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Image File Format using MATLAB

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Abstract

Mathematics plays a vital role in day to day life. The image file format is applied in various areas of engineering and science. The aim of this project is based on JPEG image using Singular Value Decomposition (SVD). Moreover, linear algebra (SVD) is used to compress the image in MATLAB.

Keywords: Image Compression, MATLAB, Singular Value Decomposition

1. Introduction

The generality becomes very popular with taking selfies. Promptly, they urge to laying hold of their unforgettable incident. Due to this, the count of photocopy and videos were raised. This results in need of large amount of storage to save all these items. Comprehensively, it stand insert of transmission capacity. On deck of this, the image compression technique is urged. This execution truncate the cache engrossed by the reflection left out any depletion of nature of the picture. So that, the photography proportions can be contracted by choosing appropriate squeezing usage buildup on the essential of patron or ultilization.

2. Preliminaries

Definition 2.1: Matrix: Here, m × n is a considered with rectangular array of mn real or complex numbers. Where, horizontal row is denoted by m and vertical column is denoted by n.

Example 2.1

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

Definition 2.2: Orthogonal: A square matrix A of order n is said to be Orthogonal, if

$$A^T A = AA^T = I_n$$

Example 2.2

$$A = \begin{bmatrix} cosx & sinx \\ -sinx & cosx \end{bmatrix}$$

Definition 2.3: Singular Value Decomposition: If A is a matrix with singular values $\sqrt{\sigma_1}, \sqrt{\sigma_2}, \ldots, \sqrt{\sigma_r},$ Where r is the rank of A^*A.

Define, $V = [x_1 | x_2 | \ldots | x_n], U = [y_1 | y_2 | \ldots | y_n]$ where x_i is an orthonormal basis of eigenvectors for A^*A and $y_i = \frac{1}{\sqrt{\sigma_i}} Ax_i$. Additionally, $s_i = \sqrt{\sigma_i}$

Thus $AV = US$, $A = USV^*$.

Definition 2.4: M-files: M-files are ordinary American Standard Code for Information Interchange text files written in MATLAB language. They are called M-files because they must have an '.m' at the end of their name (like unit .m). M-files can be created using any editor or word processing application.

There are two types of M-files:
- Script files
- Function files.

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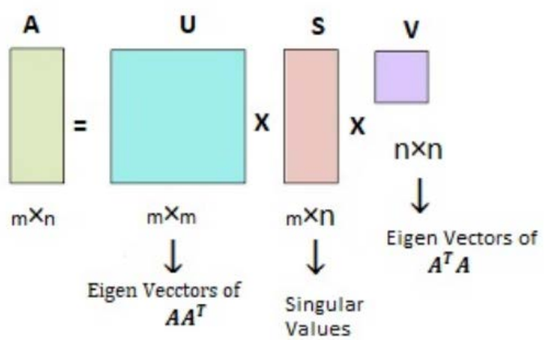
Definition 2.5: Image Compression: Image Compression is the process of encoding or converting an image file in such a way that it consumes less space than the original file. It is a type of compression technique that reduces the size of an image file without affecting or degrading its quality to a great extend.

There are two types of image compression

- Lossy Compression (i.e., JPEG, Webp)
- Lossless Compression (i.e., TIFF, GIF)

3. Solving SVD in Image Compression using MATLAB

Memory Utilization



Mathematical Analysis

Let A_{mXn}. Where, A_{mXn} is the image matrix

Applying Singular Value Decomposition, A can be written as

A = USV^T

$$A = [u_1 \;\; u_2 \cdots u_m]\begin{bmatrix} s_1 & 0 & \ldots & 0 \\ 0 & s_2 & \ldots & 0 \\ \vdots & 0 & \ddots & \vdots \\ 0 & 0 & \ldots & s_n \end{bmatrix}\begin{bmatrix} v^T \\ v_2^T \\ \vdots \\ v_n^T \end{bmatrix}$$

Take, $s_1 > s_2 > \cdots s_n > 0$

Now, A can be written as

A = US^1V^T

$$A = [u_1 \;\; u_2 \cdots u_m]\begin{bmatrix} s_1 & 0 & \cdots & \cdots & 0 \\ 0 & \ddots & \cdots & \cdots & \vdots \\ \vdots & \cdots & s_r & \cdots & \vdots \\ \vdots & \cdots & \cdots & \ddots & \vdots \\ 0 & \cdots & \cdots & \cdots & \vdots \end{bmatrix}\begin{bmatrix} v^T \\ v_2^T \\ \vdots \\ v_n^T \end{bmatrix}$$

"In the above matrix, S values after r terms are approximated to zero. So, the multiplication of the r term > 0".

If $m = n$, the matrix A can be written as

$$A = [s_1u_1 \;\; s_2u_2 \cdots s_ru_r \; 0 \ldots 0]\begin{bmatrix} v_1^T \\ v_2^T \\ \vdots \\ v_n^T \end{bmatrix}$$

$$A = s_1u_1v_1^T + s_2u_2v_2^T + \ldots + s_ru_rv_r^T$$

$$A = \Sigma_{i=1}^{r} s_iu_iv_i^T$$

Example 3.1

To find the solution of Singular Value Decomposition,

$$A = \begin{pmatrix} 2 & -2 \\ 1 & 1 \end{pmatrix}$$

Solution:

Given: $A = \begin{pmatrix} 2 & -2 \\ 1 & 1 \end{pmatrix}$

To Find: Singular Value Decomposition

i.e., $A = USV^T$

$$A^T = \begin{pmatrix} 2 & 1 \\ -2 & 1 \end{pmatrix}$$

$$A^TA = \begin{pmatrix} 2 & 1 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} 2 & -2 \\ 1 & 1 \end{pmatrix}$$

$$A^TA = \begin{pmatrix} 5 & -3 \\ -3 & 5 \end{pmatrix}$$

Characteristic Equation is,

$$|A^TA - \lambda I| = 0$$

$$\left|\begin{pmatrix} 5 & -3 \\ -3 & 5 \end{pmatrix} - \lambda I\right| = 0 \qquad (1)$$

$$\begin{vmatrix} 5-\lambda & -3 \\ -3 & 5-\lambda \end{vmatrix} = 0 \qquad (2)$$

$(5 - \lambda)(5 - \lambda) - 9 = 0$

$25 + \lambda^2 - 10\lambda - 9 = 0$

The characteristic equation is,

$\lambda^2 - 10\lambda + 16 = 0$

The **Eigen values** is,

$$\lambda_1 = 2, \lambda_2 = 8$$

i. e, The **Singular Value** is,

$$S_1 = \sqrt{8},\ S_2 = \sqrt{2} \tag{3}$$

Using diagonal matrix in equation (3), we get

$$S = \begin{pmatrix} 2\sqrt{2} & 0 \\ 0 & \sqrt{2} \end{pmatrix}$$

When $\lambda = 8$ in equation (2), we get

$$|A^T A - \lambda I| V_1 = 0$$

$$\begin{pmatrix} -3 & -3 \\ -3 & -3 \end{pmatrix} V_1 = 0$$

$$-3x_1 - 3x_2 = 0$$

Therefore, $x_1 = x_2$

Solving x_1 and x_2,

Put $x_2 = 1$, then $x_1 = -1$

$$V_1 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

Now, we have to find an orthonormal basis for $V_1 V_1$,

$$V_1 = \begin{pmatrix} -0.7071 \\ 0.7071 \end{pmatrix}$$

When $\lambda = 2$ in equation (2), we get

$$|A^T A - \lambda I| V_2 = 0$$

$$\begin{pmatrix} 3 & -3 \\ -3 & 3 \end{pmatrix} V_2 = 0$$

$$3x_1 - 3x_2 = 0$$

$$-3x_1 + 3x_2 = 0$$

i.e.,

$$x_1 = x_2$$

Solving x_1 and x_2,

Put $x_2 = 1$, then $x_1 = 1$

$$V_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

We have to find an orthonormal basis for V_2,

$$V = \begin{pmatrix} 0.7071 \\ 0.7071 \end{pmatrix}$$

Therefore, the **Eigen vector** is

$$V = \begin{pmatrix} -0.7071 & 0.7071 \\ 0.7071 & 0.7071 \end{pmatrix}$$

Finally, **To Find:** Singular Value Decomposition i.e., $A = USV^T$

$$\Rightarrow U = AVS^{-1}$$

To Find: U

$$U = AVS^{-1} = \begin{pmatrix} 2 & -2 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} -0.7071 & 0.7071 \\ 0.7071 & 0.7071 \end{pmatrix} \begin{pmatrix} 0.3535 & 0 \\ 0 & 0.7071 \end{pmatrix}$$

$$U = AVS^{-1} = \begin{pmatrix} 2 & -2 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} -0.2499 & 0.4999 \\ 0.2499 & 0.4999 \end{pmatrix}$$

Therefore,

$$U = AVS^{-1} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

Now, we have to find the **Singular Value Decomposition.**

$$A = USV^T = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 2\sqrt{2} & 0 \\ 0 & \sqrt{2} \end{pmatrix} \begin{pmatrix} -0.7071 & 0.7071 \\ 0.7071 & 0.7071 \end{pmatrix}$$

$$A = USV^T = \begin{pmatrix} 1.999 & -1.999 \\ 0.999 & 0.999 \end{pmatrix} \approx \begin{pmatrix} 2 & -2 \\ 1 & 1 \end{pmatrix}$$ Hence,

$$A = \begin{pmatrix} 2 & -2 \\ 1 & 1 \end{pmatrix}$$

Example 3.2

MATLAB M-file coding for Singular Value Decomposition

```
>>A= [2 -2; 1 1]
A = 2 −2
 1 1
>> [U S V] = svd (A)
```

OUTPUT:

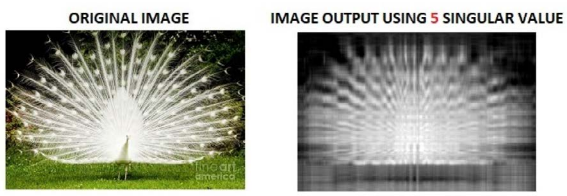
```
U = −1.0000    0.0000
     0.0000    1.0000
V = −0.7071    0.7071
     0.7071    0. 7071
S =  2.8284    0.0000
     0.0000    1.4142
```

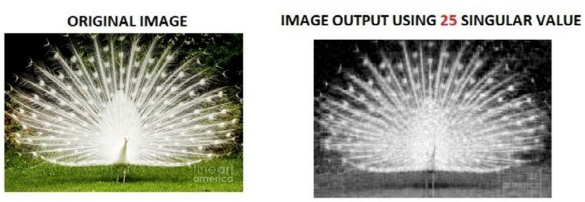
MATLAB M-file Coding for Image Compression

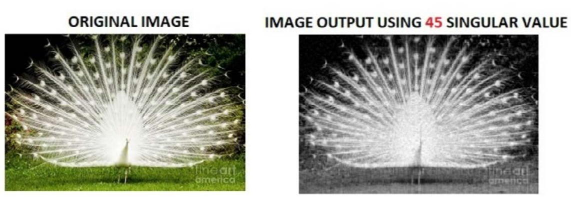
```
>>close all;
>>clear all;
A=imread ('C: /Users/User/Pictures/white-peacock-dustin-k-ryan.jpg');
B=rgb2gray(A);
D=double(B);
```

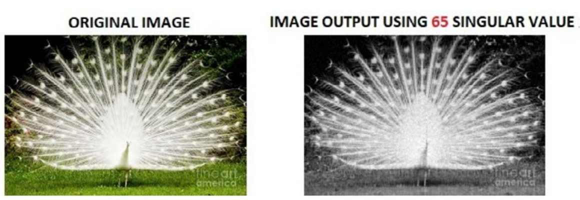
```
[U, S, V ]=svd(D);
dispEr=[];
numSVals=[];
for N=5:20:100
C=S;
C(N+1:end,:)=0;
C(:,N+1:end)=0;
D=U*C*V';
figure;
subtitle('SINGULAR VALUE DECOMPOSITION');
subplot(2,3,1),imshow(A),title('ORIGINAL IMAGE');
buffer=sprintf('IMAGE OUTPUT USING "%d"
SINGULAR VALUE',N); subplot(2,3,3),imshow(uint8(D
)),title(buffer);
error=sum(sum((D-D).^2));
dispEr=(dispEr:error);
numSVals=(numSVals:N);
end
```

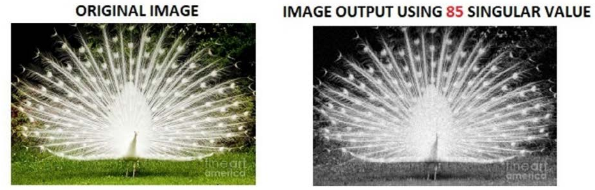
OUTPUT:
Singular Value Decomposition











4. Conclusion

In this paper, we conclude that the image is effectively calculated by using Singular Value Decomposition. "Singular Value Decomposition provides good compression ratio and also a practical solution to image compression problem". The output displayed from original image converted into JPEG image compression for different integer values using MATLAB. Also, the time compression between manual calculation and using MATLAB shows that MATLAB is time saving. JPEG is one of the best file formats in image compression because, every time the man can save his image as JPEG.

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