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Domination Parameter Characterization using Matrix Representation

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Abstract

Background: Determining all possible γ - sets of G and γ - sets satisfying different domination parameters. **Methods:** Matrix representation for determining γ - sets and MATLAB code for the same. **Results:** It will help determining the γ - sets for any graph with less effort using MATLAB code. **Application:** Can be used to characterize graphs based on domination parameter.

Keywords: Domination Dot Stable, γ - stable, Graph Domination Graphs

1. Introduction

A graph is represented by various kinds of binary matrices. Adjacency matrix, incidence matrix, cut matrix, Circuit matrix are few kinds of such matrices. Binary matrix representation is comfortable for programming purposes. Properties of matrices are easily identified by coding them. In literature of graph theory numerous results are available using matrix representation.

In [1], a method for material selection for a given engineering component using graph theory and matrix approach is provided. In [2], some methods for selection of a rapid prototyping process that best suits the end use of a given product or part using graph theory and matrix approach is presented. In [3], a method for identifying the isomorphism of topological graph by using incident matrices is provided.

In [4], Bounds related to domination number of G, energy of G and rank of the incident matrix of the graph G is discussed. In [5], a method of generating a minimum weighted spanning tree by using adjacency matrix of G is provided. In [6], a characterization of planar graphs when G and $\overline{G}$ are γ - stable graphs is discussed.

In [7], some survey on graphs which have equal domination and closed neighborhood packing numbers are done. In this paper we present a method of identifying three graph parameters. We also provide MATLAB codes to execute the same.

2. Terminology

We consider only simple connected undirected graphs G = (V, E). An adjacency matrix of a graph G with n vertices that are assumed to be ordered from v_1 to v_n is defined by,

$$A = [a_{ij}]_{n\times n} = \begin{cases} 1, & \text{if there exist an edge between } v_i \text{ and } v_j \\ 0, & \text{otherwise.} \end{cases}$$

The incidence matrix of G is a n x m matrix B where n and m are the number of vertices and edges respectively, such that

$$B = [b_{ij}]_{n\times m} = \begin{cases} 1, & \text{if } x_j \text{ is incident on } v_i \\ 0, & \text{otherwise.} \end{cases}$$

The open neighborhood of vertex $v \in V(G)$ is denoted by $N(v) = \{u \in V(G) \mid (u\ v) \in E(G)\}$
while its closed neighborhood is the set $N[v] = N(v) \cup \{v\}$. We indicate that u is adjacent to v by writing $u \perp v$. For details of on graph theory we refer to[8].

A set of vertices D in a graph G = (V, E) is a dominating set if every vertex of V – D is adjacent to some vertex of D. If D has the smallest possible cardinality of any dominating

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set of G, then D is called a minimum dominating set. The cardinality of any minimum dominating set for G is called the domination number of G and it is denoted by γ(G). γ - set denotes a dominating set for G with minimum cardinality. A dominating set D is said to be an independent dominating set if no two vertices in D are adjacent. A set of vertices D in a graph G is called a clique dominating set if every two vertices in D are adjacent. A vertex in V – D is k – dominated if it is dominated by at least 2 – vertices in D, that is | N (v) ∩ D | ≥ 2. If every vertex in V – D is k – dominated then D is called a k – dominating set. The private neighborhood of v ∈ D is denoted by pn [v, D], is defined by pn[v, D] = N(v) - N(D - {v}). For details of on domination we refer to[9].

3. Results and Discussion

Let G be any graph with n – vertices. Let A denote the adjacency matrix of G. Let N denote a n x n matrix[7], where

$$N=[n_{ij}]_{n\times n}=\begin{cases}1 & \text{if } i=j\\ a_{ij} & \text{the } (i,j)^{th} \text{ entry in the adjacency matrix.}\end{cases}$$

Let x = ⟨x (v_1), x (v_2),..., x(v_n)⟩^{T} be a {0, 1} vector. If x represents any dominating set, then Nx ≥ 1, that is in the resulting matrix Nx, all the entry values are non zero[7].

Example

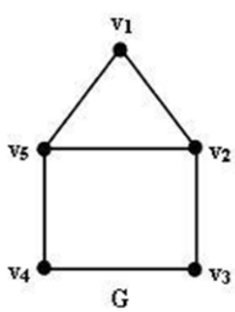


Figure 1. Graph for Nx ≥ 1.

$$\overset{\textbf{N}}{\begin{bmatrix}1&1&0&0&1\\1&1&1&0&1\\0&1&0&1&0\\0&0&1&1&0\\1&1&0&1&1\end{bmatrix}}\overset{\textbf{x}}{\begin{bmatrix}1\\0\\0\\1\\0\end{bmatrix}}=\overset{\textbf{Nx}}{\begin{bmatrix}1\\1\\1\\1\\2\end{bmatrix}}\geq\begin{bmatrix}1\\1\\1\\1\\1\end{bmatrix}$$

That is, {v_1, v_4} is a dominating set for G [7].

Nx is a column matrix. The number of non zero entries in any row of matrix N denotes N[v_i] (closed neighbors of v_i) and x denotes a dominating set. Each entry in Nx denotes the number of vertices dominating any vertex v_i. If row v_i entry in Nx is 1, then v_i ∈ v - D is a private neighbor. Similarly if row v_i entry in Nx ≥ 2. , then vertex v_i ∈ v - D is k – dominated by x.

The matrix method of finding a dominating set can be used to characterize graphs satisfying a given domination parameter. Graph characterization based on dominating set focus on γ – set and all possible γ – sets satisfying the defined property. For this purpose, since we are more focused in all possible γ – sets than all possible dominating set, we use the following notation.

Notation

- Let G be a graph with n vertices v_1,v_2,....,v_n. Let γ (G) = k. Consider all possible subsets with k vertices. Label them as S_1, S_2, ..., S_p, where p = nC_k. Let X = {x_1,x_2,....x_p} be a set of {0,1} vectors defined by x_i = ⟨x (v_1), x (v_2),..., x(v_n)⟩^{T}, where $x(v_i)=\begin{cases}1 & \text{if } v_i\in S_i\\ 0 & \text{otherwise.}\end{cases}$

Following the above notation if γ(G) = 2, n = 5, then S_1 = {v_1, v_2} , S_2 = {v_1, v_3} , S_3 = {v_1, v_4} , S_4 = {v_1, v_5} , S_5 = {v_2, v_3} , S_6 = {v_2, v_4} , S_7 = {v_2, v_5} , S_8 = {v_3, v_4} , S_9 = {v_3, v_5} ,S_{10} = {v_4, v_5} ,. So, x_1 = ⟨1, 1, 0, 0, 0⟩^{T}, x_2 = ⟨1, 0, 1, 0, 0⟩^{T}, x_3 = ⟨1, 0, 0, 1, 0⟩^{T}, x_4 = ⟨1, 0, 0, 0, 1⟩^{T}, x_5 = ⟨0, 1, 1, 0, 0⟩^{T}, x_6 = ⟨0, 1, 0, 1, 0⟩^{T}, x = ⟨0, 1, 0, 0, 1⟩^{T}, x_8 = ⟨0, 0, 1, 1, 0⟩^{T}, x_9 = ⟨0, 0, 1, 0, 1⟩^{T}, x_{10} = ⟨0, 0, 0, 1, 0⟩^{T}.

- Nx_i is a column matrix. Let us denote this as vector, nx_i = ⟨ nx_i (v_1), nx_i (v_2),..., nx_i (v_n)^T.
- Define a matrix of vectors V as V = $[v_{ij}]_{nxp}$ = [x_1, x_2,... x_p ,], where each xi, i = 1, 2, …, p denotes a vector defined in notation 1. Determine NV. This is a n x p matrix, where each column denotes vector nxi, that is the column denotes vector nx_1, nx_2, …, nx_p.

Using the matrix property for identifying dominating sets, we provide a method of identifying, Domination dot stable graphs, γ - stable graphs, Graph domination graphs. We also have provided MATLAB program for identifying the same.

3.1 Domination Dot Stable Graphs

An elementary edge contraction of a graph G is obtained by identifying two adjacent vertices u and v, that is, by removal of u and v and addition of a new vertex w adjacent to those points which u or v was adjacent. We say that G.uv is obtained by contracting (u, v), where u adjacent to v. A graph G is said to be domination dot stable (DDS) if $\gamma(G.uv) = \gamma(G)$, for all $u,v \in V(G)$, $u \perp v$[10].

In all figures the circled vertices represent a γ - set for G.

Example

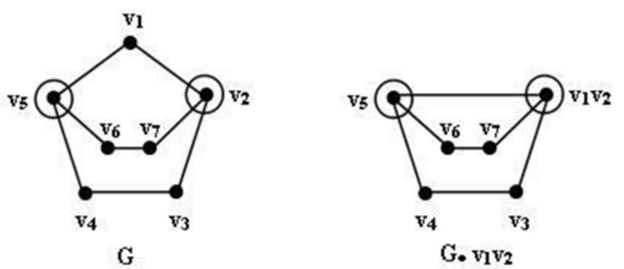


Figure 2. $\gamma(G) = \gamma.(G.v_1v_2) = 2$. Similarly $\gamma(G) = \gamma.(G.uv) = 2$, for all $u,v \in V(G)$, $u \perp v$, implies G is domination dot stable graph.

The following result is proved in[10].

A graph G is DDS if and only if every γ - set of G is an independent dominating set.

Matrix Representation for Domination Dot Stable Graphs

If matrix NV contains no zero entry, then every x_i, i = 1, 2, …p are γ – sets for G, which implies there is atleast one non independent γ – set for G.

If matrix NV has atleast one zero entry, then consider the non zero columns of NV. Let $S \subseteq X$ be the set of all vectors such that $Nx_i \geq 1$, that is $NS \geq 1$. Let $|S| = q$, $q < p$. Consider the i^{th} column of S. Comparing $x_i = \langle x(v_1), x(v_2),..., x(v_n)\rangle^T$. and $nx_i = \langle nx_i(v_1), nx_i(v_2),..., nx_i(v_n)\rangle^T$., for all i = 1, 2, …, q, if for every non zero entry in x_i, the nx_i entry in the corresponding position is also 1, then no two vertices in x_i are adjacent, that is x_i an independent γ – set.

Example

For the graph in Figure 2

$$N = \begin{bmatrix} 1&1&0&0&1&0&0\\ 1&1&1&0&0&0&1\\ 0&1&1&1&0&0&0\\ 0&0&1&1&1&0&0\\ 1&0&0&1&1&1&0\\ 0&0&0&0&1&1&0\\ 0&1&0&0&0&1&1 \end{bmatrix}; Nx = \begin{bmatrix} 1&1&0&0&1&0&0\\ 1&1&1&0&0&0&1\\ 0&1&1&1&0&0&0\\ 0&0&1&1&1&0&0\\ 1&0&0&1&1&1&0\\ 0&0&0&0&1&1&0\\ 0&1&0&0&0&1&1 \end{bmatrix}\begin{bmatrix}0\\1\\0\\0\\1\\0\\0\end{bmatrix} = \begin{bmatrix}2\\1\\1\\1\\1\\1\\1\end{bmatrix} \geq \begin{bmatrix}1\\1\\1\\1\\1\\1\\1\end{bmatrix}$$

$\gamma(G) = 2$. We consider all possible subsets with 2 vertices and label them as $\{S_1, S_2,....,S_{21}\} = \{\{v_1, v_2\}, \{v_1, v_3\}, \{v_1, v_4\}, \{v_1, v_5\}, \{v_1, v_6\}, \{v_1, v_7\}, \{v_2, v_3\}, \{v_2, v_4\}, \{v_2, v_5\}, \{v_2, v_6\}, \{v_2, v_7\}, \{v_3, v_4\}, \{v_3, v_5\}, \{v_3, v_6\}, \{v_3, v_7\}, \{v_4, v_5\}, \{v_4, v_6\}, \{v_4, v_7\}, \{v_5, v_6\}, \{v_5, v_7\}, \{v_6, v_7\}\}$.

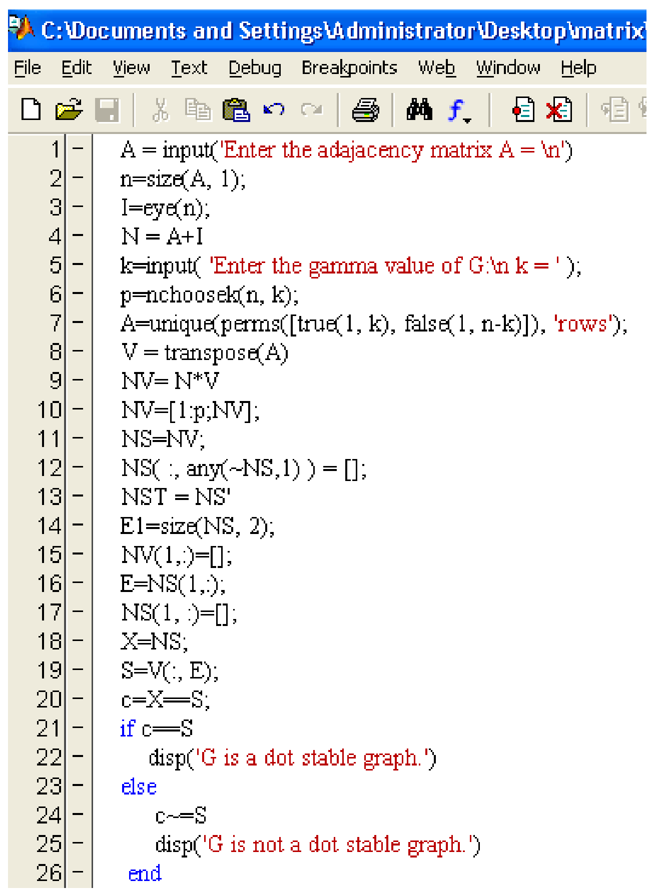
$$NV = \begin{bmatrix} 1&1&0&0&1&0&0\\ 1&1&1&0&0&0&1\\ 0&1&1&1&0&0&0\\ 0&0&1&1&1&0&0\\ 1&0&0&1&1&1&0\\ 0&0&0&0&1&1&0\\ 0&1&0&0&0&1&1 \end{bmatrix}\begin{bmatrix} 1&1&1&1&1&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\ 1&0&0&0&0&0&1&1&1&1&1&0&0&0&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&1&0&0&0&0&1&1&1&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&1&0&0&0&1&0&0&0&1&1&1&0&0&0\\ 0&0&0&1&0&0&0&0&1&0&0&0&1&0&0&1&0&0&1&1&0\\ 0&0&0&0&1&0&0&0&0&1&0&0&0&1&0&0&1&0&1&0&1\\ 0&0&0&0&0&1&0&0&0&0&1&0&0&0&1&0&0&1&0&1&1 \end{bmatrix}$$

$$= \begin{bmatrix} 2&1&1&2&1&1&1&1&2&1&1&0&1&0&0&1&0&0&1&1&0\\ 2&2&1&1&1&2&2&1&1&1&2&1&1&1&2&0&0&1&0&1&1\\ 1&1&1&0&0&0&2&2&1&1&1&2&1&1&1&1&1&1&0&0&0\\ 0&1&1&1&0&0&1&1&1&0&0&2&2&1&1&2&1&1&1&1&0\\ 1&1&2&2&2&1&0&1&1&1&0&1&1&1&0&2&2&1&2&1&1\\ 0&0&0&1&1&0&0&0&1&1&0&0&1&1&0&1&1&0&2&1&1\\ 1&0&0&0&1&1&1&1&1&2&2&0&0&1&1&0&1&1&1&1&2 \end{bmatrix}$$

In the above matrix NV, identifying the non zero columns, $S = \{X_9\} = [0\ 1\ 0\ 0\ 1\ 0\ 0]^T$, implies $NS = [2\ 1\ 1\ 1\ 1\ 1\ 1]^T$. In NS, for every non zero entry in x_9, the entry in the corresponding position of Nx_9 is also 1, implies x_9 is an independent γ – set, implies G is DSS.

MAT Lab Program for Dot Stable Graphs

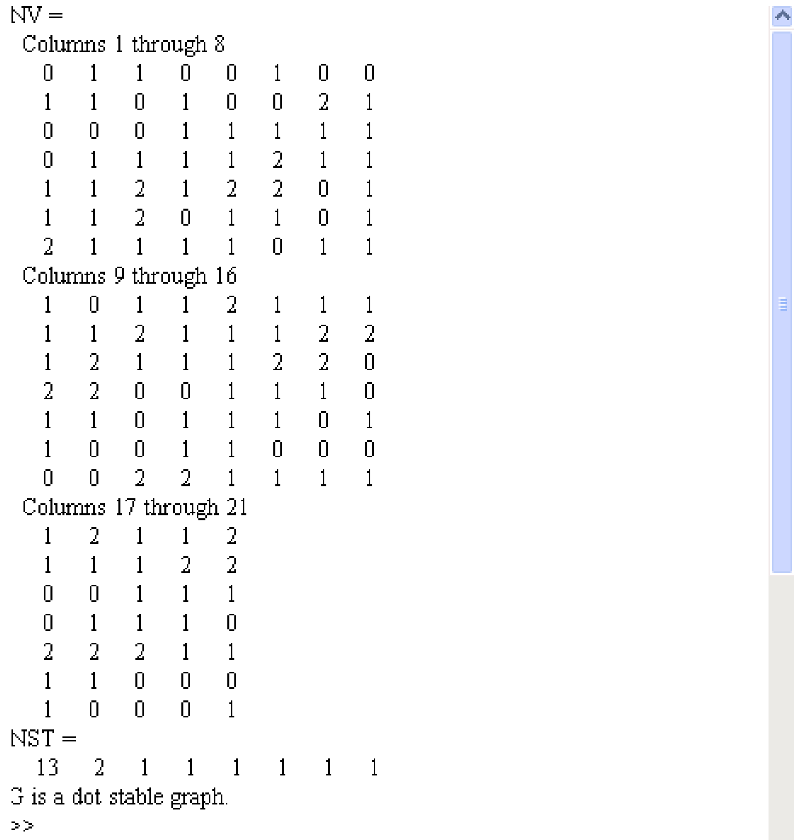
Snapshot – 1 provides the MATLAB program code for DDS graphs. Snapshot – 2 provides the output generated for the graph in Figure 2.



Snapshot – 1

Output

```
MATLAB
File  Edit  View  Web  Window  Help
Current Directory: C:\Documents and Settings\Administrator\Desktop\m
Enter the adajacency matrix A =
[ 0 1 0 0 1 0 0; 1 0 1 0 0 0 1; 0 1 0 1 0 0 0; 0 0 1 0 1 0 0 ; 1 0 0 1 0 1 0; 0 0 0 0 1 0 0; 0 1 0 0 0 1 0;]
A =
     0     1     0     0     1     0     0
     1     0     1     0     0     0     1
     0     1     0     1     0     0     0
     0     0     1     0     1     0     0
     1     0     0     1     0     1     0
     0     0     0     0     1     0     0
     0     1     0     0     0     1     0
N =
     1     1     0     0     1     0     0
     1     1     1     0     0     0     1
     0     1     1     1     0     0     0
     0     0     1     1     1     0     0
     1     0     0     1     1     1     0
     0     0     0     0     1     1     0
     0     1     0     0     0     1     1
Enter the gamma value of G:
 k = 2
V =
  Columns 1 through 8
     0     0     0     0     0     0     0     0
     0     0     0     0     0     0     0     0
     0     0     0     0     0     0     1     1
     0     0     0     1     1     1     0     0
     0     1     1     0     0     1     0     0
     1     0     1     0     1     0     0     1
     1     1     0     1     0     0     1     0
  Columns 9 through 16
     0     0     0     0     0     0     0     1
     0     0     1     1     1     1     1     0
     1     1     0     0     0     0     1     0
     0     1     0     0     0     1     0     0
     1     0     0     0     1     0     0     0
     0     0     0     1     0     0     0     0
```



```
NV =
 Columns 1 through 8
   0   1   1   0   0   1   0   0
   1   1   0   1   0   0   2   1
   0   0   0   1   1   1   1   1
   0   1   1   1   1   2   1   1
   1   1   2   1   2   2   0   1
   1   1   2   0   1   1   0   1
   2   1   1   1   1   0   1   1
 Columns 9 through 16
   1   0   1   1   2   1   1   1
   1   1   2   1   1   1   2   2
   1   2   1   1   1   2   2   0
   2   2   0   0   1   1   1   0
   1   1   0   1   1   1   0   1
   1   0   0   1   1   0   0   0
   0   0   2   2   1   1   1   1
 Columns 17 through 21
   1   2   1   1   2
   1   1   1   2   2
   0   0   1   1   1
   0   1   1   1   0
   2   2   2   1   1
   1   1   0   0   0
   1   0   0   0   1
NST =
  13   2   1   1   1   1   1   1
G is a dot stable graph.
>>
```

Snapshot – 2

3.2 γ - Stable Graphs

For a given non – adjacent pair {x, y} of vertices in a graph G, we denote by G_{xy} the graph obtained by deleting x and y and adding a new vertex xy adjacent to precisely those vertices of G – x – y which were adjacent to at least one of x or y in G. We say that G_{xy} is obtained by contracting on {x, y}[11].

A graph G is said to be γ - stable if $\gamma(G_{xy}) = \gamma(G)$, for all x, y ∈ V(G), x is not adjacent to y, where G_{xy} denotes the graph obtained by merging the vertices x, y.

Example

The following result is proved in[11].

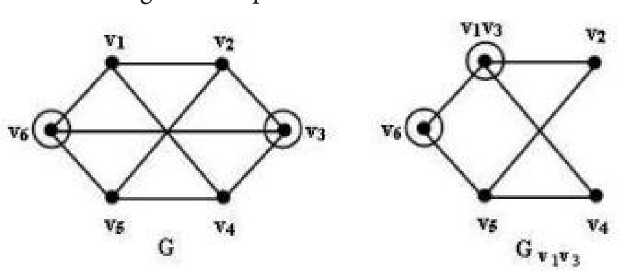


Figure 3. $\gamma(G) = \gamma(G_{v1v3}) = 2$. Similarly $\gamma(G) = \gamma(G_{uv}) = 2$ for all u, v ∈ V(G), u is not adjacent to v, implies G is γ - stable graph.

A graph G is γ - stable if and only if every γ - set D of G is a clique.

Matrix Representation for γ – Stable Graphs

If matrix NV contains no zero entry, then every x_i, i = 1, 2, …p are γ – sets for G, implies atleast one γ – set is not a clique (k > 1) for G, implies G is not γ - stable.

If matrix NV has atleast one zero entry, then consider the non zero columns of NV. Let S ⊆ Xbe the set of all vectors such that $Nx_i \geq 1$, that is NS ≥ 1. Consider the i[th] column of S. Comparing $x_i = \langle x(v_1), x(v_2), ..., x(V_n)\rangle^T$ and $nx_i = \langle nx_i(v_1), nx_i(v_2), ..., nx_i(V_n)\rangle^T$, for all i = 1, 2, …, q (q < p), if for every non zero entry in x_i, the entry in the corresponding position of nx_i is k, then any two vertices in the γ – set x_i are adjacent, that is x_i is a clique.

Example

For the graph in Figure 3

$$N = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

$\gamma(G) = 2$. We consider all possible subsets with 2 vertices and label them as $\{S_1, S_2, \ldots, S_{15}\} = \{\{v_1, v_2\}, \{v_1, v_3\}, \{v_1, v_4\}, \{v_1, v_5\}, \{v_1, v_6\}, \{v_2, v_3\}, \{v_2, v_4\}, \{v_2, v_5\}, \{v_2, v_6\}, \{v_3, v_4\}, \{v_3, v_5\}, \{v_3, v_6\}, \{v_4, v_5\}, \{v_4, v_6\}, \{v_5, v_6\}\}$.

$$NV = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

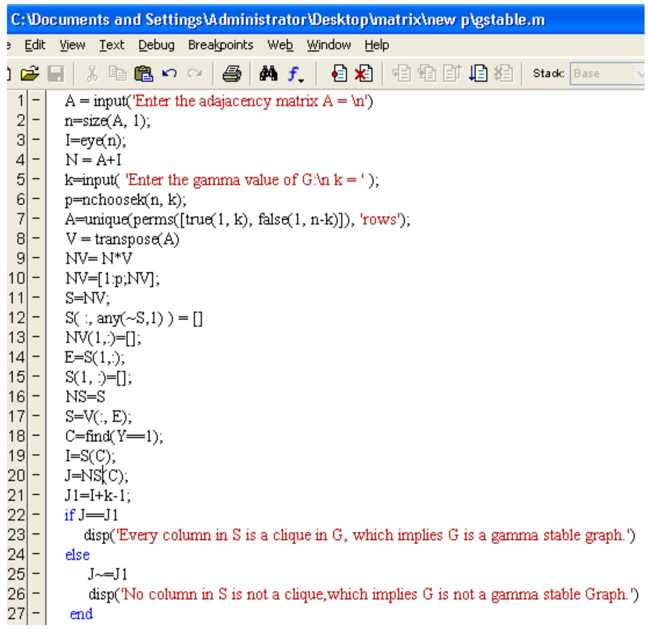
$$= \begin{bmatrix} 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 0 & 1 & 1 & 2 & 1 \\ 2 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 2 & 2 & 1 & 2 & 2 & 1 & 2 & 1 & 2 & 1 \\ 1 & 2 & 2 & 2 & 1 & 1 & 1 & 1 & 0 & 2 & 2 & 1 & 2 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 1 & 1 & 1 & 2 & 2 & 2 \\ 1 & 2 & 1 & 2 & 2 & 1 & 0 & 1 & 1 & 1 & 2 & 2 & 1 & 1 & 2 \end{bmatrix}$$

$$S = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix}; NS = \begin{bmatrix} 2 & 2 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 1 & 1 & 2 & 2 & 2 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 2 & 2 & 1 & 2 & 2 & 1 & 1 \\ 1 & 2 & 1 & 1 & 1 & 1 & 2 & 1 & 2 & 1 \\ 1 & 1 & 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 \\ 1 & 1 & 2 & 1 & 1 & 1 & 1 & 2 & 1 & 2 \end{bmatrix}$$

In NS, for every non zero entry in x_i, the entry in the corresponding position of Nx_i is 2, $i = 1, 2, \ldots, 10$, implies x_i is a clique, implies G is γ - stable.

MAT Lab Program for γ - Stable Graphs

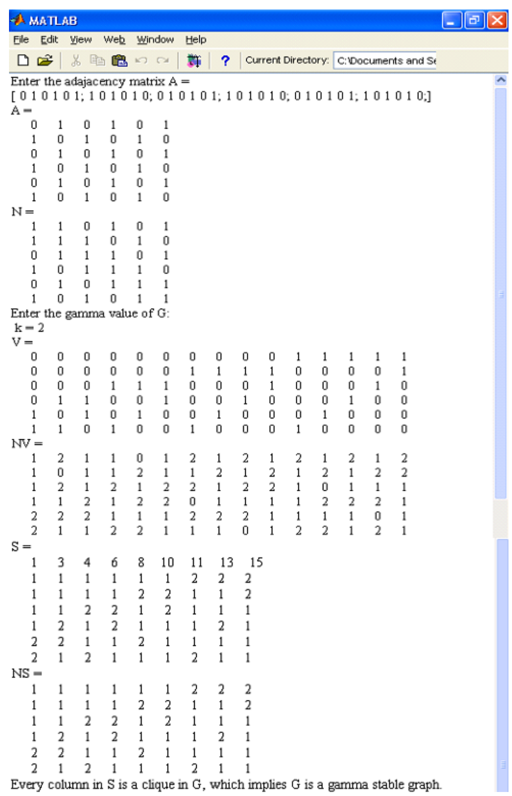
Snapshot – 3 provides the MATLAB program code for γ stable graphs. Snapshot – 4 provides the output generated for the graph in Figure 3.



```
C:\Documents and Settings\Administrator\Desktop\matrix\new p\gstable.m
A = input('Enter the adjacency matrix A = \n')
n=size(A, 1);
I=eye(n);
N = A+I
k=input( 'Enter the gamma value of G:\n k = ' );
p=nchoosek(n, k);
A=unique(perms([true(1, k), false(1, n-k)]), 'rows');
V = transpose(A)
NV= N*V
NV=[1:p;NV];
S=NV;
S( :, any(~S,1) ) = []
NV(1,:)=[];
E=S(1,:);
S(1, :)=[];
NS=S
S=V(:, E);
C=find(Y==1);
I=S(C);
J=NS(C);
J1=I+k-1;
if J==J1
   disp('Every column in S is a clique in G, which implies G is a gamma stable graph.')
else
   J~=J1
   disp('No column in S is not a clique,which implies G is not a gamma stable Graph.')
 end
```

Snapshot – 3

Output



```
Enter the adajacency matrix A =
[ 0 1 0 1 0 1; 1 0 1 0 1 0; 0 1 0 1 0 1; 1 0 1 0 1 0; 0 1 0 1 0 1; 1 0 1 0 1 0;]
A =
  0  1  0  1  0  1
  1  0  1  0  1  0
  0  1  0  1  0  1
  1  0  1  0  1  0
  0  1  0  1  0  1
  1  0  1  0  1  0
N =
  1  1  0  1  0  1
  1  1  1  0  1  0
  0  1  1  1  0  1
  1  0  1  1  1  0
  0  1  0  1  1  1
  1  0  1  0  1  1
Enter the gamma value of G:
 k = 2
V =
  0  0  0  0  0  0  0  0  0  0  1  1  1  1  1
  0  0  0  0  0  0  1  1  1  1  0  0  0  0  1
  0  0  0  1  1  1  0  0  0  1  0  0  0  1  0
  0  1  1  0  0  1  0  0  1  0  0  0  1  0  0
  1  0  1  0  1  0  0  1  0  0  0  1  0  0  0
  1  1  0  1  0  0  1  0  0  0  1  0  0  0  0
NV =
  1  2  1  1  0  1  2  1  2  1  2  1  2  1  2
  1  0  1  1  2  1  1  2  1  2  1  2  1  2  2
  1  2  1  2  1  2  2  1  2  2  1  0  1  1  1
  1  1  2  1  2  2  0  1  1  1  1  2  2  2  1
  2  2  2  1  1  1  2  2  2  1  1  1  1  0  1
  2  1  1  2  2  1  1  1  0  1  2  2  1  2  1
S =
  1  3  4  6  8  10  11  13  15
  1  1  1  1  1  1   2   2   2
  1  1  1  1  2  2   1   1   2
  1  1  2  2  1  2   1   1   1
  1  2  1  2  1  1   1   2   1
  2  2  1  1  2  1   1   1   1
  2  1  2  1  1  1   2   1   1
NS =
  1  1  1  1  1  1  2  2  2
  1  1  1  1  2  2  1  1  2
  1  1  2  2  1  2  1  1  1
  1  2  1  2  1  1  1  2  1
  2  2  1  1  2  1  1  1  1
  2  1  2  1  1  1  2  1  1
Every column in S is a clique in G, which implies G is a gamma stable graph.
```

Snapshot – 4

3.3 Graph Domination Graphs

A γ - set D ⊆ Vis said to graph domination set if D covers all the vertices and edges of G. We shall denote a graph domination set D by γ_G (G)[5].

Example

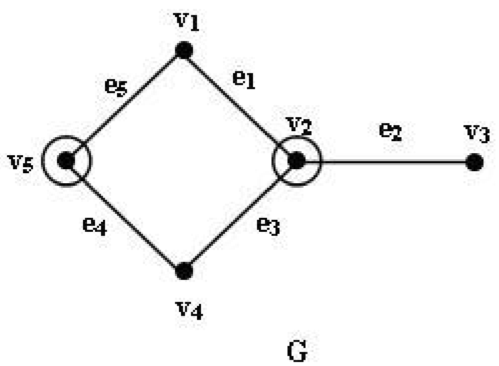


Figure 4. G is graph domination graph. The γ – set {v_2, v_5}covers all the vertices and edges of G.

The following result is proved in[5].

A γ - set D is a graph domination set if and only if V – D is independent.

Matrix Representation for Graph Domination Graph

Let B be the incidence matrix of G. Any row x_i in S^T represents a vector, which is a γ – set for G. Any column in B represents an edge e in G. There are three possibilities,

1. Both end vertices of e ∈ x_i.
2. One end vertex of e belongs to x_i.
3. Both do not belong to x_i.

With this in mind, consider the i^{th} row of S^T and j^{th} column in B. If both the end vertices of e are included in x_i, then the dot product of i^{th} row and j^{th} column is 2. Similarly if one end vertex of e is include in x_i, the dot product value is 1 and if no end vertex of e is included, the dot product value is 0.

S^TB can be defined as follows.

$$S^T B = [s_{ij}]_{q\times n} = \begin{cases} 2, & if\ (x, y) \in x_i \\ 1, & if\ x\ or\ y \in x_i \\ 0, & if\ (x, y) \notin x_i \end{cases}$$

For all e=(x, y) ∈ E (G).

Each column of S^TB represents an edge and each row of S^TB represents a γ – set. The ij^{th} entry of S^TB specifies if the edge is covered by the corresponding γ – set, that is any non zero entry in S^TB means that, that a particular edge is dominated by the corresponding γ – set. If S^TB has a row with all non zero entries, then it means that all the edges of G are covered by the corresponding γ – set D, which implies D is a graph dominating set for G, implies G is a graph domination graph.

Example

For the graph in Figure 4

$$N = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \end{bmatrix}$$

γ(G)=2. We consider all possible subsets with 2 vertices and label them as {S_1, S_2,.....,S_{10}} = {{v_1, v_2}, {v_1, v_3}, {v_1, v_4}, {v_1, v_5}, {v_2, v_3}, {v_2, v_4}, {v_2, v_5}, {v_3, v_4}, {v_3, v_5}, {v_4, v_5}}.

$$NV = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 & 1 & 2 & 1 & 1 & 2 & 0 & 1 & 1 \\ 2 & 2 & 2 & 1 & 2 & 2 & 1 & 2 & 1 & 1 \\ 1 & 1 & 0 & 0 & 2 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 & 2 & 2 & 1 & 1 & 2 \\ 1 & 1 & 2 & 2 & 0 & 1 & 1 & 1 & 1 & 2 \end{bmatrix}$$

$$S = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix};\ S^T \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix};=\ S^TB = \begin{bmatrix} 2 & 1 & 1 & 0 & 1 \\ 1 & 1 & 2 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

The third row of S^TB is non zero, implies all the edges are covered by the γ – set in row three of S^T = { v_2, v_5 }, implies G is a graph domination graph.

MAT Lab Program for Graph Domination

Snapshot – 5 provides the MATLAB program code for graph domination graphs. Snapshot – 6 provides the output generated for the graph in Figure 4.

C:\Documents and Settings\Administrator\Desktop\new p\gdomination.m

```
A = input('Enter the adajacency matrix of a graph A = \n')
n=size(A, 1);
I=eye(n);
N = A+I
k=input( 'Enter the gamma value of G:\n k = ' );
p=nchoosek(n, k);
A=unique(perms([true(1, k), false(1, n-k)]), 'rows');
V = A'
NV= N*V
NV=[1:p;NV];
M1=NV;
NS=NV;
NS( :, any(~NS,1) ) = [];
NV(1,:)=[];
K=NS(1,:);
NS(1, :)=[];
X=NS;
S=V(:, K)
ST=S'
B = input('Enter the inceidence matrix of graph B = \n');
S1=ST*B
m=size(S1, 1);
L1=[1:m];
L=L1';
S1=[L S1];
S1(any(S1==0,2),:)=[]
S2=S1;
if S2(:,1)>=1;
   L2=S2(:,1);
disp('There is atleasst one non zero row in S1.')
disp('The corresponding row in S:')
R1=ST(L2,:)
disp('G is graph domination')
else
   disp('There is no non zero rows in S1, which implies G is not graph domination.')
end
```

Snapshot – 5

Output

MATLAB

```
Enter the adjacency matrix of a graph A =
[ 0 1 0 0 1; 1 0 1 1 0; 0 1 0 0 0; 0 1 0 0 1; 1 0 0 1 0; ]
A =
   0   1   0   0   1
   1   0   1   1   0
   0   1   0   0   0
   0   1   0   0   1
   1   0   0   1   0
N =
   1   1   0   0   1
   1   1   1   1   0
   0   1   1   0   0
   0   1   0   1   1
   1   0   0   1   1
Enter the gamma value of G:
 k = 2
V =
   0   0   0   0   0   0   1   1   1   1
   0   0   0   1   1   1   0   0   0   1
   0   1   1   0   0   1   0   0   1   0
   1   0   1   0   1   0   0   1   0   0
   1   1   0   1   0   0   1   0   0   0
NV =
   1   1   0   2   1   1   2   1   1   2
   1   1   2   1   2   2   1   2   2   2
   0   1   1   1   1   2   0   0   1   1
   2   1   1   2   2   1   1   1   0   1
   2   1   1   1   1   0   2   2   1   1
S =
   0   0   0   1
   0   1   1   1
   1   0   0   0
   0   0   1   0
   1   1   0   0
```

```
ST =
   0   0   1   0   1
   0   1   0   0   1
   0   1   0   1   0
   1   1   0   0   0
Enter the inceidence matrix of graph B =
[1 0 0 0 1; 1 1 1 0 0; 0 1 0 0 0; 0 0 1 1 0; 0 0 0 1 1;]
S1 =
   0   1   0   1   1
   1   1   1   1   1
   1   1   2   1   0
   2   1   1   0   1
S1 =
   2   1   1   1   1   1
There is atleasst one non zero row in S1.
The corresponding row in S:
R1 =
   0   1   0   0   1
G is graph domination
>>
```

Snapshot – 6

4. Conclusion

This paper has presented a MATLAB code for verifying and characterizing domination parameters. This provides an easy method of determining these properties. Once the adjacency matrix and γ – value is known the program easily verifies the graph parameter. This method can further be implemented for verifying these kinds of parameters and is specifically more useful, when the size of the graph is large. So the proposed method is efficient for characterizing graphs based on the domination parameters.

5. References

1. Venkata Rao R. A material selection model using graph theory and matrix approach. Materials Science and Engineering. 2006; A 431:248–55.
2. Venkata Rao R, Padmanabhan KK. Rapid prototyping process selection using graph theory and matrix approach. Journal of Materials Processing Technology. 2007; 194: 81–8.
3. Yang F, Deng Z, Tao J, Li L. A new method for isomorphism identification in topological graphs using incident matrices. Mechanism and Machine Theory. 2012; 49:298–307.
4. Kamal Kumar M. Relation between domination number, energy of graph and rank. International Journal of Mathematics and Scientific Computing. 2011; 1(1):58–61.
5. Yamuna M, Karthika K. Minimal spanning tree from a minimum dominating set, WSEAS Transactions on Mathematics. 2013 Nov; 12(11):1055–64.
6. Yamuna M, Karthika K. Planar graph characterization using γ - stable graphs. WSEAS Transactions on Mathematics. 2014; 13:493–504.
7. Rubalcaba RR, Schneider A, Slater PJ. A survey on graphs which have equal domination and closed neighborhood

packing numbers. AKCE J Graphs Combin 2006; 3(2):93–114.

8. West DB. Introduction to Graph Theory. Second ed. Englewood Cliffs, NJ: Prentice-Hall; 2001.
9. Haynes TW, Hedetniemi ST, Slater PJ. Fundamental of Domination in Graphs. New York: Marcel Dekker; 1998.
10. Yamuna M, Karthika K. Excellent – domination dot stable graphs. International Journal of Engineering Science, Advanced Computing and Bio – Technology. 2011; 2(4):209–16.
11. Yamuna M, Karthika K. γ - Stable tree. International Journal of Pure and Applied Mathematics. 2013; 87 (3):453–8.