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Fitting and Forecasting Mortality Rates for Egypt Applying the Lee-Carter Model

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Abstract:
This paper aims at using an extrapolative stochastic projection model namely "Lee-Carter Model" which is applied on mortality experience of general population in the context of Egypt, fitting the model to historical data, then estimating the model's parameters using Singular Value Decomposition (SVD), finally forecasting mortality trends in an Auto-Regressive Integrated Moving Average (ARIMA) framework. As regards to the longevity risk, I consider the possibility of changing the annuity benefits by relating the benefits to the updated mortality forecasts based on experienced mortality taking factors age and time into consideration therefore calculating the expected present values for pricing and reserving life annuities where the effect of mortality improvement is especially obvious in life annuity products.

The paper findings can benefit the actuary to deal with longevity risk in pricing & valuation of annuity products by measuring life expectancy where actuaries used mortality forecasts for cash flow projections and the assessment of premiums and reserves in life insurance and pension annuities. Setting the appropriate assumptions used in pricing & reserving which affecting the financial position of the annuity providers, as omission or miscalculation of the longevity risk could lead to many financial issues.

Keywords: *Lee-carter model, life annuities, longevity risk, mortality projection, and singular value decomposition*

1. Introduction

Mortality assumptions are important to many areas of actuarial practice such as life insurance business and maintenance of private and public pension programs. In recent years the actuaries' problem regarding mortality assumptions is that people are living longer than they were expected to live according to the life tables being used for actuarial computations by life insurance companies and pension funds for decades, which is known as "Longevity Risk" where unexpected level of mortality improvement has become an increasing challenge for life annuities business.

Longevity risk; is the risk to which annuity providers are exposed to pay out higher amount of benefits (life annuity payments) than expected in the future, this risk exists due to increasing life expectancy trends among policyholders and pensioners, due to many factors such as global economic development, improved standards of living, improved awareness of health behaviors and medical discoveries, where unexpected improvements in life expectancies may lead to severe solvency issues and many financial problems for annuity providers enough to quantify longevity as a major risk. Traditionally, longevity risk is viewed as non-hedge able leading to underestimating of life annuity products' premium(Cui, 2007).

Mortality assumptions used by actuaries in pricing and reserving life annuity products in Egypt is based on foreign life tables and ignoring the improvement in mortality rates overtime that could lead to severe solvency issues for annuity providers in Egypt. As a result, the need for robust and reliable models for mortality projection has become a growing issue among actuaries and policy makers.

Egyptian life expectancies at birth for both males and females were less than (45) years at the beginning of the twentieth century and have increased rapidly to be higher than (65) at the end of the twentieth century, and continues to reach around (70) in 2016, life expectancy increment is about 20 years over 100 years which is about (0.2) years annually, as shown in the following figure;

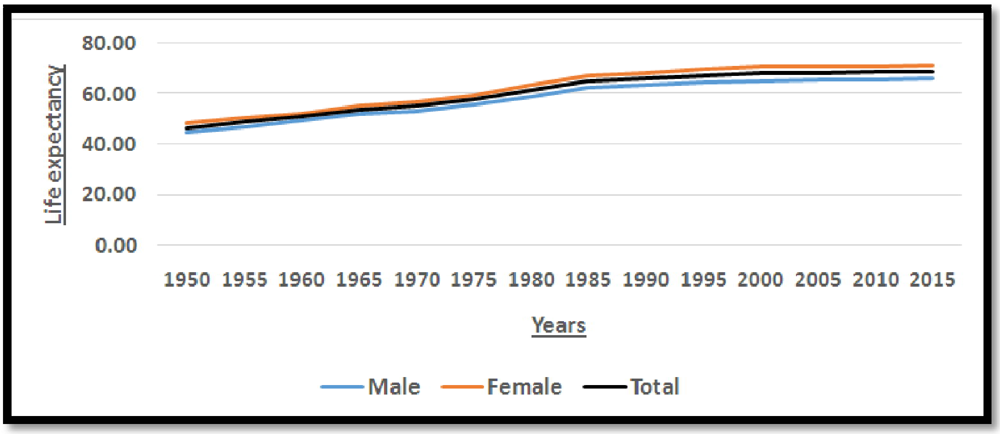

Figure 1: "History of the Egyptian Life Expectancy at Birth" (WHO, 2015)

From the above figure (1), it is clear that;

- Higher life expectancy for females relative to males.
- It also shows a consistent increase in life expectancies over the years, many believed that the lifetime will be (75) soon and so life annuity products should be designed taking age and time factors into consideration.
- There are no any extreme jumps in the Egyptian life expectancies either up or down due to extreme events which could harm the representativeness of the dataset for future estimations.

For example, a person aged 50-years old had a life expectancy of 29.5 years {e_{50}= 29.5} based on deterministic life table depending on factor age only. While based on projected life table taking factors age and time into consideration had a life expectancy of 33.6 years {e_{50}= 33.6}, representing an increase by about 4 years out of 20 years, i.e. more than 2 months annually. The trends in improving mortality among the elderly are significantly challenging public pension plans as well as private pension funds and life insurers.

In order to reduce longevity risk, actuaries are trying to develop better models for mortality improvement by being more aware for levels of uncertainty involved in the forecasts, and also policy makers must ensure that mortality assumptions adequately reflect the mortality of the population and encourage active assessment and monitoring of longevity assumptions by pension funds and annuity providers in order to avoid any unexpected increase in future payments related to the underestimation of longevity.

Therefore, this paper is concerned at using an extrapolative stochastic projection model namely "Lee-Carter Model" which is applied on the mortality experience in the context of Egypt.

2. The Data

The mortality experience for the Egyptian population is obtained from Central Agency for Public Mobilization and Statistics (CAPMS) which is the official statistical agency of Egypt that collects, process and analyzes statistical data and conducts the census, established since 1964.Also, the mortality data is given by five-year age groups, covering the period from 1990 to 2015.

It is believed that population data are the most reliable for tracking the historical improvement in mortality rates, especially in countries where the amount of data on annuitants is relatively small and not sufficient for mortality trend studies.

At a specific point of time, the insured and general populations could share a similar shape of mortality profile, but the level of mortality for the insured is typically lower than that for the general population, mainly due to the underwriting procedures employed prior the inception of the insurance policies.

The following figure shows the Egyptian log mortality rates of the general population over all age groups for the past twenty-six years (1990 – 2015) (Male, Female & Total) used to fit the Lee-Carter model;

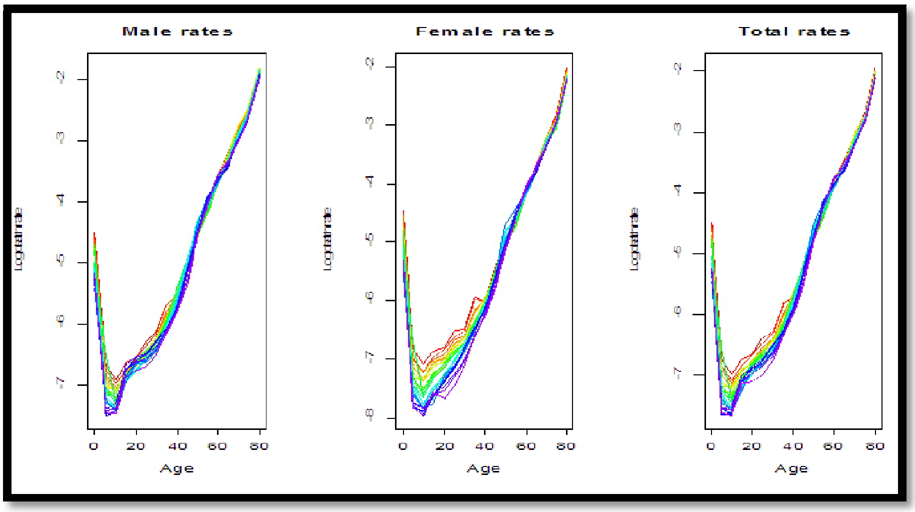

Figure 2: Log Mortality Rates for the General Population in Egypt (Male, Female, and Total)

From the above figure (2), it is clear that;

- The pattern of mortality rates for the male, female and total is similar.
- The mortality rates are increasing smoothly with age, the exception to this generalization is in the age group (0–5), due to infant mortality.

The following figures show the trend of the Egyptian mortality rates from 1990 to 2015 for age groups (30-35) & (40-45), classified into male, female and total;

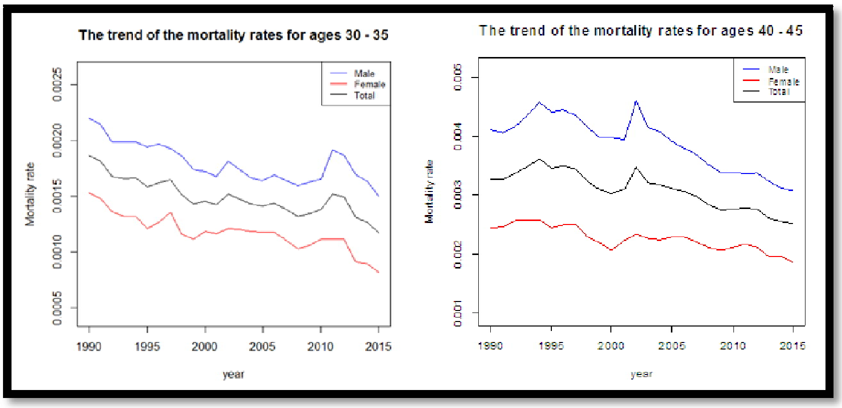

Figure 3

From the above figure 3, it is clear that;

- The mortality rates are deteriorated overtime, where mortality improvements have been noted at advanced ages.
- Egyptian data confirms that mortality is declining at all ages overtime with different behavior according to different ages.

2. The Lee-Carter Model

This model can be described as a "Statistical association model which combines a demographic model with statistical time series analysis", which is the most widely used model all over the world for the extrapolation of trends and age patterns in mortality(Maxwell & Andera, 2004).

The Lee-Carter methodology for forecasting mortality rates is a simple bilinear model in the variables x (age) and t (time). The model is defined as (Wang J. Z., 2007):

$$\ln(m_{x,t}) = \alpha_x + \beta_x k_t + \epsilon_{x,t}$$

Where:

$m_{x,t}$ → The central death rate at age (x) in year (t).

α_x → Age-specific parameter that represent the average pattern of mortality by age overtime.

β_x → Age-specific parameter that capture the sensitivity of $\ln(m_{x,t})$ to changes in the mortality index (k_t).

k_t → Time varying parameter that signifies the speed of mortality improvement. (k_t) is an index of the level of mortality at time (t).

$\epsilon_{x,t}$ → The residual term at age (x) and time (t), is there to capture that part of the death rate which is not captured by the model, and has a mean of zero and variance σ_ϵ^2.

The time component (k_t) captures the main time trend on the logarithmic scale in mortality rates at all ages, the model includes no assumption about the nature of the trend in (k_t).

The age component (β_x) modifies the main time trend according to whether change at a particular age is faster or slower than the main trend. In principle, not all the (β_x) need have the same sign, in which case movement in opposite directions could occur. The model assumes that (β_x) is invariant overtime (Debón, Martínez-Ruiz, & Montes., 2011).

In order to obtain a unique solution for the system of equations of the model, (α_x) is set equal to the averages overtime of the $\ln(m_{x,t})$, the square values of (b_x) sum to unity, and (k_t) values sum to zero, these are of the forms (Wang & Yue, 2015):

$$\alpha_x = \frac{\sum_t \ln(m_{x,t})}{T} \sum_x b_x^2 = 1 \sum_t k_t = 0$$

The implementation of the model for mortality forecasting can be performed in two stages:

2.1. In the First Stage "Fitting the Model"

In the first stage, $\{\alpha_x\}$, $\{\beta_x\}$ and $\{k_t\}$ have been estimated using historical mortality data. Since all parameters on the right-hand side of the model $\{\alpha_x, \beta_x$ and$k_t\}$ are unobservable, as a result the model cannot be fitted by simple methods like simple regression or ordinary least squares (Lee, 2000).

To solve this problem, alternative approaches have been proposed as Singular Value Decomposition (SVD).

2.2. In the Second Stage "Forecasting"

When the parameters have been determined, a forecast can be made. Since $\{\alpha_x, \beta_x\}$ are time-invariant, the focus will be on (k_t).

Fitted values of (k_t) are interpreted as a time series which can be modelled by an appropriate autoregressive integrated moving average process (ARIMA). Then we extrapolate (k_t) through the fitted ARIMA model to obtain a forecast of future death rates (Nocito).

3. Estimation Approach

Lists of approaches have been proposed on how the parameters in the Lee-Carter model can be estimated. This model cannot be fit by ordinary regression methods as none of the variables on the right-hand side of the model are observable. Singular Value Decomposition (SVD)is used as a widely used method for estimation.

In this section the SVD is considered, which is easier to implement than the other methods and give satisfactory results. The Lee-Carter is applied in a two-stage estimation procedure:

3.1. The First Stage

Where the singular value decomposition (SVD) is applied to the matrix of $\{\ln(m_{x,t}) - \hat{\alpha}_x\}$ to obtain estimates of (β_x) and (k_t). The whole application of the SVD method algorithm is relatively simple and follows six steps (Wang J. Z., 2007):

- Step (1): Using this method of SVD, the parameter estimate ($\hat{\alpha}_x$) is obtained as the arithmetic average of $\ln(m_{x,t})$ overtime, as shown below;

$$\ln(m_{x,t}) = \alpha_x + \beta_x k_t + \epsilon_{x,t}$$

$$\sum_t \ln(m_{x,t}) = \sum_t (\alpha_x + \beta_x k_t + \epsilon_{x,t})$$

$$\sum_t \ln(m_{x,t}) = T.\alpha_x + \beta_x.\sum_t k_t + \sum_t \epsilon_{x,t}$$

$$\{\epsilon_{x,t} \sim N(0, \sigma^2)\}$$

(Since it is assumed that the residual term average is Zero)

$$\{\sum_t \kappa_t = 0\}$$

$$\sum_t \ln(m_{x,t}) = T.\alpha_x + \beta_x.(0) + (0)$$

$$\sum_t \ln(m_{x,t}) = T.\alpha_x$$

Then,

$$\hat{\alpha}_x = \frac{\sum_t \ln(m_{x,t})}{T}$$

- Step (2): Create a matrix $\mathbb{Z} = \{\mathbb{Z}_{x,t}\}$ for estimating (β_x) and (k_t), where;

$$\mathbb{Z}_{x,t} = \ln(m_{x,t}) - \hat{\alpha}_x$$

$$\mathbb{Z}_{x,t} = \beta_x.k_t$$

- Step (3):Apply the SVD to matrix ($\mathbb{Z}_{x,t}$) which decomposes the matrix of ($\mathbb{Z}_{x,t}$) into the product of three matrices:

$$ULV^{\backslash} = SVD(\mathbb{Z}_{x,t}) = L_1 U_{x1} V_{t1} + \dots + L_X U_{xX} V_{tX}$$

Where,

U→ represents the age component.
L→ represents the singular values.
V→ represents the time component.

- Step (4): The parameter estimate ($\hat{k}_t$) is derived from the first vector of the time component matrix and the first singular value ($\hat{k}_t = L_1 V_{t1}$) and ($\hat{\beta}_x$) is derived from the first vector of the age-component matrix ($\hat{\beta}_x = U_{x1}$).
- Step (5): Lee-Carter approximate a new matrix ($\widehat{Z}_{x,t}$)by the product of the estimated parameters ($\hat{\beta}_x$) and ($\hat{k}_t$), to get

$$\hat{Z}_{x,t} = \hat{\beta}_x \times \hat{k}_t.$$

$$\hat{Z}_{x,t} = \hat{\beta}_x \times \underbrace{\begin{bmatrix} \hat{Z}_{x_1t_1} & \hat{Z}_{x_1t_2} & \cdots & \hat{Z}_{x_1t_n} \\ \hat{Z}_{x_2t_1} & \hat{Z}_{x_2t_2} & \cdots & \hat{Z}_{x_2t_n} \\ \cdots & \cdots & \cdots & \cdots \\ \hat{Z}_{x_mt_1} & \hat{Z}_{x_mt_2} & \cdots & \hat{Z}_{x_mt_n} \end{bmatrix}}_{\hat{k}_t}$$

- Step (6): Estimate of the logarithm central death rate;

$$\ln(\hat{m}_{x,t}) = \hat{a}_x + \hat{Z}_{x,t}$$

$$\ln(\hat{m}_{x,t}) = \hat{a}_x + \hat{\beta}_x . \hat{k}_t$$

Estimating Lee-Carter model parameters by using SVD method is implemented in the R-package.

3.2. The Second Stage

The time series of (k_t) is re-estimated by the method of so called "second stage estimation". Lee and Carter noticed that once (β_x) and (k_t) have been estimated, the observed total number of deaths $D_t \equiv \sum_x D_{x,t}$ is not guaranteed to be equal to the fitted number of deaths. Therefore, they made a second stage estimation of (k_t) by finding a value that makes the observed number of deaths equal to the predicted number of deaths. That is, searching for k_t such that [4]:

$$D_t = \sum_x \{ (e^{\hat{\alpha}_x + \hat{\beta}_x . \hat{k}_t}) * N_{x,t} \}$$

Where,

D_t → is the total number of deaths in year (t).
$N_{x,t}$ → is the population (exposure to risk) of age (x) in year (t).
$\hat{\alpha}_x, \hat{\beta}_x$ And $\hat{k}_t$ are the estimates of α_x, β_x, and k_t respectively.

There are several advantages to make a second stage estimate of the (k_t) parameter in this way. First, this guarantees that the life tables fitted over the sample years will fit the total number of deaths and the population age distributions. Second, the empirical time series of (k_t) can be extended to include years for which age-specific data on mortality are not available because the second stage estimate of (k_t) permits an indirect estimate of mortality which is very useful for some developing countries with less complete data (Wang J. Z., 2007).

However, the reason for the differences in the observed number of deaths and the fitted number is that the estimates of (k_t) are computed by minimizing the least square error over log-mortality rather than mortality itself. As a result, age groups with small numbers of death had the same weight as age groups with large numbers of death, even though they contributed very little to the overall mortality (Lee, 2000).

4. Fitting the Lee-Carter Model

In this section the Lee-Carter model parameters (α_x, β_x and k_t) is estimated using SVD, the following results is obtained based on the equations previously explained using R-software, demography and forecast packages to fit and forecast Lee-Carter model

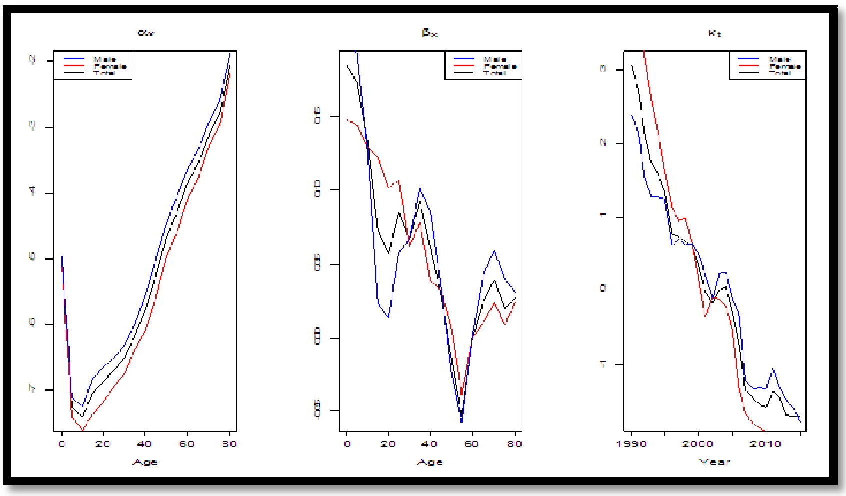

Figure 4: Fitted Values for the Parameters (α_x, β_x and k_t)
Classified Into Male, Female & Total

From the above figures, it is clear that;

- The average mortality rate grows for male, female and total as the age increases, indicated by the ($\hat{\alpha}_x$) pattern except for the hump that is as a result of the accidental.
- The ($\hat{\beta}_x$)describes the tendency of mortality at age x to change as the general level of mortality k_t changes. This indicates that when β_x is large for some x, the death rate at age x various a lot than the general level of mortality and vice versa.
- The shape of the total curve is more similar to that of the female curve. That is, the magnitude of the fitted values differs, but the pattern of increase or decrease in mortality rates with age, is similar. The above analysis of the β_x values reveals that the shape of the total curve has been influenced to a greater extent of the change in the female rates at various ages.
- The ($\hat{k}_t$) Captures the main trend on logarithmic scale in death rates at all ages and as expected, has a decreasing trend with increment with time.

5. Forecasting Mortality Rates

One advantage of the Lee-Carter approach is that once the data are fitted to the model and the values of the vectors ($\hat{\alpha}_x$, $\hat{\beta}_x$ and $\hat{k}_t$) are found; only the mortality index $\hat{k}_t$ needs to be predicted.

In order to forecast future mortality rates, Lee and Carter assumes that $\hat{\alpha}_x$ and $\hat{\beta}_x$ remains constant over time and the mortality index $\hat{k}_t$ is intrinsically viewed as a stochastic process where the dynamics of $\hat{k}_t$ are modeled by Auto-Regressive Integrated Moving Average (ARIMA) models (Gitau, 2017).

In this paper, the mortality index ($\hat{k}_t$) will be forecasted by a standard univariate time series model, ARIMA (0, 1, 0), which is sometimes known as Random Walk with Drift (RWD) model, that can be expressed as follows(Butt & Haberman, 2009);

$\hat{k}_t = \hat{k}_{t-1} + \theta + \varepsilon_t$

Where,

θ → Is the constant drift term, known as the drift parameter measures the drift in the form of average annual deviations, and its maximum likelihood estimate is; $\hat{\theta} = \frac{\hat{k}_T - \hat{k}_1}{T-1}$, which means that $\hat{\theta}$ only depends on the first and last of $\hat{k}_t$ estimates.

$\boldsymbol{\varepsilon_t}$ →Does the error term represent the white noise in the stochastic process, independent identically distributed ~$N(0, \sigma^2)$.

Now using the fitted time series model for k_t using SVD to forecast it over the desired time period. The following figure illustrates fitted mortality index ($\hat{k}_T$) for Egypt from 1990 to 2015 and forecasted ones from 2015 to 2035 with its 80% confidence interval;

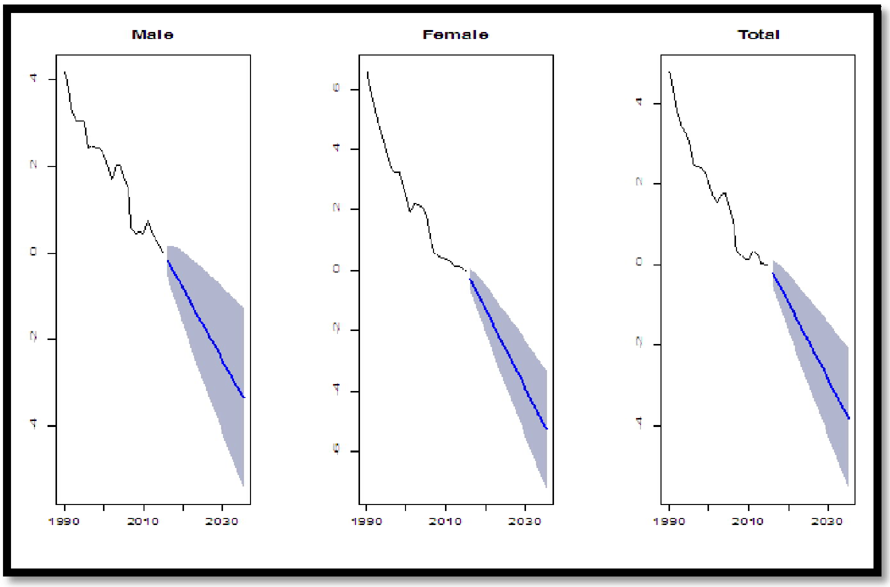

Figure 5: Forecasted ($\hat{k}_t$) Classified into Male, Female & Total

Finally, it's easy to derive the full pattern of mortality rates, past and forecasted mortality rates for people aged (30-35) & (40-45), the expected improvement is clearly visible as shown in the following figure;

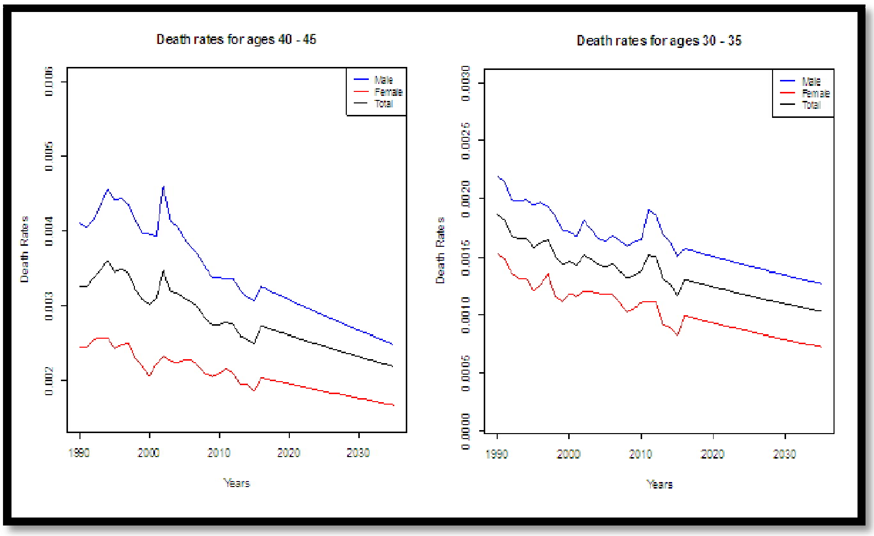

Figure 6: Mortality Rates Full Pattern for Age Groups (40 – 45) and (30 – 35)

6. Model Evaluation

Using the mean percentage error to examine the model goodness of fit, classified into male, female and total;

$$\text{MAPE} = \frac{1}{n}\sum_{i}\frac{[y_i - \hat{y}_i]}{y_i} * 100\%$$

6.1. Lee-Carter Analysis

ME	MSE	MPE	MAPE
-0.00196	0.00578	-0.00014	0.00974
Averages across years			
IE	ISE	IPE	IAPE
0.15172	0.42327	-0.00988	0.72691

Table 1: Female Data
Percentage Variation Explained: 85.4%
Error Measures Based On Log Mortality Rates
Averages across Ages

ME	MSE	MPE	MAPE
0.00154	0.00470	-0.00014	0.00976
Averages across years:			
IE	ISE	IPE	IAPE
0.11543	0.34837	-0.00918	0.73367

Table 2: Male Data
Percentage Variation Explained: 73.1%
Error Measures Based On Log Mortality Rates
Averages across Ages

ME	MSE	MPE	MAPE
0.00158	0.00445	-0.00013	0.00896
Averages across years			
IE	ISE	IPE	IAPE
0.12010	0.32711	-0.00849	0.66987

Table 3: Total Data
Percentage Variation Explained: 80.2%
Error Measures Based On Log Mortality Rates
Averages across Ages

From the above results, it is clear that;

- The MAPE of the fitted log mortality rates for males is approximately 1%, therefore the model fits reasonably well.
- The MAPE of the fitted log mortality rates for females is approximately 1%, therefore the model fits reasonably well.
- The MAPE of the fitted log mortality rates for totals is also approximately 1%, therefore the model fits reasonably well.

3. Forecasting Life Expectancy

Life expectancies are often used by demographers to measure the evolution of mortality. Specifically, $(e_{x,t})$is the average number of years that an x-aged individual in year (t) will survive, allowing for the evolution of mortality rates with time after (t). The formula giving $(e_{x,t})$is: (Girosi & King, 2007)

$$e_{x,t} = \int_{\omega \geq 0} \exp\left(- \int_{0}^{\omega} m_{x+h,t}\, dh \right)$$

$$= \frac{1 - \exp(-m_{x,t})}{m_{x,t}} + \sum_{i \geq 1} \left(\prod_{j=0}^{i-1} \exp(-m_{x+j,t}) \right) \frac{1 - \exp(-m_{x+i,t})}{m_{x+i,t}}$$

Where;

- The ratio $\left[\frac{1-\exp(-m_{x,t})}{m_{x,t}}\right]$is the average fraction of the year lived by an individual alive at age $(x+i)$.
- The product $\left[\prod_{j=0}^{i-1} \exp(-m_{x+j,t})\right]$ is the probability(${}_{i}p_{x,t}$) of reaching age $(x+i)$.

Figure (7) illustrates the Egyptian fitted life expectancy at birth (e_o) from 1990 to 2015 and the forecasted ones from 2016 to 2035 with 80% confidence interval based on the Lee-Carter model, for males. The values obtained with different models are very close with a slightly larger values coming from the Lee-Carter approach for projection of life expectancies.

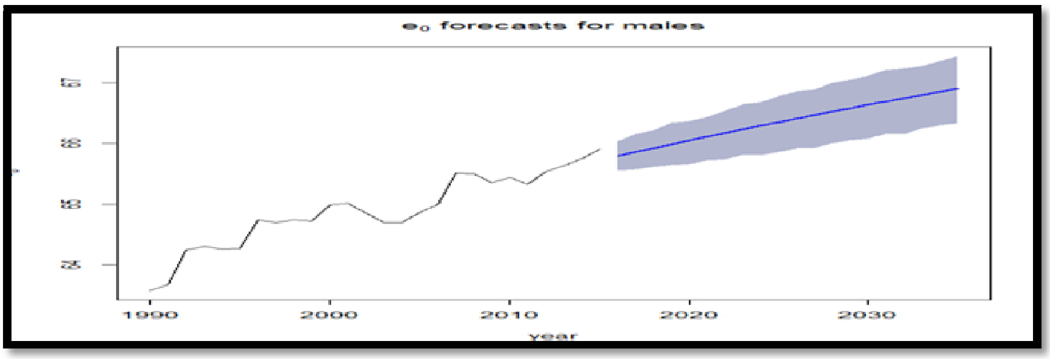

Figure 7: "Forecasting of (e_o) from 2016 to 2035 with 80% Confidence Interval for Males"

Figure (8) illustrates the fitted Egyptian life expectancy at birth (e_o) from 1990 to 2015 and the forecasted ones from 2016 to 2035 with 80% confidence interval, for females.

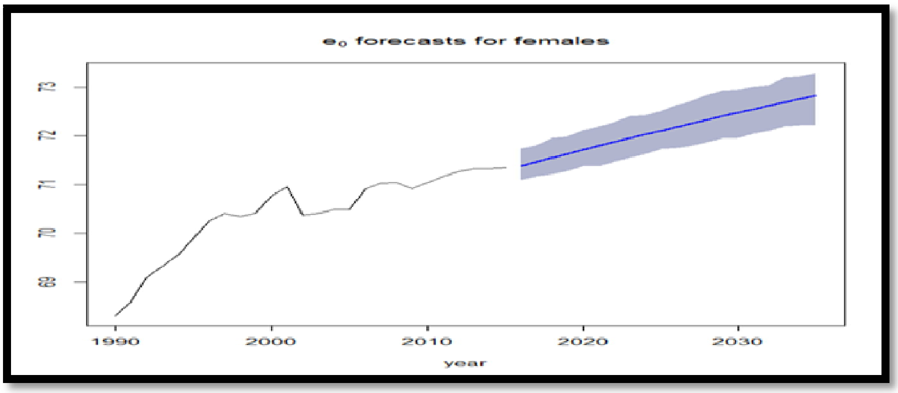

Figure 8: "Forecasting of (e_o) from 2016 to 2035 with 80% Confidence Interval for Females"

Figure (9) illustrates the Egyptian fitted life expectancy at birth (e_o) from 1990 to 2015 and the forecasted ones from 2016 to 2035 with 80% confidence interval, for both combined.

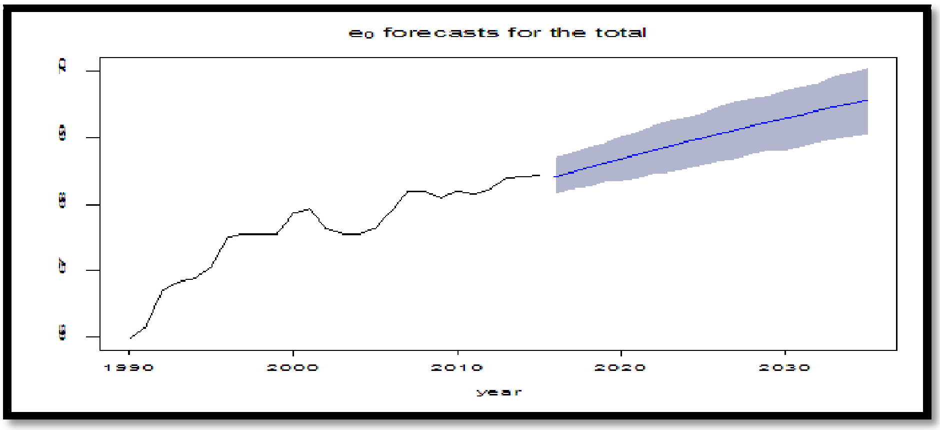

Figure 9: "Forecasting of (e_o) from 2016 to 2035 with 80% Confidence Interval for Both Sexes Combined"

From the above figures (7&8&9), it is clear that;

- The life expectancies at birth for females are higher than that for males keeping all other factors constant.
- It also shows a consistent increase in life expectancies over the years.
- There are no any extreme jumps in the Egyptian life expectancies either up or down due to extreme events which could harm the representativeness of the dataset for future estimations.
- For males, the life expectancy at birth was less than (63) at 1990 and have increased rapidly to be higher than (66) at 2015, life expectancy increment is about one month and half annually, and is expected to increase in the future.
- For females, the life expectancy at birth was less than (68) at 1990 and have increased rapidly to be higher than (72) at 2015, life expectancy increment is about two months annually, and is expected to increase in the future.

4. Conclusions

- The global phenomenon of longevity has been affecting Egypt over the last decade and is predicted to affect the country over the next decades.
- Life expectancy for males and females has been increasing steadily over the last several decades. Both genders have been experiencing an acceleration of mortality improvement over the last decade, many social and economic factors have contributed to this situation and are likely to continue to do so.
- The paper reveals a potential shortfall of provisions for future annuity and pension payments for several of the standard mortality tables used in the Egyptian market based on this expected evolution and improvement in mortality and life expectancy, this implies that the current contributions to the unfunded social security scheme, will not be sufficient to pay future pensions.
- In this paper, the Lee-Carter model is used to investigate mortality patterns in Egypt, which is viewed as the most efficient and transparent methods of modelling and projecting mortality improvements. Projection of life expectancies and actuarial present value is derived.
- Finally, the paper considers the possibility of changing the annuity benefits by relating the benefits to the updated mortality forecasts after implementing of Lee-Carter model on the experienced Egyptian mortality, this will enhance the calculation of the expected present values for pricing and reserving life annuities where the effect of mortality improvement is especially obvious in life annuity products.

5. Recommendations

- Based on the conclusions above, it is observed clearly that longevity risk exists in pension schemes and for annuity providers. In this regard, it is recommended that longevity risk management ideas should be implemented to enable management to measure and manage the changes and progress in future mortality.
- Since longevity risk is a major concern to annuity providers and pension plans, it is recommended to apply the Lee-Carter approach as it considered a significant departure from previous approaches: in particular it involves age and time factors and uses matrix decomposition to extract a single time-varying index for the level of mortality.
- The strengths of the Lee Carter model are its simplicity and robustness in the context of linear trends in age-specific death rates allowing changes in the time-varying component, $\{\kappa_t\}$. Therefore, it is recommended to monitor the experience and re-calibrate the model from time to time reflecting the latest experience on trends in mortality, so it should be regularly updated overtime to reflect the emerging experience.
- In order to avoid applying the result of a developed country to developing country, the annuity providers and pension plans in Egypt should use the Egyptian mortality data in fitting and forecasting the mortality rates. This will bring substantial benefit to the Egyptian insurance market in which regulators and insurance companies will finally have a benchmark that can be used in pricing and reserving life annuities.

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