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A New Procedure for Solving Direct Insignificant Programming Problem

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Abstract

In this paper, an effort has been taken to develop an algorithm for solving Intuitionistic Fuzzy Direct Insignificant Programming Problem (IFLFPP). Here, the cost coefficients in the objective function of IFLFPP are taken as Symmetric Triangular Intuitionistic Fuzzy Numbers (STriIFNs). The problem has been solved in the crisp environment by using direct ranking function. A numerical example is given to illustrate the importance of the proposed method.

Keywords: Intuitionistic Fuzzy Direct Insignificant Programming Problem, Ranking Function, Symmetric Triangular Intuitionistic Fuzzy Number

MSC (2010):

1. Introduction

Insignificant programming compacts with the optimization problem of one or several parts of functions subject to some constraints. Direct insignificant programming method[11–12] is applied in many real life models such as building planning, economic and business planning, health care and hospital planning. There are many kinds of formulations to the objective function of problems, may be direct programming, quadratic programming, multi objective and insignificant programming. In these kinds of problems, it is possible that some coefficient of the problem in objective function, technical coefficient or decision making variable may be Intuitionistic fuzzy number.

Attanassov[1] presented the theory of Intuitionistic Fuzzy Set (IFS) theory by the enumeration of non-membership function and uncertainty. In many applications, ranking of Intuitionistic Fuzzy Numbers[10,14] is an essential part of the decision making process. Many real world problems involve handling and concern of intuitionistic fuzzy data for making decision[13]. To analyze and estimate altered replacements, it is necessary to rank Intuitionistic Fuzzy Numbers.

Farhana Akand Prany [3] developed a technique to explain a fuzzy multi objective direct insignificant programming problem with fuzzy coefficient. He used the Graded Mean Representation Method for defuzzification and also comparisons were made with existing methodology.

Sujeet Kumar Singh and shiv Prasad Yadav[4] suggested a process to solve Intuitionistic Fuzzy Direct Insignificant Programming Problem (IFLFPP) which is converted into corresponding crisp Multi-Objective Direct Insignificant Programming Problem (MOLFPP). The transformed MOLFPP is altered to a LPP using fuzzy mathematical programming method.

Moumita Deb and P. K. De[5,15] proposed a new technique to solve fully Fuzzy Direct Insignificant Programming Problem by means of graded mean integration representation method where the variables and parameters are taken as Trapezoidal fuzzy numbers. Sapan kumar das and Tarni Mandal[7] introduced a new methodology for solving direct insignificant programming problem using Homotopy Perturbation Method and factorization technique. Farhana Ahmed Simi and Md. Shahjalal Talukder[6] proposed a

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new methodology for solving the Direct Insignificant Programming Problem. The LFPP has been converted into LPP and the transformed LPP has been solved algebraically using the concept of duality.

The rest of the paper is prearranged as follows: Certain elementary definitions and notations are existing in Section 2. In Section 3, the general form of IFLFPP and the direct ranking function are discussed. In Section 4, an algorithm is generated for the proposed method. In Section 5, validity and applicability of the proposed method are demonstrated with a numerical example followed by conclusion in Section 6.

2. Preliminaries

2.1 Definition

An Intuitionistic Fuzzy Set (IFS) A in X is defined as an objective of the system:

$$A=\{\langle x,\mu_A(x),\nu_A(x)\rangle : x\in \mathrm{X}\}$$

Where the functions $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ define degree of belongingness and the degree of non-belongingness of the component $x \in X$ respectively and for every $x \in X$ in A, $0 \le \mu_A(x)+\nu_A(x) \le 1$ holds.

2.2 Definition

For each mutual fuzzy subset A on X, the degree of uncertainty or intuitionistic fuzzy index of the component x in A is defined as:

$$\pi_A(x)=1-\mu_A(x)+\nu_A(x),0\le \pi_A(x)\le 1.$$

2.3 Definition

The set of all curved combinations of a finite number of opinions is known as the curved polyhedron spanned by these opinions.

2.4 Definition

An Intuitionistic Fuzzy Number (IFN) $\tilde{A}^I$ is:

- An Intuitionistic fuzzy subset on R,
- Normal, that is, there is some $x_0 \in R$ such that $\mu_{\tilde{A}^I}(x_0)=1, \nu_{\tilde{A}^I}(x_0)=0,$
- The belongingness function $\mu_{\tilde{A}^I}(x)$ is convex, i.e. $\mu_{\tilde{A}^I}(\lambda x_1+(1-\lambda)x_2)\ge \min(\mu_{\tilde{A}^I}(x_1),\mu_{\tilde{A}^I}(x_2))$ for every $x_1,x_2\in R,\lambda\in[0,1]$
- The non-belongingness function $\nu_{\tilde{A}^I}(x)$ is concave, i.e. $\nu_{\tilde{A}^I}(\lambda x_1+(1-\lambda)x_2)\le \max(\nu_{\tilde{A}^I}(x_1),\nu_{\tilde{A}^I}(x_2))$ for every $x_1,x_2\in R,\lambda\in[0,1]$

2.5 Definition

An IFS $\tilde{A}^I$ in R is said to be a Symmetric Triangular Intuitionistic Fuzzy Number (STriIFN), if there exist real numbers a, h and h' where $h \le h'$ and $h, h' \ge 0$ such that the belongingness functions and non-belongingness functions are defined as follows:

$$\mu_{\tilde{A}^I}(x)=\begin{cases}\frac{x-(a-h)}{h} & for\, x\in[a-h,a]\\ \frac{a+h-x}{h} & for\ x\in[a,a+h]\\ 0 & otherwise\end{cases}$$ and

$$\nu_{\tilde{A}^I}(x)=\begin{cases}\frac{a-x}{h'} & for\ x\in[a-h',a]\\ \frac{x-a}{h'} & for\ x\in[a,a+h']\\ 0 & otherwise\end{cases}$$

A STriIFN can be represented as $\tilde{A}^I_{STriIFN}=[a;h,h;h',h']$

3. Mathematical Model for Intutionistic Fuzzy Direct Insignificant Programming Problem

3.1 Intuitionistic Fuzzy Direct Insignificant Programming Problem

General structure of Intuitionistic Fuzzy Direct Insignificant Programming Problem is defined as:

$$\text{Maximize}\,\tilde{Z}^I=\frac{\tilde{c}^I x+\tilde{\alpha}^I}{\tilde{d}^I x+\tilde{\beta}^I}$$

subject to $\tilde{A}^I x \le \tilde{B}^I, x \ge 0.$

Where $\tilde{c}^I = \left(\tilde{c}_1^I, \tilde{c}_2^I, \ldots, \tilde{c}_n^I\right), \tilde{d}^I = \left(\tilde{d}_1^I, \tilde{d}_2^I, \ldots, \tilde{d}_n^I\right),$

$\tilde{B}^I = \left(\tilde{b}_1^I, \tilde{b}_2^I, \ldots, \tilde{b}_m^I\right)^T, X \in R^n, x \in X$

$\tilde{\alpha}^I$ and $\tilde{\beta}^I$ are scalar and $\tilde{A}^I = \left[\tilde{a}_{ij}^I\right]_{n \times m}$

3.2 Ranking Function

An efficient method for ordering the components of F(R), the set of STriIFNs on the real line, is to define a ranking function $\Re : F(R) \to R$ which maps each STriIFN into the real number.

Orders on *F(R)* are defined as follows

$$\tilde{A}^I \succeq \tilde{B}^I \text{ if and only if } \Re(\tilde{A}^I) \ge \Re(\tilde{B}^I)$$

$$\tilde{A}^I \approx \tilde{B}^I \text{ if and only if } \Re(\tilde{A}^I) > \Re(\tilde{B}^I)$$

$$\tilde{A}^I \approx \tilde{B}^I \text{ if and only if } \Re(\tilde{A}^I) = \Re(\tilde{B}^I),$$

where $\tilde{A}^I, \tilde{B}^I$ are in $F(R)$.

Note:

For any two STriIFNs $\tilde{A}^I = [a; h, h; h', h']$ and $\tilde{B}^I = [a; k, k; k', k']$, the relation $\preceq$ is a partial order relation and is defined as $\tilde{A}^I \preceq \tilde{B}^I$ if and only if

$$2a + \frac{1}{2}(h' - h) \le 2b + \frac{1}{2}(k' - k)$$

4. Proposed Methodology

Theorem:

If the curved set of possible result of $Ax \le b, x \ge 0$ is a convex polyhedron, then at least one of the exciting opinions gives an Intuitionistic Fuzzy (IF) optimal result.

Proof:

Let $x^{(1)}, x^{(2)}, x^{(3)}, \ldots\ldots\ldots, x^{(k)}$ be the exciting opinions of possible region F of IF direct insignificant problem.

$$Max\, \tilde{Z}^I = \frac{\tilde{u}^I(x)}{\tilde{v}^I(x)} X$$

subject to constraints $Ax \le B$,

and non − negative restriction $x \ge 0$.

Suppose x(m) is the exciting opinion among $x^{(1)}, x^{(2)}, x^{(3)}, \ldots\ldots\ldots, x^{(k)}$ at which the value of IF objective function is max (say z*).

$$\tilde{z}^I * = \max \left\{\frac{\tilde{u}^I(x)}{\tilde{v}^I(x)}\right\} x^{(k)}$$

$$\tilde{z}^I * = \left\{\frac{\tilde{u}^I(x)}{\tilde{v}^I(x)}\right\} x^{(m)}$$

We now consider a point $x^{(0)}$, in the possible section F which is not an exciting opinion and let $\tilde{z}^{I(0)}$ be the corresponding value of IF objective function. Then;

$$\tilde{z}^{I(0)} = \frac{\tilde{u}^I(x)}{\tilde{v}^I(x)} x^{(0)}$$

Since $x^{(0)}$, is not an exciting opinion, it can be expressed as a curved combination of the exciting opinions $x^{(1)}, x^{(2)}, x^{(3)}, \ldots\ldots\ldots, x^{(k)}$ of the possible section F, where F is assumed to be bounded set. Then;

$$x^{(0)} = \lambda_1 x^{(1)} + \lambda_2 x^{(2)} + \ldots\ldots\ldots + \lambda_k x^{(k)}$$

$$\text{where } \lambda_1, \lambda_2, \lambda_3, \ldots\ldots\ldots, \lambda_k \ge 0, \sum_{t=1}^{k} \lambda_k = 1.$$

Now substitute the value of $x^{(0)}$, in the Equation (2), we get;

$$\tilde{z}^{I(0)} = \frac{\tilde{u}^I(x)}{\tilde{v}^I(x)} \left[\lambda_1 x^{(1)} + \lambda_2 x^{(2)} + \ldots\ldots\ldots + \lambda_k x^{(k)}\right]$$

$$\tilde{z}^{I(0)} \le \frac{\tilde{u}^I(x)}{\tilde{v}^I(x)} x^{(m)}$$

$$\tilde{z}^{I(0)} \le \tilde{z}^I *, \quad \text{since } cx^{(m)} = \tilde{z}^I *$$

which implies that at IF optimal result, the exciting opinion result is atleast as good as any other IF possible result.

Theorem

If the IF optimal results occur at more than one exciting opinion, the value of the IF object function will be the same for all curved combinations of this exciting opinion.

Proof

Let $x^{(1)}, x^{(2)}, x^{(3)}, \ldots\ldots\ldots, x^{\Theta}\ (r \le k)$ be the exciting opinions of the possible section F at which the IF object function assumes the same IF optimum value. This means:

$$\tilde{z}^{I*} = \frac{\tilde{u}^I(x)}{\tilde{v}^I(x)} x^{(1)} = \frac{\tilde{u}^I(x)}{\tilde{v}^I(x)} x^{(2)} = \ldots\ldots\ldots = \frac{\tilde{u}^I(x)}{\tilde{v}^I(x)} x^{(r)}$$

Let $x = \lambda_1 x^{(1)} + \lambda_2 x^{(2)} + \ldots\ldots\ldots + \lambda_r x^{(r)}$

$$\lambda_1, \lambda_2, \lambda_3, \ldots\ldots\ldots, \lambda_r > 0,$$

$$\sum_{j=1}^{r} \lambda_j = 1$$

be the curved combination of $x^{(1)}, x^{(2)}, x^{(3)}, \ldots\ldots\ldots, x^{(r)}$ Then,

$$\frac{\tilde{u}_x^I}{\tilde{v}_x^I} x = \frac{\tilde{u}_x^I}{\tilde{v}_x^I}\left[\lambda_1 x^{(1)} + \lambda_2 x^{(2)} + \ldots\ldots\ldots + \lambda_r x^{(r)}\right]$$

$$= \lambda_1\left(\frac{\tilde{u}_x^I}{\tilde{v}_x^I} x^{(1)}\right) + \lambda_2\left(\frac{\tilde{u}_x^I}{\tilde{v}_x^I} x^{(2)}\right) + \ldots\ldots\ldots + \lambda_r\left(\frac{\tilde{u}_x^I}{\tilde{v}_x^I} x^{(r)}\right)$$

$$= \lambda_1 \tilde{z}^I * + \lambda_2 \tilde{z}^I * + \ldots\ldots\ldots + \lambda_k \tilde{z}^I *$$

$$= (\lambda_1 + \lambda_2 + \ldots\ldots\ldots + \lambda_k)\tilde{z}^I *$$

$$\frac{\tilde{u}_x^I}{\tilde{v}_x^I} x \le \tilde{z}^I *, \quad \text{since } \sum_{j=1}^{r} \lambda_j = 1.$$

4.1 Intuitionistic Fuzzy Direct Insignificant Programming Algorithm

Step 1: Convert the given data into a general IFLFPP which is of the form,

$$\text{optimize } \tilde{Z}^I = \frac{\tilde{c}^I x + \tilde{\alpha}^I}{\tilde{d}^I x + \tilde{\beta}^I} \text{ subject to } Ax \le B, x \ge 0.$$

Step 2: Check whether the objective function is to be minimized or maximized. If it is to be decreased, now transform to a problem of increasing it by means of the relation.

$$Minimum\,\tilde{Z}^I = -\,Maximum(-\tilde{Z}^I)$$

Step 3: Checked whether all b_i (i=1, 2,.., m) are non-negative. If anyone of b_i is destructive, then reproduce the equivalent inequality of the restrictions by -1.

Step 4: Convert all the inequalities of the constraints into equalities by presenting intuitionistic fuzzy relaxed and/or excess variables in the restrictions.

Step 5: Using Ranking Function, the Symmetric Triangular Intuitionistic Fuzzy Number is converted into its equivalent crisp value.

Step 6: Compute an initial basic feasible solution for the LFPP.

Step 7: Determine the value of $z^1 = c_B x_B + \alpha$, $z^2 = d_B x_B + \beta$ and $z = \frac{z^1}{z^2}$

Step 8: Obtain $\Delta_j = z^2(z_j^1 - c_j) - z^1(z_j^2 - d_j)$

where $z_j^1 - c_j = c_B y_j - c_j$ and $z_j^2 = d_B y_B - d_j$

Step 9: Analyze the sign of Δ_j

- If all $\Delta_j \ge 0$, then the initial basic feasible solution x_B is an optimal solution.
- Otherwise go to following stage.

Step 10: If there are additional destructive Δ_j, then choose the most destructive of them. Let it be Δ_j

- If all $y_i \ge 0$, then there is an uncontrolled result to the given problem.
- If at least one $y_i \ge 0$, then the corresponding vector enters the basis.

Step 11: Compute the ratios for the vectors and select the minimum value. Let the minimum value will leave the basis.

Step 12: Transform the primary component to union by separating the row by itself and all other components in its column to zeros.

Step 13: Go to step 7 and repeat the process upto the optimum result is gained.

5. Numerical Example

$$\text{Min } Z = \frac{-(2;0.5,0.5;1,1)\xi_1 + (1;0.5,0.5;0.75,0.75)\xi_2 + (2;0.5,0.5;1,1)}{(1;0.5,0.5;0.75,0.75)\xi_1 + (3;0.5,0.5;2,2)\xi_2 + (4;1,1;2,2)}$$

subject to,

$$-\xi_1 + \xi_2 \le 4,$$
$$2\xi_1 + \xi_2 \le 14,$$
$$\xi_2 \le 6,$$
$$\xi_1, \xi_2 \ge 0$$

Solution:

$$\text{Max } Z = \frac{(2;0.5,0.5;1,1)\xi_1 - (1;0.5,0.5;0.75,0.75)\xi_2 - (2;0.5,0.5;1,1)}{(1;0.5,0.5;0.75,0.75)\xi_1 + (3;0.5,0.5;2,2)\xi_2 + (4;1,1;2,2)}$$

Using Direct Ranking function, the Intuitionistic fuzzy cost coefficients in the objective function of the IFLFPP are converted into equivalent crisp values and the transformed LFPP is as follows:

$$\text{Max } Z = \frac{4.3\xi_1 - 2.1\xi_2 - 4.3}{2.1\xi_1 + 6.8\xi_2 + 8.5}$$

Subject to

$$-\xi_1 + \xi_2 + \zeta_1 = 4$$
$$2\xi_1 + \xi_2 + \zeta_2 = 14$$
$$\xi_2 + \zeta_3 = 6$$
$$\xi_1, \xi_2 \ge 0$$

Initial Iteration

dj		cj	4.3	-2.1	0	0	0	
		2.1	6.8	0	0	0		
dB	cB	ξ_B	ξ_1	ξ_2	ζ_1	ζ_2	ζ_3	θ
0	0	ζ_1 =4	-1	1	1	0	0	-4
0	0	ζ_2 =14	2*	1	0	1	0	7
0	0	ζ_3 =6	0	1	0	0	1	0
z2 =8.5	z1= -4.3	z=-0.5						
z2j - dj Δj		z1j – cj	-4.3	2.1	0	0	0	
		-2.1	-6.8	0	0	0		
		-95.6	48.8	0	0	0		

First Iteration

d_j		c_j	4.3	-2.1	0	0	0
		2.1	6.8	0	0	0	
d_B	c_B	ξ_B	ξ_1	ξ_2	ζ_1	ζ_2	ζ_3
0	0	ζ_1 =11	0	1.5	1	0.5	0
0	0	ξ_1 =7	1	0.5	0	0.5	0
0	0	ζ_3 =6	0	1	0	0	1
z^2 =23.2	z^1= 25.8	z=1.1					
z^2_j - d_j Δ_j		z^1_j − c_j	0	4.3	0	2.2	0
		0	-5.7	0	1.1	0	
		0	246.9	0	22.6	0	

Since all,

$\ddot{A}_j \ge 0$, the optimality has been attained at this iteration.

Hence, the optimal solution to the given IFLFPP is:

$\max Z = 1.1$ and $\xi_1 = 7, \xi_2 = 0.$

6. Conclusion

This paper extends the traditional research on the foundation of the theory of IFSs. Here, a Direct Insignificant Programming Problem is explained in which the cost coefficients of the objective function are considered as Symmetric Triangular Intuitionistic Fuzzy Numbers. An efficient direct ranking function has been applied for defuzzification. The results are verified by means of a numerical example. In several areas, manufacture development, commercial and industry development, marketing and broadcasting collection, academy development and student admittances, healthiness and hospital development, etc. every so frequently aspect problems to proceed results that improve income/total, account/transactions, real cost/average cost, production/worker, cherish/persistent ratio, etc. such problems can be explained efficiently by formulating the mathematical model of the data as a LFPP in an Intuitionistic Fuzzy environment. In future, the proposed methodology can be solve Intuitionistic Fuzzy Multi-Objective Direct Insignificant Programming Problem. Also, potential efforts have to be taken to model and solve real life oriented problems as IFLFPP.

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