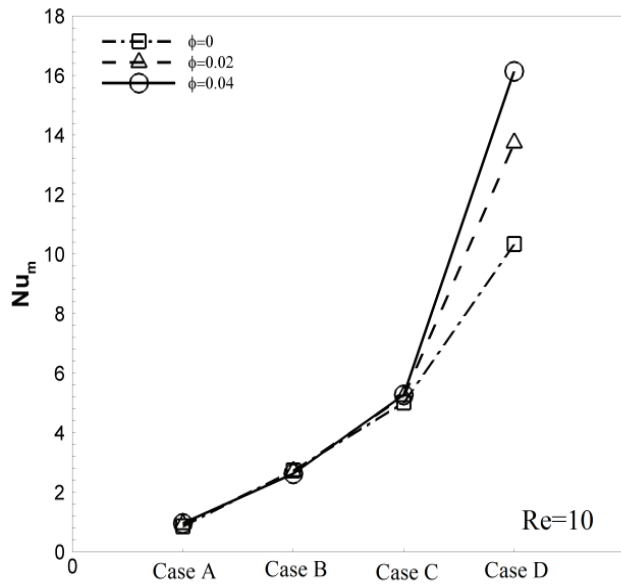




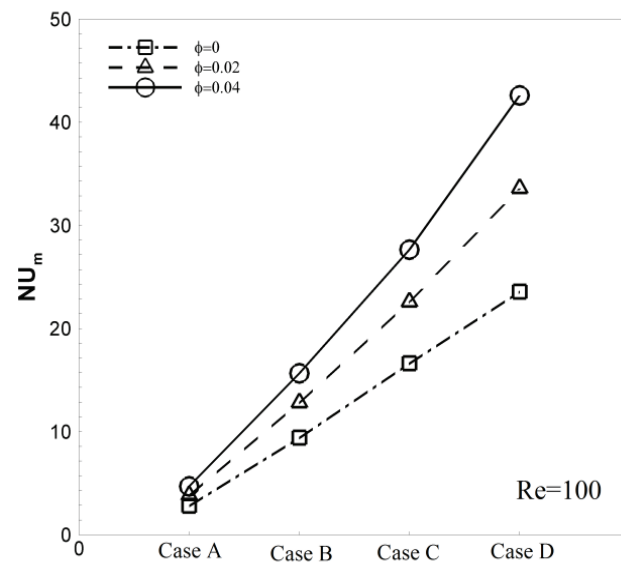
**Figure 9.** Graph of local Nusselt number along the dimensionless micro channel for cases A, B, C, D.

channel. It is mainly due to the increased number of teeth, which increases the contact surface between the wall and fluid. The other reason is due to the greater chance of passing the effects of tooth in the fluid layer near the wall in low Reynolds that increases the coefficient of friction.

Values of the average Nusselt number for the Reynolds are  $Re = 10$  and  $Re = 100$ , and the volume percent  $\phi = 0, 0.02, 0.04$  for scenarios A, B, C, D are given in Table 3. It is observed that increasing number of teeth and increasing



**Figure 11.** Comparison of mean  $C_f$  along the x and y for scenarios A, B, C, D.

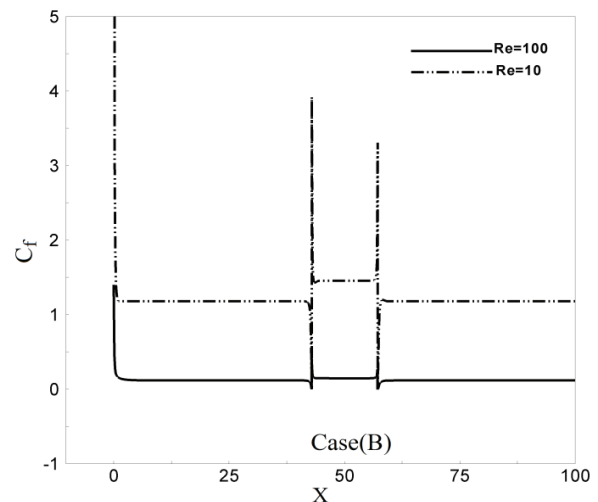


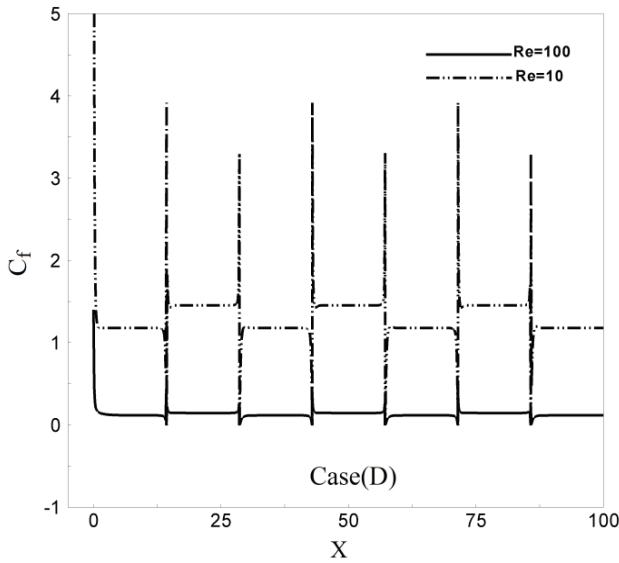
**Figure 10.** The average Nusselt number in cases A, B, C, D.

Reynolds number increase the average Nusselt number and this increase in heat transfer rate due to better mixing of the fluid layers near the surface is higher in Reynolds numbers due to teeth and strengthening the components of the fluid velocity. But on the other hand, with the significant increase in heat transfer due to the increase in the number of teeth increases the friction coefficient.

## 5. Conclusion

With increasing volume fraction of the solid particles, the thermal performance of nano fluids increases owing to the volume of particles with relatively higher thermal conductivity. Significant increase in Nusselt number is more pronounced with increasing volume percent of solid particles, and an increase in the number of teeth caused by micro channels in high Reynolds. The presence of a tooth in the micro channel increases the speed and





**Figure 12.** Comparison of friction factor  $C_f$  for the states of D, B and Reynolds numbers  $Re = 10$  &  $Re = 100$ , and  $\phi = 0,0$ .

**Table 2.** Different values  $C_{fm}(x)$ ,  $C_{fm}(y)$  for modes A, B, C, D the Reynolds number  $Re = 10$  and  $Re = 100$ , and  $\phi = 0,0$

		$C_{fm}(x)$ ( $\phi=0.00$ )	$C_{fm}(y)$ ( $\phi=0.00$ )
Re = 10	Case (A)	0.2522	$1.0216 \times 10^{-8}$
Re = 100		0.2601	$6.7108 \times 10^{-5}$
Re = 10	Case (B)	3.8495	0.0831
Re = 100		0.6447	$6.4029 \times 10^{-3}$
Re = 10	Case (C)	8.9949	0.1740
Re = 100		0.9032	0.0335
Re = 10	Case (D)	9.2751	0.6435
Re = 100		0.9292	0.0768

the dimensionless temperature of the central line.

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**Table 3.** Different values  $Nu_m$  for modes A,B,C,D the Reynolds number  $Re=10$  and  $Re=100$   $\phi = 0,0.02,0.04$

		$Nu_m$ ( $\phi=0.00$ )	$Nu_m$ $\phi=0.02$ )	$Nu_m$ $\phi=0.04$ )
Re=10	Case (A)	0.8673	0.9318	0.9644
Re=100		2.8146	3.8287	4.6834
Re=10	Case (B)	2.7452	2.7125	2.6236
Re=100		9.4297	12.8163	15.6655
Re=10	Case (C)	5.0180	5.2327	5.2728
Re=100		16.6163	22.6077	27.6377
Re=10	Case (D)	10.3473	13.7429	16.1369
Re=100		23.5645	33.6020	42.6044

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## Appendix Nomenclature

A	Area, m <sup>2</sup>
c	Experimental constant, Equation (10)
C <sub>f</sub>	friction factor
C <sub>p</sub>	Heat capacity, Jkg <sup>-1</sup> K <sup>-1</sup>
d	Diameter, nm
g	Gravity acceleration, ms <sup>-2</sup>
h, l	Microchannel height and length, m
H = h/h, L = l/h	Dimensionless microchannel height and length
k	Thermal conductivity coefficient, Wm <sup>-1</sup> K <sup>-1</sup>
Nu <sub>x</sub> , Nu <sub>m</sub>	Local and averaged Nusselt number
$\bar{p}$	Fluid pressure, Pa
$\bar{p}$	Modified pressure, Pa
$P = (\bar{p} / \rho_{nf} u_c^2)$	Dimensionless pressure
$Pe = (u_s d_s / \alpha_f)$	Peclet number
$Pr = \nu_f / \alpha_f$	Prandtl number
$Re = \rho_f u_c h / \mu_f$	Reynolds number
T <sub>H</sub> , T <sub>C</sub>	Hot and cold Temperature, K
u, v	Velocity components in x , y directions, ms <sup>-1</sup>
uc	nlet flow velocity, ms <sup>-1</sup>
u <sub>s</sub>	Brownian motion velocity, ms <sup>-1</sup>
(U, V) = (u/U <sub>0</sub> , v/U <sub>0</sub> )	Dimensionless flow velocity in x-y direction
x, y	Cartesian coordinates, m
(X, Y) = (x/h, y/h)	Dimensionless coordinates
<b>Greek symbols</b>	
α	Thermal diffusivity, m <sup>2</sup> s <sup>-1</sup>

$\phi$	Nano particles volume fraction	c	Cold
$\kappa_b$	Boltzmann constant, $\text{JK}^{-1}$	f	Base fluid (pure water)
$\mu$	Dynamic viscosity, Pas	h	Hot
$\theta = (T-T_c)/(T_H - T_c)$	Dimensionless temperature	m	mean
$\rho$	Density, $\text{kgm}^{-3}$	nf	Nano fluid
$\nu$	Kinematics viscosity, $\text{m}^2\text{s}^{-1}$	s	Solid nano particles

**Super- and Sub-scripts**