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An Approach for Solving Fuzzy Game Problem

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Abstract

In this paper we consider a two person zero sum game with imprecise values in payoff matrix. All the imprecise values are assumed to be triangular or trapezoidal Fuzzy Numbers. An approach for solving problems by using ranking of the fuzzy numbers has been considered to solve the fuzzy game problem. By using ranking to the payoffs we convert the fuzzy valued game problem to crisp valued game problem, which can be solved using the traditional method.

Keywords: Fuzzy Set, Fuzzy Number, Triangular Fuzzy Number, Trapezoidal Fuzzy Number, Ranking Of Fuzzy Numbers

1. Introduction

The history of game theory dates back to the early twentieth century but a new turn for its wider applicability took only in 1944, when John von Newmann and Oscar Morgenstern published the famous article 'Theory of games and economic Behavior'[1]. A game is a model of a situation where two or more groups are in dispute over some issues [2]. The participants in a game are called the players. In classical game theory it is assumed that all data of a game are known exactly by players. Using the notion of fuzzy sets, each component in a game (set of players, set of strategies, set of payoffs, etc) can be fuzzified. Fuzzy set theory was formally introduced by Lotfi. A. Zadeh in his classic paper "Fuzzy sets" in the year 1965[3]. Fuzzy set is defined with the help of grades of membership of elements in the set which is described in the interval [0,1]. Among various types of fuzzy sets, there is a special significance for the fuzzy sets that are defined on the set R of real numbers. Membership functions of these sets, which have the form $A : R \rightarrow [0, 1]$ can be viewed as fuzzy numbers or fuzzy intervals under certain conditions. Fuzzy numbers play an important role in many applications, including decision making, approximate reasoning, optimization etc. Of all shapes of fuzzy numbers, triangular fuzzy number and trapezoidal fuzzy number are widely applied in Optimization problems. (Dubois and Prade[4], 1980, Zimmermann[5], 1996) In fuzzy game problems, all parameters are fuzzy numbers. Fuzzy numbers may be normal or abnormal, triangular or trapezoidal and these fuzzy numbers are not directly comparable. Several methods are introduced for ranking of fuzzy numbers. After ranking, these fuzzy numbers will be converted into crisp numbers and become comparable. Here we want to use a method which is introduced for ranking of fuzzy numbers, by Basirzadeh et al.[6]. We apply this method for all fuzzy game problems where the entries of pay-off matrix are trapezoidal fuzzy number and triangular fuzzy number. Using this ranking, the fuzzy Game problem is converted to a crisp value problem, which can be solved using the traditional method.

2. Basic Definitions

2.1 Definition:(Fuzzy Set)

A fuzzy set A defined on a non empty set X is the family $A=\{(x,\mu_A(x))/ x \in X\}$ where $\mu_A: X\rightarrow I$ is the membership function. In classical fuzzy set theory the set I is usually defined on the interval $[0,1]$ such that $\mu_A(x) = 0$ if x does not belong to A, $\mu_A(x) = 1$ if x strictly belongs to A and any intermediate value represents the degree in which

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x could belong to A. $\mu_A(x) < \mu_A(x')$ indicates that the degree of membership of x to A is lower than the degree of membership of x'.

2.2 Definition: (Fuzzy Number)

A fuzzy number $\tilde{A}$ is a fuzzy set on the real line R, must satisfy the following conditions.

1. There exist atleast one $x_o \in R$ with $\mu_{\tilde{A}}(x_o) = 1$
2. $\mu_{\tilde{A}}(x)$ is piecewise continuous.
3. $\tilde{A}$ must be normal and convex.

2.3 Definition: (An Arbitrary Fuzzy Number)

A fuzzy number has been defined in various forms. We present an arbitrary fuzzy number $\breve{A}_\omega$ by an ordered pair of functions $\left(\underline{A}(r), \overline{A}(r)\right)$, where 0≤r≤ ω and ω is an arbitrary constant between zero and one (0≤ ω ≤1), in a parametric form which satisfies the following requirements:

1. $\underline{A}(r)$ is a bounded left continuous non-decreasing function over [0, ω].
2. $\overline{A}(r)$ is a bounded left continuous non-increasing function over [0, ω].
3. $\underline{A}(r) \leq \overline{A}(r)$, 0≤r≤ ω

A crisp number "k" is simply represented by $\underline{A}(r) = \overline{A}(r) = k$, 0≤r≤ ω

If $\tilde{A}$ be an arbitrary fuzzy number then the α-cut of $\tilde{A}$ is $\left[\tilde{A}\right]_\alpha = \left[\underline{A}(\alpha), \overline{A}(\alpha)\right]$, 0≤r≤ ω

If ω =1, then the above –defined number is called a normal fuzzy number.

2.4 Definition : (Ranking of Fuzzy Numbers)[7]

Let = $\tilde{A}_\omega = \left(\underline{A}(r), \overline{A}(r)\right)$, (0≤r≤ ω) be a fuzzy number, then the value $M_\alpha\left(\tilde{A}_\omega\right)$ is assigned to $\tilde{A}_\omega$ for a decision level higher than "α" which is calculated as follows:

$$M_\alpha\left(\tilde{A}_\omega\right) = \frac{1}{2}\int_0^\omega \left(\underline{A}(r) + \overline{A}(r)\right) dr, \text{ where } 0 \leq \alpha \leq 1$$

The measure of a fuzzy number is obtained by the average of two side areas, left side area and right side area, from membership function to α axis.

3. Triangular and Trapezoidal Fuzzy Numbers

3.1 Triangular Fuzzy Number[8]

A fuzzy number $\tilde{A}$ = (a, b, c) is said to be triangular fuzzy number if its membership function is given by ,where a≤b≤c are real numbers

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < a \\ \dfrac{x-a}{b-a}, & a \leq x \leq b \\ \dfrac{c-x}{c-b}, & b \leq x \leq c \\ 0, & x > c \end{cases}$$

3.2 Ranking of Triangular Fuzzy Number

Let, $\tilde{A} = \left(\underline{A}(r), \overline{A}(r)\right)$ (0≤r≤1 be a fuzzy number. The value $M_0^{Tri}\left(\tilde{A}\right)$ called the measure of $\tilde{A}$ is calculated as follows:

$$M_0^{Tri}\left(\tilde{A}\right) = \frac{1}{2}\int_0^1 \left(\underline{A}(r) + \overline{A}(r)\right) dr, \text{ where } 0 \leq r \leq 1.$$
$$= \frac{1}{4}[2b + a + c]$$

If $\tilde{A}_\omega = \left(\underline{A}(r), \overline{A}(r)\right) = (a + \frac{b-a}{\omega} r, c + \frac{b-c}{\omega} r)$ be an arbitrary triangular fuzzy number at decision level higher than "α" and α, $\omega \in$ **[0,1].**

3.3 Trapezoidal Fuzzy Number[9]

A fuzzy number $\tilde{A}$ = (a, b, c, d) is said to be trapezoidal fuzzy number if its membership function is given by, where a≤b≤c≤d are real numbers

$$\mu_{\underline{A}}(x) = \begin{cases} 0 & \text{for } x < a \\ \dfrac{(x-a)}{(b-a)} & \text{for } a \leq x \leq b \\ 1 & \text{for } b \leq x \leq c \\ \dfrac{(d-x)}{(d-c)} & \text{for } c \leq x \leq d \\ 0 & \text{for } x > d \end{cases}$$

3.4 Ranking of Trapezoidal Fuzzy Number

Let $\tilde{B}=\left(\underline{B}(r),\overline{B}(r)\right)$, $(0\le r\le 1)$ be a fuzzy number. The value $M_0^{Tra}\left(\tilde{B}\right)$ called the measure of $\tilde{A}$ is calculated as follows:

$$M_0^{Tra}\left(\tilde{B}\right)=\frac{1}{2}\int_0^1\left(\underline{B}(r)+\overline{B}(r)\right)\,dr\ ,\ \text{where}\ \ 0\le r\le 1$$
$$=\frac{1}{4}\left[a+b+c+d\right]$$

If $\tilde{B}_{\omega}=\left(\underline{B}(r),\overline{B}(r)\right)=(a+\frac{b-a}{\omega}r,c+\frac{d-c}{\omega}r)$ be an arbitrary trapezoidal fuzzy number at decision level higher than "α" and α, $\omega \in$ **[0,1].**

4. Crisp Game Value of the Matrix

A game can be expressed as a matrix for example consider

$$\begin{array}{cc} & \textit{Player B} \\ \textit{Player A} & \begin{bmatrix} 5 & 4 & 3 & 4 \\ 4 & 2 & 1 & 3 \\ 3 & 0 & -2 & 1 \end{bmatrix} \end{array}$$

This is a 3×4 game i.e., there are two players, Player A and B. Player A has 3 strategies and player B has 4 strategies. The values of the matrix are gain for player A and loss for player B.

The minimum value in each row represents the least gain (pay off) to player A if he chooses this particular strategy. So player A's best strategy will be the strategy that maximizes his minimum gain. The maximum value in each column represents the maximum loss to player B if he chooses this particular strategy. So player B's best strategy will be the strategy that minimizes his maximum losses.

4.1 Saddle Point

If in a game the maximin value is equal to the minimax value, then that point is called as a saddle point or equilibrium point and the corresponding strategies of the saddle point are called optimal strategies. The payoff at the saddle point is called the crisp game value of the game matrix.

$$\begin{array}{cc} & \textit{Player B} \\ \text{Ex:}\ \textit{Player A} & \begin{bmatrix} 5 & 4 & 3 & 4 \\ 4 & 2 & 1 & 3 \\ 3 & 0 & -2 & 1 \end{bmatrix} \end{array}$$

Here a_{13} = 3 is the saddle point of the game. The crisp game value is 3.

4.2 Solution of all 2×2 Matrix Game

Consider the general 2×2 game matrix $A=\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$

To solve this game we proceed as follows:

- Test for a saddle point.
- If there is no saddle point, solve by finding equalizing strategies.
  The Optimal mixed strategies for player A = (p_1,p_2) and for player B = (q_1,q_2)

where $p_1=\dfrac{a_{22}-a_{12}}{\left(a_{11}+a_{22}\right)-\left(a_{12}+a_{21}\right)}$; $p_2=1-p_1$

$$q_1=\frac{a_{22}-a_{21}}{\left(a_{11}+a_{22}\right)-\left(a_{12}+a_{21}\right)}\ \ ;\ q_2=1-q_1\ \text{ and}$$

$$\text{Value of the game V}=\frac{a_{11}a_{22}-a_{12}a_{21}}{\left(a_{11}+a_{22}\right)-\left(a_{12}+a_{21}\right)}$$

5. Pay off Matrix

The problem that we are aiming to solve is a two player zero sum fuzzy game in which the entries in the payoff matrix A are triangular fuzzy number.

The pay off matrix is

$$A=\begin{bmatrix} \tilde{a}_{11} & \tilde{a}_{12} & \tilde{a}_{13} & \cdots & \tilde{a}_{1n} \\ \tilde{a}_{21} & \tilde{a}_{22} & \tilde{a}_{23} & \cdots & \tilde{a}_{2n} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ \tilde{a}_{m1} & \tilde{a}_{m2} & \tilde{a}_{m3} & \cdots & \tilde{a}_{mn} \end{bmatrix}_{m\times n}$$

In the above game Player A has 'm' strategies and Player B has 'n' strategies. If Player A chooses the ith strategies and Player B chooses jth strategies, the Player A win an amount $x \in \tilde{a}_{ij}$.

5.1 Fuzzy Game with Payoffs as Triangular Fuzzy Number

The Payoffs in a matrix are not fixed numbers. Here we use Triangular Fuzzy Numbers as the entries in the Payoff matrix.

Consider a Fuzzy game between two Players X and Y, the fuzzified Payoff matrix is given by A as follows:

$$A=\begin{bmatrix} (1,2,3) & (6,7,8) & (-4,-3,-3) \\ (5,6,7) & (0,3,5) & (-1,2,5) \\ (-6,-5,-4) & (-2,-1,0) & (-1,0,1) \end{bmatrix}$$

In this game, if X chooses row one and Y chooses column two then X wins an amount x ∈ (6, 7, 8) is a triangular fuzzy number and Y losses the same amount.

5.2 Fuzzy Game with Payoffs as Trapezoidal Fuzzy Number

The Payoffs in a matrix are not fixed numbers. Here we use Trapezoidal Fuzzy Numbers as the entries in the Payoff matrix.

Consider a Fuzzy game between two Players X and Y, the fuzzified Payoff matrix is given by A as follows:

$$A = \begin{bmatrix} (1,4,5,6) & (1,2,4,5) & (3,4,5,8) & (4,5,7,8) \\ (5,10,12,17) & (4,5,9,14) & (5,7,10,14) & (7,10,11,12) \\ (-1,0,2,3) & (-1,2,3,4) & (8,17,21,30) & (5,6,7,10) \end{bmatrix}$$

In this game, if X chooses row two and Y chooses column two then X wins an amount x ∈ (4,5,9,14) is a trapezoidal fuzzy number and Y losses the same amount.

6. General Rules For Dominance[10, 11]

If all elements in a row are less than are equal to the corresponding elements in another row then that row is dominated. If all elements in a column are greater than are equal to the corresponding elements in another column then that column is dominated. Dominated rows and columns may be deleted to reduce the pay-off matrix.

7. General Rules for Matrix Oddment Method for n×n games[12]

Step 1: Let A= (a_{ij})be nxn pay off matrix. Obtain a new matrix C, Whose first column is obtained from A by subtracting 2nd column from 1st; second column is obtained by subtracting A's 3rd column from 2nd and so on till the last column of A is taken care of. Thus C is a n × (n−1) matrix.

Step 2: Obtain a new matrix R, from A, by subtracting its successive rows from the preceding ones, in exactly the same manner as was done for columns in step1. Thus R is a (n−1) × n matrix.

Step 3: Determine the magnitude of oddments corresponding to each row and each column of A. The oddment corresponding to ith row of A is defined as the determinant $|C_i|$ where C_i is obtained from C by deleting ith row. Similarly oddment corresponding jth column of A=$|R_j|$, defined as determinant where R_j is obtained from R by deleting its jth column.

Step 4: Write the magnitude of oddments (after ignoring negative signs, if any) against their respective rows and columns.

Step 5: Check whether the sum of row oddments is equal to the sum of column oddments. If so, the oddments expressed as fractions of the grand total yields the optimum strategies. If not, the method fails.

Step 6: Calculate the expressed value of the game corresponding to the optimum mixed strategy determined above for the row player (against any move of the column player)

8. Numerical Example

Consider the following game problem with payoff as triangular fuzzy numbers. We want to solve it by traditional method.

Player B

Player A $\begin{pmatrix} (-4,-1,2) & (-2,1,8) & (-2,1,4) \\ (-2,1,4) & (-8,-2,4) & (-4,2,8) \\ (2,3,4) & (3,4,5) & (-6,-5,4) \end{pmatrix}$

Let $\tilde{A} = (\underline{A}(r), \overline{A}(r)).(0 \le r \le 1)$ be a fuzzy number, and then the value $M(\tilde{A})$ is assigned to $\tilde{A}$ is calculated as follows:

$$M_0^{Tri}(\tilde{A}) = \frac{1}{2}\int_0^1 (\underline{A}(r) + \overline{A}(r))\, dr$$
$$= \frac{1}{4}[2b + a + c]$$

Step: 1 We obtain the values $M(\widetilde{a_{ij}})$ for each $\widetilde{a_{ij}}$ of the fuzzy game which is given in table-1

Step:2

The reduced fuzzy game problem is

Player B

Player A $\begin{pmatrix} -1 & 2 & 1 \\ 1 & -2 & 2 \\ 3 & 4 & -3 \end{pmatrix}$

It has no saddle point. So, we can apply Matrix oddment method.

Step: 3

We compute the matrices C and R by subtracting the successive columns (rows) from the Proceeding column (row).

Table.1 Rank values of the triangular fuzzy numbers in the fuzzy game problem of numerical example : 8

$\widetilde{a_{11}}=(-4,-1,2)$	$M_0^{Tri}(\widetilde{a_{11}})=\frac{1}{4}[-2-4+2]=-1$
$\widetilde{a_{12}}=(-2,1,8)$	$M_0^{Tri}(\widetilde{a_{12}})=\frac{1}{4}[2-2+8]=2$
$\widetilde{a_{13}}=(-2,1,4)$	$M_0^{Tri}(\widetilde{a_{13}})=\frac{1}{4}[-2+2+4]=1$
$\widetilde{a_{21}}=(-2,1,4)$	$M_0^{Tri}(\widetilde{a_{21}})=\frac{1}{4}[-2+2+4]=1$
$\widetilde{a_{22}}=(-8,-2,4)$	$M_0^{Tri}(\widetilde{a_{22}})=\frac{1}{4}[-4,-8+4]=-2$
$\widetilde{a_{23}}=(-4,2,8)$	$M_0^{Tri}(\widetilde{a_{23}})=\frac{1}{4}[4-4+8]=2$
$\widetilde{a_{31}}=(2,3,4)$	$M_0^{Tri}(\widetilde{a_{31}})=\frac{1}{4}[6+2+4]=3$
$\widetilde{a_{32}}=(3,4,5)$	$M_0^{Tri}(\widetilde{a_{32}})=\frac{1}{4}[8+3+5]=4$
$\widetilde{a_{33}}=(-6,-5,4)$	$M_0^{Tri}(\widetilde{a_{33}})=\frac{1}{4}[-10-6+4]=-3$

$|R_1| = 17$ $|R_2| = -20$ $|R_3| = 9$

$|C_1| = 14$ $|C_2| = -12$ $|C_3| = 20$

The Augmented pay off matrix is

				Row Oddments
	−1	2	2	17
	1	−2	2	20
	3	4	−3	9
Column Oddments	14	12	20	46

The Optimum Strategies are

$$\text{Player A}:(\frac{17}{46},\frac{20}{46},\frac{9}{46});\ \text{Player B}:(\frac{14}{46},\frac{12}{46},\frac{20}{46})$$

$$\text{Value of the game } =\frac{17}{46}(-1)+\frac{20}{46}(1)+\frac{9}{46}(3)=\frac{15}{23}$$

9. Numerical Example

Consider the following game problem with payoff as trapezoidal fuzzy numbers. We want to solve it by traditional method.

Player B

$$\text{Player A}\begin{pmatrix}(1,4,5,6) & (1,2,4,5) & (3,4,5,8) & (4,5,7,8)\\(5,10,12,17) & (4,5,9,14) & (5,7,10,14) & (7,10,11,12)\\(-1,0,2,3) & (-1,2,3,4) & (8,17,21,30) & (5,6,7,10)\end{pmatrix}$$

According to the definition of a trapezoidal fuzzy number, Let $\tilde{B}=(\underline{B}(r)+\overline{B}(r)).(0\le r\le 1)$ be a fuzzy number, then the value $M(\tilde{B})$ is assigned to $\tilde{B}$ is calculated as follows:

$$M_0^{Tra}(\tilde{B})=\frac{1}{2}\int_0^1(\underline{B}(r)+\overline{B}(r))\,dr$$
$$=\frac{1}{4}[m+n+\delta+\beta]$$

Step: 1 We obtain the values $\mathrm{M}(\widetilde{a_{ij}})$ for each $\widetilde{a_{ij}}$ of the fuzzy game which is given in table-2

Step: 2

We change the fuzzy game problem into a crisp game problem. So, we have the following reduced fuzzy game problem.

Player B

$$\text{Player A}\begin{pmatrix}4 & 3 & 5 & 6\\11 & 12 & 9 & 10\\1 & 2 & 16 & 19\end{pmatrix}$$

$$\text{Max}(\text{min})=9;\ \text{Mini}(\text{max})=11$$

It has no saddle point. So, we can apply Dominance principle.

Step: 3

Row I is dominated by Row II, So we omit Row I.

$$\begin{pmatrix}11 & 12 & 9 & 10\\1 & 2 & 16 & 19\end{pmatrix}$$

Column II is dominated by Column I, So we omit Column II

$$\begin{pmatrix}11 & 9 & 10\\1 & 16 & 19\end{pmatrix}$$

Column IV is dominated by Column III, So we omit Column IV

$$\begin{pmatrix}11 & 9\\1 & 16\end{pmatrix}$$

Here, Max(min)=9; Mini(max)=11

It has no saddle point.

Table 2. Rank values of the trapezoidal fuzzy numbers in the fuzzy game problem of numerical example : 9

$\widetilde{a_{11}} = (1,4,5,6)$	$M_0^{Tra}(\widetilde{a_{11}}) = \frac{1}{4}[1+4+5+6] = 4$
$\widetilde{a_{12}} = (1,2,4,5)$	$M_0^{Tra}(\widetilde{a_{12}}) = \frac{1}{4}[1+2+4+5] = 3$
$\widetilde{a_{13}} = (3,4,5,8)$	$M_0^{Tra}(\widetilde{a_{13}}) = \frac{1}{4}[3+4+5+8] = 5$
$\widetilde{a_{14}} = (4,5,7,8)$	$M_0^{Tra}(\widetilde{a_{14}}) = \frac{1}{4}[4+5+7+8] = 6$
$\widetilde{a_{21}} = (5,10,12,17)$	$M_0^{Tra}(\widetilde{a_{21}}) = \frac{1}{4}[5+10+12+17] = 11$
$\widetilde{a_{22}} = (8,10,11,19)$	$M_0^{Tra}(\widetilde{a_{22}}) = \frac{1}{4}[8+10+11+19] = 12$
$\widetilde{a_{23}} = (5,7,10,14)$	$M_0^{Tra}(\widetilde{a_{23}}) = \frac{1}{4}[5+7+10+14] = 9$
$\widetilde{a_{24}} = (7,10,11,12)$	$M_0^{Tra}(\widetilde{a_{24}}) = \frac{1}{4}[7+10+11+12] = 10$
$\widetilde{a_{31}} = (-1,0,2,3)$	$M_0^{Tra}(\widetilde{a_{31}}) = \frac{1}{4}[-1+0+2+3] = 1$
$\widetilde{a_{32}} = (-1,2,3,4)$	$M_0^{Tra}(\widetilde{a_{32}}) = \frac{1}{4}[-1+2+3+4] = 2$
$\widetilde{a_{33}} = (12,14,18,20)$	$M_0^{Tra}(\widetilde{a_{33}}) = \frac{1}{4}[12+14+18+20] = 16$
$\widetilde{a_{34}} = (8,17,21,30)$	$M_0^{Tra}(\widetilde{a_{34}}) = \frac{1}{4}[8+17+21+30] = 19$

Step: 4 To find Optimum mixed strategy and value of the game:

Here $a_{11} = 11$, $a_{12} = 9$, $a_{21} = 1$, $a_{12} = 16$

$$p_1 = \frac{7}{17}; p_2 = 1 - p_1 = 1 - \frac{7}{17} = \frac{10}{17}$$

$$q_1 = \frac{15}{17}; q_2 = 1 - q_1 = 1 - \frac{15}{17} = \frac{2}{17}$$

Strategy for player $A = (0, \mathbf{\frac{7}{17}}, \mathbf{\frac{10}{17}})$ **;** Strategy for player $B = (\frac{15}{17}, \mathbf{0}, \frac{2}{17}, \mathbf{0})$ and

Value of the game $V = \frac{167}{17}$

10. Conclusion

In this paper, a method of solving fuzzy game problem using ranking of fuzzy numbers has been considered. This method can be used to solve any m x n two person game with its values as triangular or trapezoidal fuzzy numbers. These imprecise values are converted into crisp values and the reduced crisp value game is then solved by any of the available traditional method like dominance rule, graphical method etc.

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