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Complement of Interval Valued Intuitionistic Fuzzy Graphs

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Abstract

In this paper, the Complement of Interval Valued Intuitionistic Fuzzy Graph (CIVIFG) is introduced and a few properties are analyzed. Several characteristics of self-complement, self weak complement and self co-weak complement of interval-valued intuitionistic fuzzy graphs are also investigated.

Keywords: Complement of Fuzzy Graph, Complement of Interval-valued Intuitionistic Fuzzy Graph (CIVIFG), Fuzzy Graph, Interval-valued Intuitionistic Fuzzy Graph (IVIFG)

1. Origin of the Study

Zadeh [1] introduced the concept of uncertainty in 1965, which was described by a fuzzy set. Now-a-days the theory of fuzzy sets turned out to be a significant concept of investigation in different areas including Logic, Topology, Algebra, Analysis etc.

Rosenfeid [2] introduced fuzzy graphs and obtained several theoretical concepts. Sunitha and Vijayakumar [3] studied the properties of complementary fuzzy graphs. The interval-valued fuzzy set is an expansion of fuzzy set, in which the degrees of association are the intervals of members. Atanassov [4] initiated intuitionistic fuzzy set which is represented by association and non-association values.

Muhammad, Akram, and Wieslaw A. Dudek [5] defined the interval-valued fuzzy graphs and a few operations on them. Talebi, A. A., and H. Rashmanlou [6] studied the properties of isomorphism and CIVFG. A. Mohamed Ismayil and Mohamed Ali [10] studied the strong interval-valued intuitionistic fuzzy graphs. Section 2 includes basic definitions related to the study. Section 3 introduces the description of CIVIFGs. The features of self-complement, self weak and co-weak complement of CIVIFG are analyzed. Section 4 concludes the paper.

2. Basic Definitions

Definition 2.1 [1]
The boundaries of fuzzy sets are not accurate in the membership function which is defined by $0 \le \mu_A(x) \le 1$.

Definition 2.2 [2]
A *fuzzy graph* $G = (V, E)$ is a set of functions $V : X \to [0,1]$ and $E : X \times X \to [0,1]$ such that $\mu(x, y) \le \mu(x) \Lambda \mu(y)$ for all x and y in V. Λ represents the minimum.

Definition 2.3 [5]
Let D[0,1] be the closed sub intervals of [0,1] and Let $M = [M_L, M_U]$ which belongs to [0,1]. M_U and M_L denote the upper limits and lower limits of M.

Definition 2.4 [7]
If $G \cong \bar{G}$ then the fuzzy graph is said to be *self-complement*. That is, there exists a isomorphism f: $G \to \bar{G}$.

Definition 2.5 [5]
An IVFG is a pair of functions $G = (A, B)$ of $G^* = (V, E)$ such that, $A = [M_{AL}, M_{AU}]$ & $B = [M_{BL}, M_{BL}]$ denote the IVFSs on V and the fuzzy relation on E respectively, where M_{AL}, M_{AU}: $V \to D[0, 1]$ & M_{BL}, M_{BU}: $V \times V \to D[0,1]$ such that

$$M_{AL}(x) \le M_{AU}(x) \ \forall\, x \in V$$
$$M_{BL}(xy) \le (M_{AL}(x) \Lambda M_{AL}(y))$$
$$M_{BU}(xy) \le (M_{AU}(x) \Lambda M_{AU}(y)) \quad \forall\, x, y \in E.$$

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Example 2.1[5]

G = (A, B) be a fuzzy graph such that V = {a, b, c}, E = {ab, bc, ca}, where A and B are the IVFS of V and relation of $E \subseteq V \times V$ respectively.

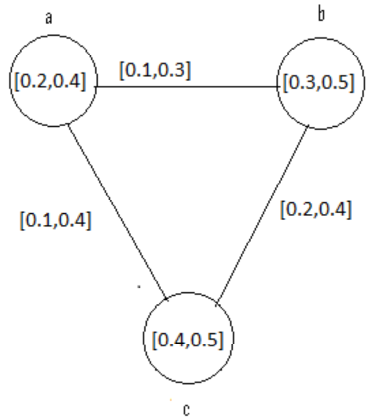

Figure 1. Interval-valued Fuzzy Graph.

Definition 2.6 [10]

An IFG $G = (A, B)$ satisfies the following conditions:

1. $M_A: V \rightarrow [0, 1]$ and $N_A: V \rightarrow [0,1]$ denote the membership and non-membership degrees of $y \in V$ such that, $\mathbf{0} \leq \boldsymbol{M_A(y) + N_A(y)} \leq \mathbf{1}\ \forall\ \boldsymbol{y}, \in \boldsymbol{V}.$
2. $M_B: V \times V \rightarrow [0.1]$ and $N_B: V \times V \rightarrow [0,1]$ are defined by, $\boldsymbol{M_B(xy) \leq (M_A(x) \wedge M_A(y))}$ and $\boldsymbol{N_B(xy) \geq max(N_A(x), N_A(y))}$

such that $\mathbf{0} \leq \boldsymbol{M_B(xy) + N_B(xy)} \leq \mathbf{1} \quad \forall\ \boldsymbol{xy} \in \boldsymbol{E}.$

Definition 2.7 [10]

An IVIFG of a graph $G = (A, B)$ holds the following conditions:

1. The functions $M_{AL}: V \rightarrow [0.1]$, $M_{AU}: V \rightarrow [0.1]$ and $N_{AL}: V \rightarrow [0,1] N_{AU}: V \rightarrow [0,1]$ are denotes the membership and non membership degrees of such that $\mathbf{0} \leq \boldsymbol{M_A(x) + N_A(x)} \leq \mathbf{1}$ for all $x \in V$.
2. The functions $M_{BL}: V \times V \rightarrow [0,1]$, $M_{BU}: V \times V \rightarrow [0,1]$, $N_{BL}: V \times V \rightarrow [0,1]$ and $N_{BU}: V \times V \rightarrow [0,1]$ are denoted by,

$$\boldsymbol{M_{BL}((xy) \leq (M_{AL}(x) \wedge M_{AL}(y))}$$
$$\boldsymbol{M_{BU}((xy) \leq (M_{AU}(x) \wedge M_{AU}(y))}$$
$$\boldsymbol{N_{BL}((xy) \geq \mathbf{max}(N_{AL}(x), N_{AL}(y))}$$
$$\boldsymbol{N_{BU}((xy) \geq \mathbf{max}(N_{AU}(x), N_{AU}(y))}$$

such that $\mathbf{0} \leq \boldsymbol{M_B(xy) + N_B(xy)} \leq \mathbf{1}\ \forall\ xy \in E.$

3. Complement of IVIFG

Definition 3.1

The CIVIFG $G = (A, B)$ is an IVIFG with $\bar{G} = (\bar{A}, \bar{B})$ where $\bar{\mathbf{A}} = \mathbf{A} = [\mathbf{M_A}, \mathbf{M_A}]$ & $\bar{\boldsymbol{B}} = [\overline{\boldsymbol{M_B}}, \overline{\boldsymbol{N_B}}]$ is defined by,

$$\overline{\boldsymbol{M_{BL}}}(\boldsymbol{xy}) = (\boldsymbol{M_{AL}}(\boldsymbol{x}) \wedge \boldsymbol{M_{AL}}(\boldsymbol{y})) - \boldsymbol{M_{BL}}((\boldsymbol{xy})$$

$$\overline{\boldsymbol{M_{BU}}}(\boldsymbol{xy}) = (\boldsymbol{M_{AU}}(\boldsymbol{x}) \wedge \boldsymbol{M_{AU}}(\boldsymbol{y})) - \boldsymbol{M_{BU}}((\boldsymbol{xy})$$

$$\overline{\boldsymbol{N_{BL}}}(\boldsymbol{xy}) = \boldsymbol{max}(\boldsymbol{N_{AL}}(\boldsymbol{x}), \boldsymbol{N_{AL}}(\boldsymbol{y})) - \boldsymbol{N_{BL}}((\boldsymbol{xy})$$

$$\overline{\boldsymbol{N_{BU}}}(\boldsymbol{xy}) = \boldsymbol{max}(\boldsymbol{N_{AL}}(\boldsymbol{x}),\ \boldsymbol{N_{AU}}(\boldsymbol{y})) - \boldsymbol{N_{BU}}((\boldsymbol{xy})$$

such that $\mathbf{0} \leq \boldsymbol{M_B(xy) + N_B(xy)} \leq \mathbf{1} \quad \forall\ \boldsymbol{xy} \in \boldsymbol{E}.$

Definition 3.2

Let G_1 and G_2 be the IVIFG of $G = (A, B)$. The homomorphism of G_1 and G_2 is defined by a map such that $f: V_1 \rightarrow V_2$ holds the following:

a. $$\boldsymbol{M_{A_1L}(x) \leq M_{A_2L}(f(x))}$$
$$\boldsymbol{M_{A_1U}(x) \leq M_{A_2U}(f(x))}$$
$$\boldsymbol{N_{A_1L}(x) \geq N_{A_2L}(f(x))}$$
$$\boldsymbol{N_{A_1U}(x) \geq N_{A_2U}(f(x))}\ \textit{and}$$

b. $$\boldsymbol{M_{B_1L}(xy) \leq M_{B_2L}(f(x), f(y))}$$
$$\boldsymbol{M_{B_1U}(xy) \leq M_{B_2U}(f(x), f(y))}$$
$$\boldsymbol{N_{B_1L}(xy) \geq N_{B_2L}(f(x), f(y))}$$
$$\boldsymbol{N_{B_1U}(xy) \leq M_{B_2U}(f(x), f(y))}$$

for every $x \in V_1$, $xy \in E_1$.

Definition 3.3

Let G_1 and G_2 be the IVIFG8 of $G = (A, B)$. The weak isomorphism of G_1 and G_2 is defined by a map such that $f: V_1 \rightarrow V_2$ holds the following:

a. f is homomorphism.

b. $$\boldsymbol{M_{A_1L}(z) = M_{A_2L}(f(z))}$$
$$\boldsymbol{M_{A_1U}(z) = M_{A_2U}(f(z))}$$
$$\boldsymbol{N_{A_1L}(z) = N_{A_2L}(f(z))}$$
$$\boldsymbol{N_{A_1U}(z) = N_{A_2U}(f(z))}\ \forall\ \boldsymbol{z \in V_1}.$$

Definition 3.4

Let G_1 and G_2 be the IVIFG of $G = (A, B)$. The co - weak isomorphism of G_1 and G_2 is defined by a map such that $f{:}V_1 \to V_2$ holds the following:

a. f is homomorphism.

b. $$M_{B_1L}(xy) = M_{B_2L}(f(x), f(y))$$
$$M_{B_1U}(xy) = M_{B_2U}(f(x), f(y))$$
$$N_{B_1L}(xy) = N_{B_2L}(f(x), f(y))$$
$$N_{B_1U}(xy) = M_{B_2U}(f(x), f(y))$$

for all $x \in V_1$, $xy \in E_1$.

Definition 3.5

$G_1 = (A_1, B_1)$ & $G_2 = (A_2, B_2)$ be the IVIFG of $G = (V, E)$. An isomorphism of and is a mapping such that $f{:}V_1 \to V_2$ holds the following:

a. $$M_{A_1L}(x) = M_{A_2L}(f(x))$$
$$M_{A_1U}(x) = M_{A_2U}(f(x))$$
$$N_{A_1L}(x) = N_{A_2L}(f(x))$$
$$N_{A_1U}(x) = N_{A_2U}(f(x))$$

b. $$M_{B_1L}(xy) = M_{B_2L}(f(x), f(y))$$
$$M_{B_1U}(xy) = M_{B_2U}(f(x), f(y))$$
$$N_{B_1L}(xy) = N_{B_2L}(f(x), f(y))$$
$$N_{B_1U}(xy) = N_{B_2U}(f(x), f(y))$$

for all $x \in V_1$, $xy \in E_1$.

3.1 Self-complement IVIFGs

Definition 3.1.1

An IVIFG $G = (A, B)$ is said to be a self-complement then. $G \cong \bar{G}$ i.e., there exist a bijective homomorphism $f : G_1 \to \bar{G}$ such that for all $x, y \in V$.

i. $$M_{AL}(x_1) = \overline{M_{AL}}(f(x_1))$$
$$M_{AU}(x_1) = \overline{M_{AU}}(f(x_1))$$
$$N_{AL}(x_1) = \overline{N_{AL}}(f(x_1))$$
$$N_{AU}(x_1) = \overline{N_{AU}}(f(x_1))$$

ii. $$M_{BL}(x_1y_1) = \overline{M_{BL}}(f(x_1), f(y_1))$$
$$M_{BU}(x_1y_1) = \overline{M_{BU}}(f(x_1), f(y_1))$$
$$N_{BL}(x_1y_1) = \overline{N_{BL}}(f(x_1), f(y_1))$$
$$N_{BU}(x_1y_1) = \overline{N_{BU}}(f(x_1), f(y_1))$$

for all $x_1, y_1 \in V$, $x_1y_1 \in E$.

Theorem 3.1.2

The self-complement IVIFG $G = (V, E)$ satisfies the following conditions:

$$\sum M_{BL}(xy) = \frac{1}{2}\sum (M_{AL}(x) \wedge M_{AL}(y))$$
$$\sum M_{BU}(xy) = \frac{1}{2}\sum (M_{AU}(x) \wedge M_{AU}(y))$$
$$\sum N_{BL}(xy) = \frac{1}{2}\sum \max(N_{AL}(x), N_{AL}(y))$$
$$\sum N_{BU}(xy) = \frac{1}{2}\sum \max(N_{AU}(x), N_{AU}(y)) \quad \text{for every } x \neq y$$

Proof

If G be a self-complement of IVIFG, then (3.1.1) exists for all $x \in V$, $xy \in E$.

From Definition 3.1, the complement of IVIFGs can be written as,

$$M_{BL}(xy) = (M_{AL}(x) \wedge M_{AL}(y)) - M_{BL}(xy)$$
$$M_{BU}(xy) = (M_{AU}(x) \wedge M_{AU}(y)) - M_{BU}(xy)$$
$$N_{BL}(xy) = max(N_{AL}(x), N_{AL}(y)) - N_{BL}(xy)$$
$$N_{BU}(xy) = max(N_{AU}(x), N_{AU}(y)) - N_{BU}(xy)$$

That is,

$$\sum M_{BL}(xy) + \sum M_{BL}(xy) = (M_{AL}(x) \wedge M_{AL}(y))$$
$$\sum M_{BU}(xy) + \sum M_{BU}(xy) = (M_{AU}(x) \wedge M_{AU}(y))$$
$$\sum N_{BL}(xy) + \sum N_{BL}(xy) = \max(N_{AL}(x), N_{AL}(y))$$
$$\sum N_{BU}(xy) + \sum N_{BU}(xy) = \max(N_{AU}(x), N_{AU}(y))$$

From these equations, theorem 3.1.2 holds.

3.2 Self Weak Complement of IVIFGs

Definition 3.2.1

An IVIFG is self weak complement, then G is a weak isomorphism with its complement $\overline{G}$. That is, there exists a mapping which is bijective homomorphism $f: G \to \overline{G}$ such that

i. $M_{AL}(x) = \overline{M_{AL}}(f(x))$

$$M_{AU}(x) = \overline{M_{AU}}(f(x))$$

$$N_{AL}(x) = \overline{N_{AL}}(f(x))$$

$$N_{AU}(x) = \overline{N_{AU}}(f(x))$$

ii. $M_{BL}(xy) \leq \overline{M_{BL}}(f(x), f(y))$

$$M_{BU}(xy) \leq \overline{M_{BU}}(f(x), f(y))$$

$$N_{BL}(xy) \geq \overline{N_{BL}}(f(x), f(y))$$

$$N_{BU}(xy) \geq \overline{N_{BU}}(f(x), f(y))$$

for every $x \in V$ and $xy \in E$.

Example 3.2.1

Let $V = \{h, i, j\}$ and $E = \{hi, ij\}$ are the components of G = (V, E). The self weak complement IVIFG $G = (A, B)$, where

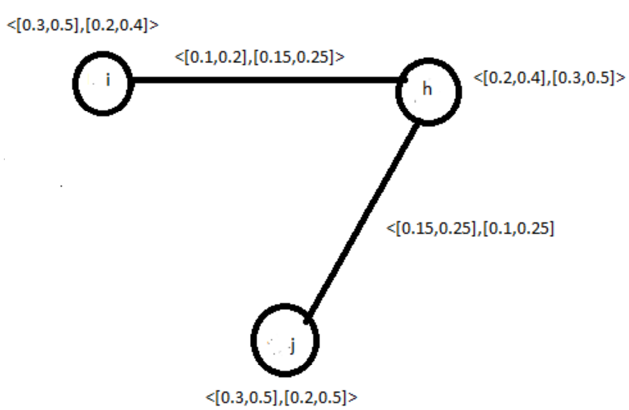

Figure 2. Self weak complement IVIFG.

Theorem 3.2.2

Let G be a self weak complement IVIFG then for every $x \neq y$,

$$\sum M_{BL}(xy) \leq \frac{1}{2} \sum (M_{AL}(x) \wedge M_{AL}(y))$$

$$\sum M_{BU}(xy) \leq \frac{1}{2} \sum (M_{AU}(x) \wedge M_{AU}(y))$$

$$\sum N_{BL}(xy) \geq \frac{1}{2} \sum \max(N_{AL}(x), N_{AL}(y))$$

$$\sum N_{BU}(xy) \geq \frac{1}{2} \sum \max(N_{AU}(x), N_{AU}(y))$$

Proof

If G is a self weak complement IVIFG, then Definition 3.2.1 holds for all $x, y \in V$.

From Definition 3.1, the complement of IVIFGs is defined by,

$$M_{BL}(xy) \leq \left(M_{AL}(f(x) \wedge M_{AL}(f(y)) \right) - M_{BL}(f(x), f(y))$$

$$M_{BU}(xy) \leq \left(M_{AU}(f(x) \wedge M_{AU}(f(y)) \right) - M_{BU}(f(x), f(y))$$

$$N_{BL}(xy) \geq max\left(N_{AL}(f(x), N_{AL}(f(y)) \right) - N_{BL}(f(x), f(y))$$

$$N_{BU}(xy) \geq max\left(N_{AU}(f(x), N_{AU}(f(y)) \right) - N_{BU}(f(x), f(y))$$

That is,

$$\sum M_{BL}(xy) + \sum M_{BL}(xy) \leq (M_{AL}(x) \wedge M_{AL}(y))$$

$$\sum M_{BU}(xy) + \sum M_{BU}(xy) \leq (M_{AU}(x) \wedge M_{AU}(y))$$

$$\sum N_{BL}(xy) + \sum N_{BL}(xy) \geq \max(N_{AL}(x), N_{AL}(y))$$

$$\sum N_{BU}(xy) + \sum N_{BU}(xy) \geq \max(N_{AU}(x), N_{AU}(y))$$

From these equations, theorem 3.2.2 is proved.

Remark 3.1

The co-weak complement of an IVIFG doesn't exist.

4. Conclusion

This paper introduces the complement of interval-valued intuitionistic fuzzy graphs (CIVIFG) and certain properties of self-complement, self-weak complement and self co-weak complement of IVIFGs. This paper helps to derive further results from IVIFGs and its complements.

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