

Long-term Association in Time-Series through Cointegration Analysis: A Case Study

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Abstract

The association among variables, especially when endogeneity is not defined, can be modelled using Optimum Least Square. In the case of endogeneity, Two Stage Least Square Method (TSLS) can be deployed. However, presence of Cointegration can jeopardise the existing association among the variables in both the cases. Even if, either Optimum Least Square (OLS) or TSLS can be estimated but the estimated coefficients in the presence of Cointegration will not be appropriate and may be misleading. Testing of cointegration is done by any of the three methods, which is further elaborated by Error Correction Models (ECM) to explore the nature of Cointegration relation.

Keywords: Cointegration, Endogeneity, Error Correction Model (ECM), Stationarity, Vector Auto Regression (VAR)

1. Introduction

Long-term association among the financial time-series is an interesting topic to explore and model. Simultaneity among the time-series restricts them to be modelled through Optimum least square (OLS). Such cases are modelled through systems of equation using Two Stage Least Square (TSLS) or Generalized Method of Moments (GMM) through Instrumental Variables (IVs). Vector Auto Regression (VAR) is also one particular type of systems of equation to model a set of variables (which may or may not be endogenous to one another, though VAR can be modelled among non-endogenous variables as well). Contemporaneous endogenous variables on the RHS of equation require test of endogeneity before they are modelled by VAR. Therefore, usually contemporaneous variables are avoided in VAR estimation. Mainly, lagged term of variables (pre-defines variables of endogenous variable) are taken on the RHS of VAR estimation, as a consequence test of test of endogeneity can be ignored in VAR estimation.

However, there is typical situation, when regression modelling of any type (OLS, Simultaneous Equation Modelling, VAR) gives spurious result and unreliable coefficient estimation. Time series are supposed to be

stationary before it can be modelled. Stationarity should be in level (I (0); Integrated of order zero). There are enough instances that time-series is stationary in first difference (I (1)) if it is not I (0). There is a possibility of a unique situation when some linear combination of a set of I (1) time series variables are I (0). Such situation is known as "Cointegration". I (1) times series, if having Cointegration, would give spurious regression estimations, if modelled through OLS, TSLS or VAR. Therefore, it is always recommended to check for Cointegration before going for regression modelling of any type especially, if the time series variables I (1).

Six nation's stock indices are considered in the current case for Cointegration. They are mainly South-Asian developing economies including India and China. The index values are taken from July 2000 to July 2006.

1. India
2. China
3. Indonesia
4. Argentina
5. Brazil
6. Korea

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The results of their level and first difference Augmented Dickey Fuller (ADF) tests are reported in (Table 1).

Table 1. Augmented Dickey Fuller (ADF) tests

Sl. No.	Index	At Level (t-Values)	First Difference (t-Values)
1.	India	1.0209	-17.4685*
2.	China	-1.3008	-37.3757*
3.	Indonesia	1.29598	-33.9530*
4.	Argentina	0.0775	-37.0176*
5.	Brazil	0.3290	-36.4357*
6.	Korea	-1.655	-36.4387*

Note: t-values are reported for both level and first difference, *significance at 5% level

Stock markets have developed a tendency to show the contagious effect from one stock exchange to another. The spillover effect is studied in different ways. Cointegration among the stock prices across the countries is well sought-after phenomenon to study the spillover effect from one market to another market. However, Cointegration is more apt for studying the long-term association of a unique type. In Cointegration, there is a long-term association along with flexibility of short-term fluctuations as well. The existence of Cointegration ensures long-term association whereas ECM explores the speed in which short-term disturbance restores long-term equilibrium. The present case of stock indices of six nations is also about testing the long-term association along with short-term flexibility among the stock price movement in the stock indices having Cointegration.

All the five nations undertaken in the case are in the category of developing stock markets. They are mostly in Asia along with being in the emerging economy group have all the reasons to have long-term Cointegration tendencies among them.

2. Different Methods for Testing Cointegration

There are different methods which can be used for testing Cointegration.

- Engle and Granger (Engle and Granger, 1987),
- Engle-Yoo Method (Engle and Yoo, 1987),

- Johansen’s Cointegration Test (Johansen, 1988 and 1991),

Engle and Granger (1987) presented in their two-step method to test Cointegration but unable to have any hypothesis testing to statistically or econometrically test the Cointegration. Engle and Yoo (1987) added the third step and extended the work of Engle and Granger (1987). However, Engle and Yoo (1987) could not provide a consistent approach to test Cointegration and robustness remained the perplexing issue with their model to test Cointegration. Johansen (1988 and 1991) presented a solution which was robust as well (unlike Engle and Yoo, 1987). Further Johansen and Juselius (1990) had given critical values for hypothesis testing for Cointegration through Johansen’s method.

Johansen’s method to test Cointegration works on the VAR set up. All the variables being considered for cointegration are supposed to form VAR model. The lag term of the VAR model is decided by Akaike Information Criteria (AIC) and Schwarz-Bayesian Criteria (SBC) (Ghosh, 2010; Akaike, 1974; Schwarz, 1978) (Table 2).

3. Johansen’s Cointegration Test

Johansen (1988 and 1991) used two statistics to test the cointegration among the variables: trace test statistics and maximum test statistics (eq (1) and (2)).

$$\lambda_{trace}(r) = -T \sum_{j=r+1}^p \ln(1 - \hat{\lambda}_j) \tag{1}$$

$$\lambda_{max}(r, r+1) = -T \ln(1 - \hat{\lambda}_{r+1}) \tag{2}$$

Here, r is the number of cointegration equation. λ is the eigenvalue, T is the number of observations and p is the lag term decided by the information criteria in the model (AIC and SBC) (Table 3).

4. Error Correction Models (ECM)

Cointegration is special type of association. It gives freedom as well as restriction to the time-series having Cointegration. The freedom remains for a time being and as the threshold is crossed, the series having cointegration starts correcting (Brooks, 2002). Engle and Granger (1987) proposed Error Correction Model to explore the

can be tested only when variables being modelled are I (1). In the presence of Cointegration among time series, spurious results can be identified. In addition to this Cointegration per se provide insight of long-term association among the variables. Moreover, corresponding Error correction model of Cointegration provide valuable insight about the speed of the adjustment back to the equilibrium in case of any disturbance in the long-term association comes.

6. Study Questions for the Case

1. If series are not integrated of order 1, can still Cointegration be explored?
2. VAR is systems of equation or it can be estimated individually? Can contemporaneous terms be added to VAR estimation? How VAR is different from Simultaneous Equation Modelling?
3. The stock index series of India, China, Indonesia, Brazil, Korea and Argentina should be tested for integration. If they are integrated of order 1, Cointegration can be checked among them.
4. Cointegration generalizes the results in terms of having number of cointegration equations. Can actual association between/among variables be interpreted? Describe the role of Error Correction Modelling (ECM) in exploring the causal relationship among (between) the variables.
5. Interpret the Cointegration results and compare the results for six Stock indices undertaken in the study.

7. References

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