# *THE INTERNATIONAL JOURNAL OF SCIENCE & TECHNOLEDGE*

# Mathematical Modelling of the Thermodynamical Functional Parameters of a Typical Hand-Driven Reciprocating Compressor

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*Abstract:*
*This study was based on the mathematical modelling of the thermodynamical functional parameters of a typical hand-driven reciprocating compressor. The fixed units of the compressor like: height of the cylinder, length of the piston, external diameter of the barrel, thickness of the barrel, diameter of the nozzle, internal diameter of the cylinder and radius of the cylinder were all measured using a Vernier caliper. Other functional parameters of the device such as the initial volume of the cylinder, final volume of the compressed air, area of the nozzle, area of the barrel and mass of air trapped in the barrel were computed using relevant formulas. The discharge pressure of the compressor was measured using a pressure gauge at various number of piston strokes for step of 5-unit interval. The exit temperature of air leaving the nozzle, compressive work done on the compressor system and heat dissipation quantity were computed using the ideal gas equation, first law of thermodynamics application to open system and steady-state energy equation respectively. Mathematical models were developed using exit air pressure as the response variable and number of piston stroke, exit air temperature, compressive work done and heat loss quantity as the independent variables. Statistical tools such as ANOVA and Pearson's correlation coefficient were employed in assessing the developed model's suitability. From the results obtained, each of these independent variables has significant effect on the discharge air pressure of the compressor. As the number of piston strokes increases, the pressure, temperature, compression work requirement and heat loss increase as well during the compression operation. This thus, proves that the compression operation of a reciprocating compressor is non-adiabatic. The models developed are significantly suitable for prediction of exit air pressure as endorsed by ANOVA and Pearson's correlation coefficient results.*

*Keywords: Reciprocating compressor, thermodynamics, regression analysis and open system*

## 1. Introductions

A hand-driven reciprocating compressor is a machine which functions by admitting fluids (gases) from a low-pressure region, compresses it, and delivers it at higher pressure to the point of requirement. It consists essentially of a cylinder, piston and valve system. The piston moves in a reciprocating manner in a closely fitted cylinder and the fluid quantity delivered by the compressor is equivalent to the swept volume/ fluid volume displaced by the piston. This class of compressors works by thermodynamically sucking air into the cylinder or barrel, compresses it and then issues it out to the tire. At the point of downward stroke operation, air inside the cylinder is compressed and forced down the tube of the compressor and then into the tire through the valve which is forced open by the pressure of the air. During the upward stroke, the piston is pulled up, the valve shuts off automatically so that air cannot escape from the tire, and new air is forced back into the cylinder and the cycle continues in same way for varying number of strokes of the piston.
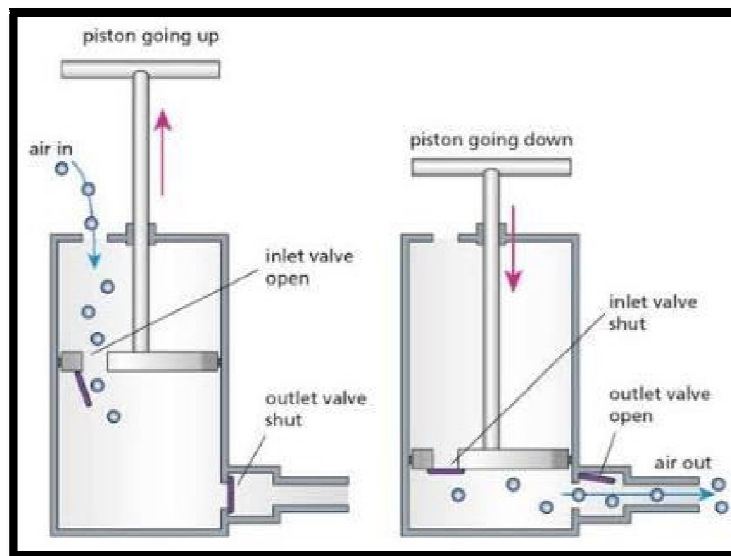
*Figure 1: Operational Mechanism of a Hand-Driven*
*Reciprocating Compressor*

Thermodynamics analysis of the operations inside the cylinder-piston unit and fluid system is an imperative sight that ought to be studied. It is a common observation that during the compression operation at higher number of piston strokes, the barrel of the compressor feels warmer indicating loss of heat which is as a result of viscous friction between the fluid (air) system and piston-cylinder unit of the machine. The rate at which the system dissipates heat solely depends on the number of piston strokes which is a function of work done on the system. In other words, the higher the work input to the system, heat energy is thus generated and loss expediently. Temperature which is the major drive of heat flow increases as well and the pressure of air in the tire being inflated rises concurrently. The mathematical modelling of this whole functional system considered at steady-state conditions is a help in understanding the parametric interactions of temperature, pressure, number of piston strokes, heat and work.

This study was focused on the mathematical modelling of the thermodynamically functional parameters of a typical hand-driven reciprocating compressor. The study was centered on the analysis of internal fluid behavioral properties of the compressor during the compression process, output effect and the development of a functional model relationship between the response variable-air pressure and each of the independent variables- number of piston strokes, fluid temperature, heat dissipation amount and work done on the system. The significance of each independence variable was determined using analysis of variance (AVOVA) statistical method.

## 2. Materials and Method

Dimensions of the fixed units of the compressor shown in figure 2 were measured. These units are: height of the cylinder, length of the piston, external diameter of the barrel, thickness of the barrel, diameter of the nozzle, internal diameter of the cylinder andradius of the cylinder. Table 1 shows these measured compressor units.



*Figure 2: A Reciprocating Hand-*
*Driven Compressor*

| S/N | Compressor Units | Values (mm) |
|---|---|---|
| 1 | Height of the cylinder (h) | 247 |
| 2 | Length of the piston (L) | 238 |
| 3 | External diameter of the cylinder | 33 |
| 4 | Thickness of the cylinder | 1 |
| 5 | Diameter of the nozzle | 3 |
| 6 | Internal diameter of the cylinder (d) | 31 |
| 7 | Radius of the cylinder (r) | 15.5 |

*Table 1: Measured Values of Some Fixed Units in the Compressor*

## 2.1. Computation of Other Essential Parameters of the Compressor

The values of other units of the compressor like: initial volume of the cylinder, final volume of the compressed air, area of the nozzle, area of the barrel and mass of air trapped in the barrel were computed using relevant formulas.

a.      Initial volume of the cylinder, $V_1 = \pi r^2 h$   (1)

$$= \pi[0.0155^2] \times 247 \times 10^{-3}$$
$$= 1.8643 \times 10^{-4} m^3$$

b.      Final volume of the compressed air, $V_2 \quad = \pi r^2[\text{barrel height} - \text{swept length}]$          (2)

$$= \pi r^2[h - L]$$
$$= \pi[0.0155^2] \times [247 - 238] \times 10^{-3}$$
$$= 6.8 \times 10^{-6} m^3$$

c.      Area of the barrel, $A_1 \quad = \pi d^2$          (3)

$$= \pi \left[\frac{0.031^2}{4}\right]$$
$$= 7.5477 \times 10^{-4} m^2$$

d.      Area of the nozzle, $A_2 = \frac{\pi d^2}{4}$

$$= \pi \left[\frac{0.003^2}{4}\right]$$
$$= 7.0686 \times 10^{-6} m^2$$

e.      Air mass inside the cylinder at the induction stroke = $\rho_{air}$ at $30°$ ambient temperature $\times$ initial volume of the cylinder
P(4)

= 1.1768 × 0.00018643
$$= 2.194 \times 10^{-4} kg$$

## 2.2. Experimental Procedures

The compressor was studied experimentally by measuring the values of air pressures inside a typical wheelbarrow tire with the aid of a pressure gauge at different number of pistons strokes up to 100 strokes at a step of 5. The fluid temperature, heat loss and work done on the compressor at the end of the compression operation were all calculated using relevant formulas. Ideal gas equation was employed in calculating the fluid temperature while the first law of thermodynamics equation applying to steady-state analysis of open systems was used in computing the values of the heat loss and work done on the compressor.

### 2.2.1. Exit Temperature Of The Fluid ($T_2$)

From the equation of state of a perfect gas, it had been proven that the product of gas pressure and volume equals the product of mass, gas constant and temperature of the gas being studied. The real gases are exemption to this rule. Hence, the air temperature leaving the nozzle of the compressor into the tire was computed at different number of piston strokes using the equation of state of an ideal gas by assuming air to behave like a perfect gas.

$$T_2 = \frac{PV}{mR}$$          (5)

### 2.2.2. Work Done on the Compressor (W)

During the compression operation, work is done on the compressor system by the operator without which the equipment is a static one. Compressive work done on the air system at different number of piston strokes were determined using equation 6.

$$W = mR(T_1 - T_2) = \frac{P_1V_1 - P_2V_2}{n-1}$$          (6)

Heat loss (Q): The cylinder of the reciprocating compressor feels warmer to touch during a repetitive compression operation. This thus proves that the process is non-adiabatic and as such, the quantity of heat dissipated was computed at various piston strokes using the steady-state energy equation. The difference between the kinetic and potential energies at inlet and outlet points of the compressor were neglected (Rogers and Mayhew, 1992)

$$Q = W + mC_p(T_1 - T_2)$$          (7)

## 3. Results and Discussion

The values of air pressure, temperature, compressive work and heat loss during the compression operation for different number of piston strokes are shown on table 2.

| S/N | NO OF STROKES (N) | DISCHARGE PRESSURE($P_2$) BAR | DISCHARGE TEMP.($T_2$)K | COMPRESSION WORK(W) J | HEAT LOSS(Q) J |
|-----|-----|-----|-----|-----|-----|
| 1.0 | 0.0 | 1.01325 | 10.94 | 0.000 | 0.000 |
| 2.0 | 10.0 | 2.80 | 30.24 | -1.215 | -5.471 |
| 3.0 | 15.0 | 4.20 | 45.40 | -2.170 | -9.768 |
| 4.0 | 20.0 | 5.60 | 60.50 | -3.121 | -14.049 |
| 5.0 | 25.0 | 7.00 | 75.60 | -4.071 | -18.328 |
| 6.0 | 30.0 | 8.30 | 89.64 | -4.956 | -22.309 |
| 7.0 | 35.0 | 9.70 | 104.80 | -5.910 | -26.606 |
| 8.0 | 40.0 | 11.10 | 119.88 | -6.860 | -30.881 |
| 9.0 | 45.0 | 12.50 | 135.00 | -7.812 | -35.167 |
| 10.0 | 50.0 | 13.90 | 150.12 | -8.764 | -39.453 |
| 11.0 | 55.0 | 15.30 | 165.24 | -9.716 | -43.739 |
| 12.0 | 60.0 | 16.70 | 180.40 | -10.671 | -48.036 |
| 13.0 | 65.0 | 18.00 | 194.40 | -11.552 | -52.004 |
| 14.0 | 70.0 | 19.40 | 209.52 | -12.504 | -56.290 |
| 15.0 | 75.0 | 20.80 | 224.64 | -13.456 | -60.576 |
| 16.0 | 80.0 | 22.20 | 239.90 | -14.417 | -64.902 |
| 17.0 | 85.0 | 23.60 | 254.90 | -15.362 | -69.154 |
| 18.0 | 90.0 | 25.00 | 270.00 | -16.312 | -73.434 |
| 19.0 | 95.0 | 26.40 | 285.12 | -17.265 | -77.721 |
| 20.0 | 100.0 | 27.80 | 300.24 | -18.217 | -82.007 |

*Table 2: Effects of Piston Stroke Variation on Air Pressure, Discharge Temperature, Compression Work and Heat Loss of a Reciprocating Compressor*

From table 2, it was inferred that as the number of piston strokes increases, the exit air pressure, temperature, compressive work and amount of heat loss increases as well. The negative sign on the values of compressive work and heat loss indicates that work was done on the compressor system and heat was lost during the process respectively.

## 4. Regression Analysis Result

A simple linear regression models were developed for each of the variables that affects the response factor – air pressure and ANOVA result were used to ascertain the significance of each model. The models developed are as follows.

$$P = 0.229 + 0.27N \tag{8}$$
$$P = 0.0926T \tag{9}$$
$$P = 1.01 - 1.47W \tag{10}$$
$$P = 1.01 - 0.327Q \tag{11}$$

| Predictor | Coef | SE Coef | T | P |
|-----------|------|---------|---|---|
| Constant | 0.22940 | 0.09604 | 2.39 | 0.028 |
| N | 0.274378 | 0.001604 | 171.09 | 0.000 |

*Table 3: Statistical Analysis between Air Pressure and Number of Piston Strokes (Model 8)*
*S = 0.209853   R-Sq. = 99.9%   R-Sq.(Adj.) = 99.9%*

| Source | DF | SS | MS | F | P |
|--------|-----|-----|-----|-----|-----|
| Regression | 1 | 1289.1 | 1289.1 | 29273.06 | 0.000 |
| Residual Error | 18 | 0.8 | 0.0 | | |
| Total | 19 | 1289.9 | | | |

*Table 4: Analysis of Variance for Model 8*

| Predictor | Coef | SE Coef | T | P |
|-----------|------|---------|---|---|
| Constant | -0.000757 | 0.001448 | -0.52 | 0.607 |
| T | 0.0925887 | 0.0000081 | 11484.87 | 0.000 |

*Table 5: Statistical Analysis between Air Pressure and Exit Temperature (Model 9)*
*S = 0.00312721   R-Sq. = 100.0%   R-Sq.(Adj.) = 100.0%*

| Source | DF | SS | MS | F | P |
|---|---|---|---|---|---|
| Regression | 1 | 1289.9 | 1289.9 | 1.31902E+08 | 0.000 |
| Residual Error | 18 | 0.0 | 0.0 | | |
| Total | 19 | 1289.9 | | | |

*Table 6: Analysis of Variance for Model 9*

| Predictor | Coef | SE Coef | T | P |
|---|---|---|---|---|
| Constant | 1.01217 | 0.00138 | 734.62 | 0.000 |
| W | -1.47040 | 0.00013 | -11434.11 | 0.000 |

*Table 7: Statistical Analysis between Air Pressure and Compressive Work (Model 10)*
*S = 0.00314109   R-Sq. = 100.0%   R-Sq.(adj.) = 100.0%*

| Source | DF | SS | MS | F | P |
|---|---|---|---|---|---|
| Regression | 1 | 1289.9 | 1289.9 | 1.30739E+08 | 0.000 |
| Residual Error | 18 | 0.0 | 0.0 | | |
| Total | 19 | 1289.9 | | | |

*Table 8: Analysis of Variance for Model 10*

| Predictor | Coef | SE Coef | T | P |
|---|---|---|---|---|
| Constant | 1.01216 | 0.00137 | 740.28 | 0.000 |
| Q | -0.326632 | 0.000028 | -11522.46 | 0.000 |

*Table 9: Statistical Analysis between Air Pressure and Heat Loss (Model 11)*
*S = 0.00311700   R-Sq. = 100.0%   R-Sq.(adj.) = 100.0%*

| Source | DF | SS | MS | F | P |
|---|---|---|---|---|---|
| Regression | 1 | 1289.9 | 1289.9 | 1.32767E+08 | 0.000 |
| Residual Error | 18 | 0.0 | 0.0 | | |
| Total | 19 | 1289.9 | | | |

*Table 10: Analysis of Variance for Model 11*

From tables 3, 5, 7 and 9, the constant terms and the coefficients of the independent variables are all significant in the models except the constant term of model 11. This is so because their respective p-values are less than the chosen $\alpha$-value of 0.05. The constant term of model 11 was removed since it was insignificant in the model. Also, from the ANOVA results shown on tables 4, 6, 8, and 10, the models are suitable for air-pressure estimations as its p-value is less than the 0.05. Sequel to the model analysis, the S-values for each model is significantly small attesting to the suitability of the models and the values of R-Sq. and R-Sq.(adj.) are all high proving the models predicting strength. Pearson's correlation coefficient between the measured and predicted pressure values for each model is unity. This thus, suggests that each of the developed model is suitable for air pressure prediction of a hand-driven reciprocating compressor.

## 5. Conclusion

The discharge air pressure of a hand-driven compressor is affected by the number of piston strokes, exit air temperature, compressive work done and heat lost during the operations. The thermodynamically properties of the air system changes as the operation process continues. The heat dissipation from the compressor is as a result of the frictional effect between the contacting surfaces of the cylinder and the piston during the course of induction and compression strokes of the compressor. The density of air at the exit section of the system is lower than that at the inlet. This is proved from the simultaneous rise of air temperature as the number of piston strokes increases, thus leading to more work requirement and heat loss quantity. Temperature and density of a working fluid/ substance are inversely related. The regression models developed are suitable for air pressure predictions of a hand-driven reciprocating compressor as endorsed by the statistical tools employed in analyzing the models.

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