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Some Properties of Double Vertex Regular Fuzzy Graph

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Abstract

In this paper, we define a new type of fuzzy graph obtained from crisp graph, named Double Vertex Fuzzy Graph (DVFG) and also discussed some properties of double vertex regular fuzzy graph.

Keywords: Double Vertex Fuzzy Graph, Order, Size, Regular Fuzzy Graph, Vertex Degree
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1. Introduction

A. Zadeh found the concept of fuzzy set and fuzzy relation to represent the fact of uncertainly in real world problem. In 1975, Rosenfeld[5] gave the concept of fuzzy graph. Nagoorgani and Radha[3] found regular fuzzy graph. Pathinathan and Jesintha Rosline[4] introduced double layered fuzzy graph. In this paper, section 2 contains the basic definitions of fuzzy graphs. In section 3, we gave a new fuzzy graph called double vertex fuzzy graph used in some theoretical concepts of regularity condition of DVFG.

2. Preliminaries

Definition 2.1 [5] A Fuzzy graph G is a pair of functions denoted by $G:(\sigma,\mu$) where $\sigma: S \rightarrow [0,1]$ is a fuzzy subset of a non-empty set S and $\mu: SXS \rightarrow [0,1]$ is a symmetric fuzzy relation on σ such that for all u, v in S the relation $\mu(u,v) = \mu(uv) \leq \sigma(u) \wedge \sigma(v)$ is satisfied. A fuzzy graph is complete if $\mu(u,v) = \sigma(u) \wedge \sigma(v)$ for all u, v in S. $G^*:(V,E)$ denotes the underlying crisp graph of $G:(\sigma,\mu)$ where $E \subseteq SXS$.

Definition 2.2 [3] Let $G:(\sigma,\mu)$ be the fuzzy graph, the order of G is defined as $o(G) = \sum_{u\in V} \sigma(u)$.

Definition 2.3 [3] Let $G:(\sigma,\mu)$ be the fuzzy graph.Tthe size of G is defined as $S(G) = \sum_{u,v\in V} \mu(u,v)$.

Definition 2.4 [3] Let $G:(\sigma,\mu)$ be the fuzzy graph. The degree of the vertex G is defined as

$$d_G(u) = \sum_{\substack{u\neq v\\ v\in V}} \mu(u,v).$$

Definition 2.5 [3] Let $G:(\sigma,\mu)$ be the fuzzy graph on $G^*:(V,E)$.If $d_G(u) = k$ for all $v \in V$, then G is said to be a regular fuzzy graph degree k or k- regular fuzzy graph.

3. Double Vertex Fuzzy Graphs (DVFG)

Definition 3.1 Let $G:(\sigma,\mu)$ be the fuzzy graph with the underlying crisp graph $G^*:(V,E)$. The pair $DV(G)$: (σ_{DV}, μ_{DV}) is defined as follows. The node set of fuzzy subset σ_{DV} is defined as

$$\sigma_{DV} = \begin{cases} \sigma(u) \text{if} u \in \sigma \\ \sigma(u) \text{if} u \in \sigma \end{cases}$$

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The fuzzy relation μ_{DV} is defined as

$$\mu_{DV} = \begin{cases} \mu(uv) \text{ if } u, v \in \mu \\ \sigma(u_i) \wedge \sigma(u_j) \quad \text{if } u_i \in \sigma \text{ and } u_j \in \sigma \\ \text{and each } u_i \text{ is adjacent with single} \\ u_j \text{ either clockwise or anticlockwise} \\ 0 \text{ otherwise} \end{cases}$$

Since $\mu_{DV} \leq \sigma_{DV}(u) \wedge \sigma_{DV}(v)$ for all u,v in $\sigma \cup \mu$ where μ_{DV} is a fuzzy relation on the subset σ_{DV}. So the pair $DV\ F\ (G)$: (σ_{DV}, μ_{DV}) is defined as double vertex fuzzy graph and the graph is labeled as $DVF\ (G)$.

Example 3.2

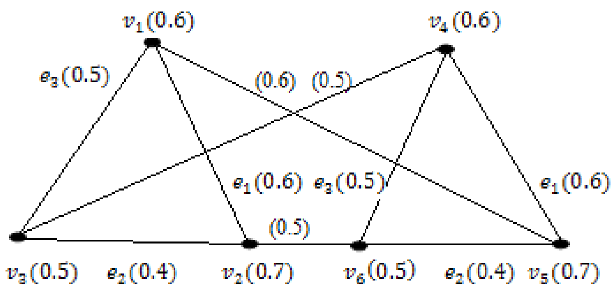
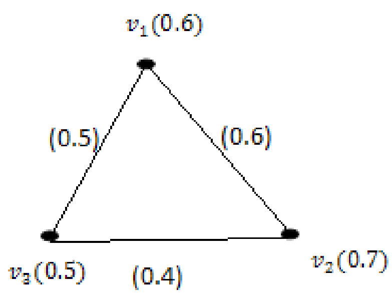


Fig 1. Double vertex fuzzy graph



Property 3.3

For a fuzzy graph H, $Order\ DVF\ (H) = 2\ Order\ (H)$.

Proof:

By definition of double vertex fuzzy graph

$$\sigma_{DV} = \begin{cases} \sigma(u) \text{ if } u \in \sigma \\ \sigma(u) \text{ if } u \in \sigma \end{cases}$$

$$OrderDVF(H) = \sum_{u \in \sigma} \sigma_{DV(H)}(u)$$

$$= \sum_{u \in \sigma} \sigma(u) + \sum_{u \in \sigma} \sigma(u)$$

$$= Order\ (H) + Order\ (H)$$

$$= 2\ Order\ (H).$$

Property 3.4

$SizeDVF(H) = 2\ Size\ (H) + Order\ (H)$, where H is a fuzzy graph.

Proof :

By definition (3.1)

$$SizeDVF\ (H) = \sum_{u,v \in \sigma} \mu_{DV(H)}(u,v)$$

$$= \sum_{u,v \in \sigma} \mu(u,v) + \sum_{u,v \in \sigma} \mu(u,v) + \sum_{u,v \in \sigma} \sigma(u_i) \wedge \sigma(u_j)$$

$$= Size\ (H) + Size\ (H) + Order\ (H)$$

$$= 2\ Size\ (H) + Order\ (H).$$

Property 3.5

Let H be a fuzzy graph, then $d_{DV(H)} = d_H(u) + \sigma(u_i) \wedge \sigma(u_j)$ if $u \in \sigma$

Proof:

Let $d_H(u) = \sum_{\substack{u \neq v \\ v \in V}} \mu(u,v)$ and let $u \in \sigma$, then

$$d_{DV(H)}(u) = \sum_{u,v \in \sigma} \mu_{DV(H)}(u,v)$$

$$= \sum_{u,v \in \sigma} \mu(u,v) + \sigma(u_i) \wedge \sigma(u_j)$$

$= d_H\ (u) + \sigma(u_i) \wedge \sigma(u_j)$ for all $u \in \sigma$.

Theorem 3.6

Let $H: (\sigma_{DV}, \mu_{DV})$ be the fuzzy graph and the underlying crisp graph is $H: (\sigma, \mu)$. Suppose if σ is a constant function and if H is regular fuzzy graph, then $DV\ (H)$ is a regular fuzzy graph.

Proof:

Suppose that σ is a constant function and H is a regular fuzzy graph.Let $\sigma(v) = c$ and $d_H(v) = k$ is a regular fuzzy graph for all $v \in \sigma$.

$d_{DV(H)}(v) = d_H\ (u) + \sigma(u_i) \wedge \sigma(u_j)$ for all $v \in \sigma$

$= k + c \wedge c = k + c$ for all $v \in \sigma$

Theorem 3.7

Let $H: (\sigma_{DV}, \mu_{DV})$ be the fuzzy graph and the underlying crisp graph is $H : (\sigma, \mu)$. If σ and μ are the constant functions on cycle, then $DVF(H)$ is a regular fuzzy graph.

Proof :

Suppose that σ and μ are constant functions. Let $\sigma(u) = c$ and $\mu\ (uv) = k \leq c$. Because H is a fuzzy graph on the cycle and only two edges are incident with each vertex for cycles, for any $u \in \sigma$.Given that μ is a constant function, $\mu(uv) = k$ for all $uv \in E$, then $d(u) = 2k$ for every $u \in \sigma$, so H is regular.

$d_{DV(H)}(u) = d_H(u) + \sigma(u_i) \blacktriangle \sigma(u_j)$ for all $u \in \sigma$

$= 2k + c \blacktriangle c = 2k + c$ for all $u \in \sigma$

Hence $DVF(H)$ is a regular fuzzy graph.

Theorem 3.8

Let $H:(\sigma_{DV}, \mu_{DV})$ be the fuzzy graph and the underlying crisp graphis $H:(\sigma, \mu)$. If σ is a constant function and μ is the alternative edge have same membership values on even cycle, then $DVF(H)$ is a regular fuzzy graph.

Proof :

Given σ is a constant function and μ is a alternative edges have same membership values.Suppose that $\sigma(u) = c$ and $\mu(e_i) = \begin{cases} k \text{ if } k < c, i \text{ is even} \\ c - k \text{ if } i \text{ is odd} \end{cases}$

Since H is a fuzzy graph on even cycle and only two edges are incident with each vertex for cycles, for any $u \in \sigma$.

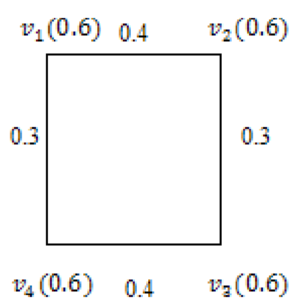
If μ is an alternative edge, say and $\mu(e_i) = \begin{cases} k \text{ if } k < c, i \text{ is even} \\ c - k \text{ if } i \text{ is odd} \end{cases}$ for all $uv \in E$, then $d(u) = k + c - k = c$ for every $u \in \sigma$, so H is regular.

$d_{DV(H)}(u) = d_H(u) + \sigma(u_i) \blacktriangle \sigma(u_j)$ for all $u \in \sigma$

$= c + c \blacktriangle c = 2c$ for all $u \in \sigma$

Therefore, $DVF(H)$ is a regular fuzzy graph.

Example 3.9



Fuzzy graph

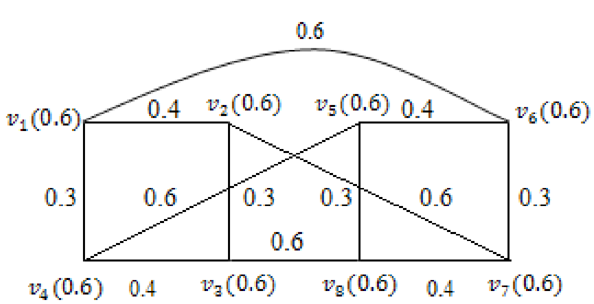


Fig 2. DVF(H) is a regular fuzzy graph

Theorem 3.10

Let $H:(\sigma_{DV}, \mu_{DV})$ be the fuzzy graph and the underlying crisp graphis $H:(\sigma, \mu)$. If σ is an alternative vertex $c_1 < c_2$ and μ is the alternative edge have same membership values on even cycle, then $DVF(H)$ is a regular fuzzy graph.

Proof :

Suppose that σ is an alternative vertex $c_1 < c_2$ and μ is alternative edges have same membership values

Let $\sigma(u_i) = \begin{cases} c_1 \text{ if } i \text{ is odd} \\ c_2 \text{ if } i \text{ is even} \end{cases}$ and $\mu(e_i) = \begin{cases} k_1 \text{ if } k_1 < c_1, i \text{ is even} \\ c_1 - k_1 \text{ if } i \text{ is odd} \end{cases}$

Since H is a fuzzy graph on even cycle and only two edges are incident with each vertex for cycles, for any $u \in \sigma$.

If μ is an alternative edge, say and $\mu(e_i) = \begin{cases} k_1 \text{ if } k_1 < c_1, i \text{ is even} \\ c_1 - k_1 \text{ if } i \text{ is odd} \end{cases}$ for all $uv \in E$, then $d(u) = k_1 + c_1 - k_1 = c_1$ for every $u \in \sigma$, so H is regular.

$d_{DV(H)}(u) = d_G(u) + \sigma(u_i) \blacktriangle \sigma(u_j)$ for all $u \in \sigma$

$= c_1 + c_1 \blacktriangle c_2 = 2c_1$ for all $u \in \sigma$

Therefore, $DVF(H)$ is a regular fuzzy graph.

Remark 3.11 In theorem 3.10 the condition $c_1 < c_2$ is necessary. Otherwise the above theorem (3.10) fails. This is illustrated with the following example.

Example 3.12

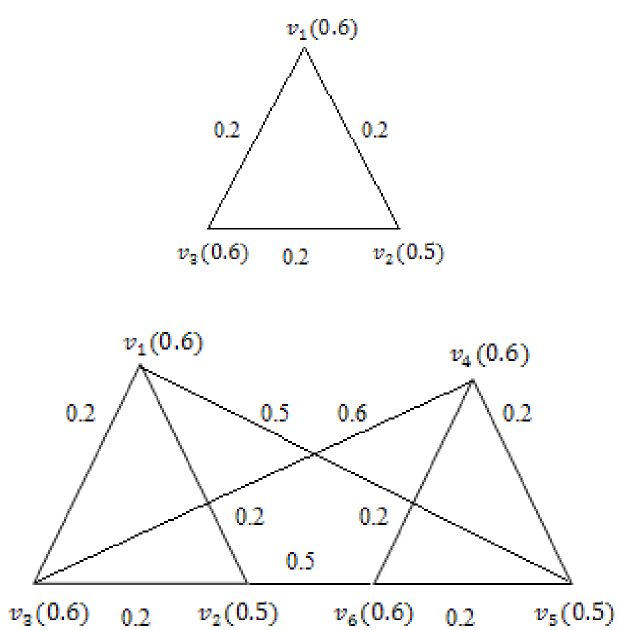


Fig 3. DVF(H) fuzzy graph is not a regular fuzzy graph

Theorem 3.13

Suppose $H:(\sigma_{DV}, \mu_{DV})$ be the fuzzy graph and the underlying crisp graphis $H:(\sigma, \mu)$. If σ is an alternative vertex $c_1 < c_2$ and μ is a constant edge have same membership values on cycle, then $DVF\ (H)$ is a regular fuzzy graph.

Proof:

Since σ is an alternative vertex $c_1 < c_2$ and μ is an constant edges have same membership values. Let $\sigma(u_i) = \begin{cases} c_1 \text{if } i \text{ is odd} \\ c_2 \text{if } i \text{ is even} \end{cases}$ and $\mu\ (e_i) = k_1 \text{if } k_1 \leq c_1$.

Since H is a fuzzy graph on cycle and only two edges are incident with each vertex for cycles, for any $u \in \sigma$.If μ is a constant edge, say and $\mu\ (e_i) = k_1 \text{ if } k_1 \leq c_1$ for all $uv \in E$, then $d(u) = 2k_1$ for every $u \in \sigma$, so H is regular.

$d_{DV(H)}(u) = d_H\ (u) + \sigma(u_i) \blacktriangle \sigma(u_j)$ for all $u \in \sigma$

$= 2k_1 + c_1 \blacktriangle c_2 = 2k_1 + c_1$ for all $u \in \sigma$

Therefore, $DVF\ (H)$ is a regular fuzzy graph.

Theorem 3.14

Let $H: (\sigma_{DV}, \mu_{DV})$ be the fuzzy graph with the underlying crisp graph $H : (\sigma, \mu)$. If $\sigma(u_i) = \begin{cases} c_1 & if\ i = 1,2,\dots,n-1 \\ c_2 \text{if } i = n \end{cases}$, $c_1 < c_2$ and μ is an constant edges have same membership values on cycle, then $DVF\ (H)$ is a regular fuzzy graph.

Proof:

Suppose that $c_1 < c_2$ and μ is a constant edges have same membership values.Since $\sigma(u_i) = \begin{cases} c_1 \text{if } i = 1,2,\dots,n-1 \\ c_2 \text{if } i = n \end{cases}$ and $\mu\ (e_i) = k_1 \text{if } k_1 \leq c_1$.

Since H is a fuzzy graph on cycle and only two edges are incident with each vertex for cycles, for any $u \in \sigma$.

If μ is an constant edges, say and $\mu\ (e_i) = k_1 \text{if } k_1 \leq c_1$ for all $uv \in E$, then $d(u) = 2k_1$ for every $u \in \sigma$, so H is regular.

$d_{DV(H)}(u_i) = d_H\ (u_i) + \sigma(\mathbf{u_i}) \blacktriangle \sigma(\mathbf{u_j})$ for all $u_i \in \sigma, i = 1,2,\dots,n-2$.

$= 2\mathbf{k_1} + \mathbf{c_1} \blacktriangle \mathbf{c_1} = 2\mathbf{k_1} + \mathbf{c_1}$ for all $u_i \in \sigma$.

$d_{DV(H)}(u_{n-1}) = d_H\ (u_{n-1}) + \sigma(\mathbf{u_{n-1}}) \blacktriangle \sigma(\mathbf{u_n})$ for $u_{n-1} \in \sigma$.

$= 2\mathbf{k_1} + \mathbf{c_1} \blacktriangle \mathbf{c_2} = 2\mathbf{k_1} + \mathbf{c_1}$ for $u_{n-1} \in \sigma$.

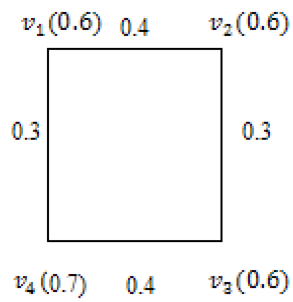
$d_{DV(H)}(u_n) = d_H\ (u_n) + \sigma(\mathbf{u_n}) \blacktriangle \sigma(\mathbf{u_1})$ for $u_n \in \sigma$.

$= 2\mathbf{k_1} + \mathbf{c_2} \blacktriangle \mathbf{c_1} = 2\mathbf{k_1} + \mathbf{c_1}$ for $u_n \in \sigma$.

Therefore, $DVF\ (H)$ is a regular fuzzy graph.

Example 3.15

$(n-1)$vertex and last vertex, μ is constant edge odd (or) even cycle, $u_1 = u_2 = \cdots = u_{n-1} < u_n$.



Fuzzy graph

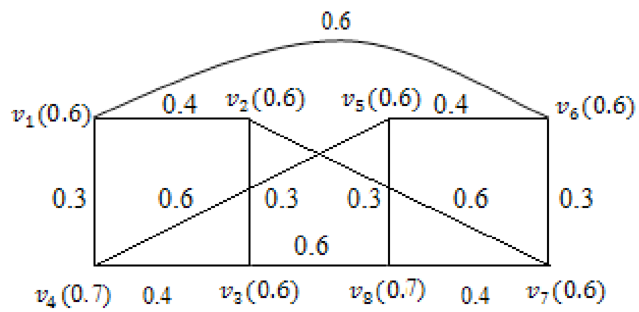


Fig.4. DVF(H) is a regular fuzzy graph

Theorem 3.16

Let $H: (\sigma_{DV}, \mu_{DV})$ be the fuzzy graph with the underlying crisp graph $H: (\sigma, \mu)$. If σ is an alternative vertex

σ(u↓i) = {(c↓1 ifiisodd@membership value @ x ≥ c↓1@where x is not a

and μ is an constant edges have same membership values on cycle, $DVF\ (H)$ is a regular fuzzy graph.

Proof :

Suppose that $c_1 \leq x$ and μ is an constant edges have same membership values. Let $\sigma(u_i) = \begin{cases} \mathbf{c_1} \text{ if } i \text{ is odd} \\ x \geq \mathbf{c_1} \text{ if } i \text{ is even} \end{cases}$

and $\mu\ (e_i) = k_1 \text{if } k_1 \leq c_1$ .Since H is a fuzzy graph on cycle and only two edges are incident with each vertex for cycles, for any $u \in \sigma$.

If μ is an constant edges, say and $\mu\ (e_i) = k_1 \text{if } k_1 \leq c_1$ for all $uv \in E$, then $d(u) = 2k_1$ for every $u \in \sigma$, so H is regular.

$d_{DV(H)}(u) = d_H\ (u) + \sigma(\mathbf{u_i}) \blacktriangle \sigma(\mathbf{u_j})$ for all $u \in \sigma$.

$= 2k_1 + c_1 \blacktriangle x = 2k_1 + c_1$ for all $u \in \sigma$.

Hence, $DVF\ (H)$ is a regular fuzzy graph.

Theorem 3.17

Let $H: (\sigma_{DV}, \mu_{DV})$ be the fuzzy graph with the underlying crisp graph $: (\sigma, \mu)$. If σ is an alternative vertex

σ(u↓i) = {(c↓1 ifiisodd@membership value @ x ≥ c↓1 where x is not and μ is an alternative edges have same membership values on even cycle, $DVF\ (H)$ is a regular fuzzy graph.

Proof :

Suppose that $c_1 \leq x$ and μ is an alternative edges have same membership values on even cycle.

Let $\sigma(u_i) = \begin{cases} c_1 \text{ if } i \text{ is odd} \\ x \geq c_1 \text{ if } i \text{ is even} \end{cases}$ and $\mu(e_i) = k_1 \text{ if } k_1 < c_1$

$$\mu(e_i) = \begin{cases} k_1 \text{ if } k_1 < c_1, i \text{ is even} \\ c_1 - k_1 \text{ if } i \text{ is odd} \end{cases}$$

Since, H is a fuzzy graph on even cycle and only two edges are incident with each vertex for cycles, for any $u \in \sigma$. Then $d(u) = k_1 + c_1 - k_1 = c_1$ for every $\in \sigma$, so H is regular.

$d_{DV(H)}(u) = d_H(u) + \sigma(u_i) \Delta \sigma(u_j)$ for all $u \in \sigma$

$= c_1 + c_1 \Delta x = 2c_1$ for all $u \in \sigma$

Therefore, $DVF\ (H)$ is a regular fuzzy graph.

Theorem 3.18

Suppose $H{:}(\sigma_{DV}, \mu_{DV})$ be the fuzzy graph and the underlying crisp graph is $H{:}(\sigma, \mu)$ is an cycle. Then σ is a constant function if and only if the following statements are equivalent:

H is a regular fuzzy graph

$DV\ F(H)$ is a regular fuzzy graph.

Proof:

Assume that σ is a constant function. Let $\sigma(u) = c$ for all $u \in \sigma$. Assume that H is a $k_1 -$ regular fuzzy graph. Then $d_H(u) = k_1$ for all $u \in \sigma$.

So, $d_{DV(H)}(u) = d_H(u) + \sigma(u_i) \Delta \sigma(u_j)$ for all $u \in \sigma$

$= k_1 + c \Delta c = k_1 + c$ for all $u \in \sigma$

Therefore, $DVF(H)$ is a regular fuzzy graph. Thus $(1) \Rightarrow (2)$ is proved.

Now, Suppose that $DVF(H)$ is a k_2 regular fuzzy graph.

Then, $d_{DV(H)}(u) = k_2$ for all $u \in \sigma$

$d_H(u) + \sigma(u_i) \Delta \sigma(u_j) = k_2$ for all $u \in \sigma$

$d_H(u) + \sigma(u_i) = k_2$ for all $u \in \sigma$

$d_H(u) + c = k_2$ for all $u \in \sigma$

$d_H(u) = k_2 - c$ for all $u \in \sigma$

So, H is a regular fuzzy graph .Hence (1) and (2) are equivalent. Conversely, suppose that H is a regular fuzzy graph and $DVF\ (H)$ is a regular fuzzy graph.

$d_{DV(H)}(u) = k + c$ and $d_H(u) = k_2 - c$ for all $u \in \sigma$.

$\Rightarrow d_H(u) + \sigma(u_i) \Delta \sigma(u_j) = k + c$ and $d_H(u) = k_2 - c$ for all $u \in \sigma$.

$\Rightarrow k_2 - c + \sigma(u_i) \Delta \sigma(u_j) = k + c$ for all $u \in \sigma$

$\Rightarrow \sigma(u_i) \Delta \sigma(u_j) = k + c - k_2 + c$ for all $u \in \sigma$

$\Rightarrow \sigma(u_i) \Delta \sigma(u_j) = k - k_2 + 2c$ for all $u \in \sigma$

$\Rightarrow \sigma(u_i) \Delta \sigma(u_j) = c_2$ for all $u \in \sigma$.

Therefore, σ is a constant function.

Theorem 3.19 Suppose that $H{:}(\sigma_{DV}, \mu_{DV})$ be the fuzzy graph on complete fuzzy graph k_n on $H : (\sigma, \mu)$. If σ is a constant function, $DVF\ (H)$ is a regular fuzzy graph.

Proof:

Suppose $H : (\sigma, \mu)$ is the fuzzy graph on complete fuzzy graph k_n on $H : (\sigma, \mu)$.Let $\sigma(u) = c$ is a constant function for all $u \in \sigma$. For any complete fuzzy graph $(n-1)$ edges are incident with a vertex.

Since H is a complete fuzzy graph.

$\mu(u, v) = \mu(uv) = \sigma(u) \Delta \sigma(v) = c \Delta c = c$

$d_G(u) = (n-1)c$

$d_{DV(G)}(u) = d_G(u) + \sigma(u_i) \Delta \sigma(u_j)$

$= (n-1)c + c = nc$ for all $u \in \sigma$.

Therefore, $DVF\ (H)$ is a regular fuzzy graph.

Theorem 3.20

Suppose that $H{:}(\sigma_{DV}, \mu_{DV})$ be the fuzzy graph on star $S_{1,n}$. If σ and μ are a constant functions $\mu \leq \sigma$. Then $d_{DV(H)}(u_1) = d_{DV(H)}(u_{n+2}) = n\mu + \sigma$ and $d_{DV(H)}(u_i) = \mu + \sigma$ for $i = 2,3,..n+1, n+3, \ldots, 2n+2$.

Proof:

Suppose that σ and μ are constant functions. Let $\sigma(u) = \sigma$ and $\mu(uv) = \mu \leq \sigma$. Since H is a fuzzy graph on the star graph and n edges are incident with single vertex.

$d(u_1) = d(u_{n+2}) = n\mu$,

$d(u_2) = d(u_3) = \cdots = d(u_{n+1}) = d(u_{n+3}) = \cdots = d(u_{2n+2}) = \mu$

$d_{DV(G)}(u_1) = d_G(u_1) + \sigma(u_1) \Delta \sigma(u_{n+3})$

$= n\mu + \sigma \Lambda \sigma = n\mu + \sigma$

For $i = 2,3, \ldots, n$,

$d_{DV(G)}(u_i) = d_G(u_i) + \sigma(u_i) \Delta \sigma(u_{n+i+2}) = \mu + \sigma$

$d_{DV(G)}(u_{n+1}) = d_G(u_{n+1}) + \sigma(u_{n+1}) \Delta \sigma(u_{n+2})$

$= \mu + \sigma$

$d_{DV(G)}(u_{n+2}) = d_G(u_{n+2}) + \sigma(u_{n+2}) \Delta \sigma(u_{n+1})$

$= n\mu + \sigma$

For all

$i = 3, \ldots, n+2, d_{DV(G)}(u_{n+i}) = d_G(u_{n+i}) + \sigma(u_{n+i}) \Lambda \sigma(u_{i-2}) = \mu + \sigma$.

4. Conclusion

In this paper, we have defined a double vertex fuzzy graph and use the concept of check the regularity conditions. We have numerical example is given to verify the regularity results. Further work in this regard would be required to study about various fuzzy networks.

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