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Non-Static Plane Symmetric Magnetized Cosmological Models in Bimetric Theory of Gravitation

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***Abstract**:*
Non-static plane symmetric cosmological models are investigated in Rosen's [Gen.Rel.Grav.Vol.4 (1973) 435] bimetric theory of gravitation in the context of cosmic string coupled with electromagnetic field, perfect fluid coupled with electromagnetic field and mesonic perfect fluid coupled with electromagnetic field. It is observed that the plane symmetric non-static cosmological model exists in 1st case but vacuum and false vacuum models are established in 2nd case. The result found in 3rd case leads to the result already obtained and studied by Mohanty and Sahoo [Czech. J.Phy. 52 (2002)1041]. It is interesting to note that there is no contribution to electromagnetic field in both 2nd and 3rd case. Some physical and geometrical behavior of the exhibited models is also discussed.
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1. Introduction

Rosen [1] has modified the general theory of relativity by introducing a second metric tensor corresponding to flat space-time. At each point of the space-time the metric tensor g_{ij} involved in the line element

$$ds^2 = g_{ij}dx^i dx^j \qquad (1)$$

Which is associated with a second metric tensor

$$d\sigma^2 = \gamma_{ij}dx^i dx^j \qquad (2)$$

Where ds is the interval between two neighbouring events as measured by a clock and a measuring rod. The interval $d\sigma$ is an abstract or a geometrical quantity which is not directly measurable. The Riemannian metric tensor g_{ij}describes the geometry of curved space-time which plays the same role as given in general relativity. The second metric tensor γ_{ij} refers to the background flat space-time, whose curvature tensor vanishes. Also it has no direct physical significance but appears in the field equations. Hence it describes a space-time of constant curvature. Moreover, the biometric theory also satisfies the covariant and equivalence principles. This theory agrees with the present observational facts pertaining to Einstein's theory of general relativity.

In recent days there has been a lot of interest in the study of cosmic strings which are topologically stable objects. These might be found during a phase transition in the early universe i.e. after the big-bang explosion as the temperature goes down below some critical temperature as predicted by Grand Unified Theories (Zel'dovich et al. [2]; Kibble [3, 4]; Everett[5], Vilenkin [6].It is believed that cosmic strings give rise to density perturbations leading to the formation of galaxies,(Zel'dovich [7]).These cosmic strings have stress energy and couple to the gravitational field. Thus it is interesting to study the effects of gravitation that arise from strings.

It is known that electromagnetic field which contains highly ionized matter plays a key role for description of the energy distribution in the universe. String magnetic fields may be created due to adiabatic compression in cluster of galaxies and cosmic anisotropies may be attributed to the large scale magnetic fields. It is believed that in anisotropic models, the presence of electromagnetic field can alter the rate of creation of the particles and directly affects the rate of expansion of the universe.

As in general relativity, the variation principle also leads to the conservation law

$$T^{ij}_{;j} = 0 \qquad (3)$$

Where (;) denotes covariant differentiation with respect to g_{ij}. Accordingly the geodesic equation of a rest particle is same as that of general relativity.
The field equations of bimetric theory of gravitation proposed by Rosen [1] are

$$N^i_j - \frac{1}{2} N \delta^i_j = 8\pi k T^i_j \qquad (4)$$

where

$$N^i_j = \frac{1}{2} \gamma^{ab} \left(g^{hi} g_{hj|a}\right)_{|b}$$

and

$$N = N^i_j, \quad (i, j=1, 2, 3, 4) ; \quad k = \sqrt{\frac{g}{\gamma}}$$

together with
g = determinant of g_{ij} and γ = determinant of γ_{ij}.
Here the vertical bar (|) denotes the covariant differentiation with respect to γ_{ij} and T^i_j is the energy momentum tensor of the matter.
Rosen[8-9], Yilmaz[10], Karade and Dhoble[11], Karade[12], Israelit[13-15], Liebscher[16], Reddy and Venkateswaralu[17], Deo and Thengane[18], Sahoo[19] and Mohanty et al.[20],Katore et al[21] ,Sahoo and Mishra[22,23] and Sahoo et al.[24] are some of the authors, who have studied various aspects of bimetric theory in different angles.
We know that Mohanty and Sahoo [25] have considered the problem of non-static plane symmetric meson field and mesonic perfect fluid in bimetric theory and found the cosmological model in 1st case where the scalar field becomes constant. However in later case they have shown that the bimetric theory does not admit perfect fluid but allows only mesonic scalar field where the scalar field is also constant.
To the best of our knowledge no author has studied the non-static plane symmetric space-time in the context of cosmic strings coupled with electromagnetic field, perfect fluid coupled with electromagnetic source and mesonic perfect fluid coupled with electromagnetic field. Therefore in this paper we are interested to study this problem in order to extend the work done by Mohanty and Sahoo [25].

2. Cosmic String coupled with Electromagnetic Field

The plane symmetric non-static metric in the general form is

$$ds^2 = e^{2h}(dt^2 - dr^2 - r^2 d\theta^2 - s^2 dz^2), \qquad (5)$$

where r, θ, z are cylindrical polar co-ordinates and h, s are functions of time 't'.
As the metric (5) is Riemannian (non-flat), it's background flat space-time metric is

$$d\sigma^2 = dt^2 - dr^2 - r^2 d\theta^2 - dz^2. \qquad (6)$$

2.1. Rosen's Field Equations

The energy momentum tensor for cosmic cloud string as Latelier [26] and Stachel [27] coupled with electromagnetic field is expressed as

$$T^i_j = T^i_{j\,string} + E^i_{j\,mag} \qquad (7)$$

where

$$T^i_{j\,string} = \rho u^i u_j - \lambda x^i x_j \qquad (8)$$

together with

$$u^i u_i = 1 = -x_i x^i \text{ and } u^i x_i = 0$$

and

$$E^i_{j\,mag} = F_{jr} F^{ir} + \frac{1}{4} F_{ab} F^{ab} g^i_j, \qquad (9)$$

where $E^i_{j\,mag}$ is electromagnetic energy tensor, F_{ij} is the electromagnetic field tensor, u^i is the four velocity vector of the string cloud, x_i represents anisotropic direction (i.e. the direction of strings) and ρ and λ are respectively the rest energy density and the tension density of the strings of cloud .
In the comoving co-ordinate system taking the string in X-axis, the energy momentum tensor (8) takes the form

$$T^1_{1\,string} = \lambda, \quad T^4_{4\,string} = \rho \quad \text{and} \quad T^i_{j\,string} = 0 \qquad (10)$$

for i, j = 2, 3 and i ≠ j.
The electromagnetic field is considered to be along X-axis, so that the only non-vanishing component of electromagnetic field tensor F_{ij} is F_{23}.
The first set of Maxwell's equation

$$F_{[ij,k]} = F_{ij,k} + F_{jk,i} + F_{ki,j} = 0 \qquad (11)$$

leads to the result F_{23}= constant = L (say).
Then

$E_{1\,mag}^{1}$ = - $E_{2\,mag}^{2}$ = - $E_{3\,mag}^{3}$ = $E_{4\,mag}^{4}$ = ζ (say) (12)
Where
$\zeta = \frac{L^2}{2s^2e^{4h}}$. (13)
So by (7), (10) and (12) the non-vanishing energy momentum tensors for a cosmic string coupled with an electromagnetic field along X-axis are
$T_1^1 = \lambda + \zeta$, $T_2^2 = T_3^3 = -\zeta$ and $T_4^4 = \rho + \zeta$. (14)
The Rosen's Field equations (4) for the metric (5) and (6) with the help of (14) are found as

$$\left(\frac{s_4}{s}\right)_4 + 2h_{44} = 16\pi k\,(\lambda + \zeta), \quad (15)$$

$$\left(\frac{s_4}{s}\right)_4 + 2h_{44} = -16\pi k\zeta, \quad (16)$$

$$\left(\frac{s_4}{s}\right)_4 - 2h_{44} = 16\pi k\zeta \quad (17)$$

And $\left(\frac{s_4}{s}\right)_4 + 2h_{44} = 16\pi k(\rho + \zeta)$. (18)

2.2. Solution of Field Equations
Equations (16) and (17) yield

$$\left(\frac{s_4}{s}\right)_4 = 0. \quad (19)$$

Integrating (19), we have

$$s = e^{a_1 t + a_2}, \quad (20)$$

where a_1 and a_2 are arbitrary constants and $a_1 \neq 0$.
On substitution of equation (19) in equations (15) to (18) we have

$$\lambda = \rho = -2\zeta \quad (21)$$

and

$$h_{44} = -8\pi k\zeta. \quad (22)$$

Substituting the value of ζ from (13) and the value of k = $\sqrt{\frac{g}{\gamma}} = s.\,e^{4h}$ in (22) and then integrating we get

$$h = a_3.\,e^{-(a_1 t + a_2)} + a_4\,t + a_5, \quad (23)$$

where a_3, a_4 and a_5 are arbitrary constants and $a_3 \neq 0$.

Also by the help of (13), (20) and (23) equation (21) yields

$$\lambda = \rho = -2\zeta = -L^2 e^{-[4a_3.e^{-(a_1 t + a_2)} + (4a_4 + 2a_1)\,t\, + (4\,a_5 + 2a_2)]}. \quad (24)$$

2.3. Model
The model for the space-time (5) corresponding to solutions (20) and (23) can be written as

$$\boxed{ds^2 = e^{2[a_3.e^{-(a_1 t + a_2)} + a_4\,t\, + a_5]}\left\{dt^2 - dr^2 - r^2 d\theta^2 - e^{2(a_1 t + a_2)} dz^2\right\}}, \quad (25)$$

where a_i ; i =1,2,3,4 and 5 are arbitrary constants and $a_1 \neq 0 \neq a_3$.
The model obtained in (25) is a Geometric string model.

2.4. Physical Properties

2.4.1. The Spatial volume V of the Universe
The spatial volume V of the universe for the model (25) is found to be

$$V = \sqrt{-g} = e^{[4a_3.e^{-(a_1 t + a_2)} + (4a_4 + a_1)\,t\, + (4\,a_5 + a_2)]}. \quad (26)$$

Now V→ constant as t→ 0 and V→∞ as t→∞.Thus the volume of universe increases as time increases. Hence it is inferred that the model starts with a constant volume and blows up at infinite future.

2.4.2. The Expansion Scalar θ
By using eqns. (20) and (23) the Expansion Scalar θ for the model (25) is found as

$$\theta = u^i_{;i} = -e^{-(a_3.e^{-(a_1 t + a_2)} + a_4\,t\, + a_5)}\left\{3a_1 a_3 e^{-(a_1 t + a_2)} - (3a_4 + a_1)\right\}. \quad (27)$$

As t → 0, θ→ constant and as t → ∞, θ → 0. Hence it is clear that the rate of expansion of the universe at initial epoch is constant. However the expansion becomes very slow as time increases and there will be no expansion at infinite time.

2.4.3. Anisotropy of the Universe

The shear scalar σ defined by Ray Choudhuri [28] as

$$\sigma^2 = \frac{1}{12}\left\{\left[\frac{g_{11,4}}{g_{11}} - \frac{g_{22,4}}{g_{22}}\right]^2 + \left[\frac{g_{22,4}}{g_{22}} - \frac{g_{33,4}}{g_{33}}\right]^2 + \left[\frac{g_{33,4}}{g_{33}} - \frac{g_{11,4}}{g_{11}}\right]^2\right\}. \quad (28)$$

Using (20), (28) yields

$$\sigma = \sqrt{\frac{2}{3}}(a_1) = \text{(constant)}. \quad (29)$$

From eqns.(27) and (29), we have

$$\frac{\sigma}{\theta} = -\frac{\sqrt{\frac{2}{3}}a_1 e^{a_3 . e^{-(a_1 t + a_2)} + a_4 t + a_5}}{3a_1 a_3 e^{-(a_1 t + a_2)} - (3a_4 + a_1)} . \quad (30)$$

Now $\frac{\sigma}{\theta} \rightarrow$ constant as $t \rightarrow 0$ and $\frac{\sigma}{\theta} \rightarrow \infty$ as $t \rightarrow \infty$

The shear scalar σ is constant throughout the evolution of the universe for all values of T. Hence this result indicates that the model is anisotropic in nature at the initial time and will continue throughout the evolution.

3. Perfect fluid coupled with an Electromagnetic source

The energy-momentum tensor for perfect fluid distribution with an electromagnetic field is given by

The energy-momentum tensor for perfect fluid distribution with an electromagnetic field is given by

$$T_j^i = T_{j(P)}^i + E_{j\,mag}^i \quad (31)$$

Where $E_{j\,mag}^i$ is described by (9) and $T_{j(P)}^i$ is the energy momentum tensor for perfect fluid distribution given by

$$T_{j(P)}^i = (p + \rho)u^i u_j - p g_j^i \quad (32)$$

together with $g_{ij} u^i u^j = 1$.

Here p and ρ are proper pressure and energy density of the fluid respectively and u^i is the four-velocity vector.

In comoving coordinate system eqn. (31) yields

$$T_1^1 = -p + \zeta, \quad T_2^2 = T_3^3 = -p - \zeta \quad \text{and} \quad T_4^4 = \rho + \zeta. \quad (33)$$

3.1. Rosen's Field Equations

The Rosen's field equations (4) for the metric (5) and (6) with the help of (33) are found as

$$\left(\frac{s_4}{s}\right)_4 + 2h_{44} = 16\pi k\,(-p + \zeta), \quad (34)$$

$$\left(\frac{s_4}{s}\right)_4 + 2h_{44} = -16\pi k\,(p + \zeta), \quad (35)$$

$$\left(\frac{s_4}{s}\right)_4 - 2h_{44} = 16\pi k\,(p + \zeta) \quad (36)$$

And $$\left(\frac{s_4}{s}\right)_4 + 2h_{44} = 16\pi k\,(\rho + \zeta). \quad (37)$$

3.2. Solution of field equations

Equations (34) and (35) yield

$$\zeta = 0. \quad (38)$$

Thus there is no contribution to magnetic field.

Also from equations (35) and (36), we get

$$\left(\frac{s_4}{s}\right)_4 = 0. \quad (39)$$

On integration, we find

$$s = e^{b_1 t + b_2}; \quad (40)$$

Where b_1 and b_2 are arbitrary constants and $b_1 \neq 0$.

Using equations (38) & (39) in equations (36) & (37) and solving we find

$$p + \rho = 0. \quad (41)$$

Case-1: p = ρ = 0. (Vacuum model)

Applying eqn. (42) in the field equations (34) to (37), we have

$h_{44} = 0.$ (43)

Integrating (43) we have

$h = b_3 t + b_4;$ (44)

Where b_3 and b_4 are arbitrary constants and $b_3 \neq 0$.

Therefore the vacuum model for the metric (5) with the help of (40) and (44) is

$ds^2 = e^{2(b_3 t + b_4)}\{dt^2 - dr^2 - r^2 d\theta^2 - e^{2(b_1 t + b_2)} dz^2\}.$ (45)

Case-2: ρ = - p ≠ 0. (False vacuum model) (46)

Here there are four field equations i.e. (34) to (37) with five unknowns. To obtain solution we take the help of the conservation law (3). Now eqn. (3) for the metric (5) reduces to

$\rho_4 + (p + \rho)\left\{3h_4 + \frac{s_4}{s}\right\} = 0.$ (47)

By substitution of (41), equation (47) yields

$\rho_4 = 0.$ (48)

On integration, (48) yields to

$\boldsymbol{\rho} = c$ (constant). (49)

By using (49), (46) yields

p = - c (constant). (50)

By $k = \sqrt{\frac{\xi}{\gamma}} = s.e^{4h}$, (40) and (50), (34) yields

$h_{44} = 8\pi c. e^{4h + b_1 t + b_2}.$ (51)

Substituting

$\beta = e^{4h + b_1 t + b_2}$ (52)

And

$\alpha = 32\pi c$ (Constant) (53)

equation (51) yields to

$\frac{1}{\beta}\frac{d}{dt}\left(\frac{1}{\beta}\frac{d\beta}{dt}\right) = \alpha.$ (54)

With the help of the substitution

$\frac{1}{\beta}\left(\frac{d\beta}{dt}\right) = \sqrt{w}.$ (55)

equation (54) gives the solution

$\beta = \frac{4}{[\sqrt{2\alpha} t + b_3]^2}.$ (56)

Where b_3 is arbitrary constant.

Equations (52) and (56) yield

$h = \frac{1}{4}\left[ln 4 - 2\ln\{\sqrt{2\alpha} t + b_3\} - b_1 t - b_2\right].$ (57)

Therefore the false vacuum model for the metric (5) with the help of (40) and (57) is

$ds^2 = e^{\frac{1}{2}[ln 4 - 2\ln\{\sqrt{2\alpha} t + b_3\} - b_1 t - b_2]}\{dt^2 - dr^2 - r^2 d\theta^2 - e^{2(b_1 t + b_2)} dz^2\}.$ (58)

3.3. Physical Properties

3.3.1. The Spatial volume V of the Universe

The spatial volume V of the universe for the model (58) is found to be

$V = \sqrt{-g} = \frac{4}{[\sqrt{2\alpha} t + b_3]^2}$ (59)

Now V→ constant as t→ 0 and V→ 0 as t→∞. This shows that the volume of the universe decreases as time increase. Thus we inferred that the universe starts from a constant volume and collapse at infinite future.

3.3.2. The Expansion Scalar θ:

The expansion scalar θ for the model (58) is found as

$$\theta = u^i_{;i} = e^{\frac{1}{4}[+2\ln\{\sqrt{2\alpha}\,t+b_3\}+\,b_1t+b_2-ln4]}\,\{\frac{b_1}{4} - \frac{3\sqrt{\alpha}}{\sqrt{2}(\sqrt{2\alpha}\,t+b_3)}\,\}. \tag{60}$$

Hence as t → 0, θ→ constant and as t → ∞, θ → ∞ . It is observed that the expansion scalar θ increases as time increase. Thus this result shows that the model has the constant rate of expansion at initial time but as time increases the rate of expansion increases.

.3.3. Anisotropy of the Universe

Using (40), (28) yields the shear scalar

$$\sigma = \sqrt{\frac{2}{3}}(b_1) = (\text{constant}). \tag{61}$$

The shear scalar (σ) is constant throughout the evolution of the universe for all values of t.

From (60) and (61) we have

$$\frac{\sigma}{\theta} = -\frac{\sqrt{\frac{2}{3}}b_1 e^{-\frac{1}{4}[-ln4+2\ln\{\sqrt{2\alpha}t+b_3\}+\,b_1t+b_2]}}{\frac{b_1}{4}-\frac{3\sqrt{\alpha}}{\sqrt{2}(\sqrt{2\alpha}t+b_3)}}. \tag{62}$$

Now $\frac{\sigma}{\theta}$ → constant as t→0 and $\frac{\sigma}{\theta}$ → 0 as t→∞.This result indicates that the model approaches to isotropy for large values of t.

4. Mesonic Perfect fluid coupled with Electromagnetic field

The energy momentum tensor for mesonic perfect fluid coupled with electromagnetic field is given by

$$T^i_j = T^i_{j(P)} + T^i_{j(s)} + E^i_{j\,mag}. \tag{63}$$

Where $T^i_{j(P)}$ and $E^i_{j\,mag}$ are defined in (32) and (9) respectively.

The energy momentum tensor for attractive scalar meson field is given by

$$T^i_{j(s)} = v^i v_j - \frac{1}{2}g^i_j(v_k\,v^k - m^2v^2) \tag{64}$$

together with

$$\sigma = g^{ij}v_{;ij} + m^2v, \tag{65}$$

Where m is the mass parameter and σ is the source density of the scalar meson field v. Here afterwards the suffix i and semicolon (;) after a field variable represent ordinary and covariant differentiations with respect to x^i and g_{ij} respectively.

4.1. Rosen's field equations

The Rosen's field equations (4) for the metric (5) and (6) with the help of (63) are found as

$$\left(\frac{s_4}{s}\right)_4 + 2h_{44} = -8\pi k\,\{2p - 2\zeta + e^{-2h}v_4^2 - m^2v^2\}, \tag{66}$$

$$\left(\frac{s_4}{s}\right)_4 + 2h_{44} = -8\pi k\,\{2p + 2\zeta + e^{-2h}v_4^2 - m^2v^2\}, \tag{67}$$

$$\left(\frac{s_4}{s}\right)_4 - 2h_{44} = 8\pi k\,\{2p + 2\zeta + e^{-2h}v_4^2 - m^2v^2\}, \tag{68}$$

$$\left(\frac{s_4}{s}\right)_4 + 2h_{44} = 8\pi k\,\{2\rho + 2\zeta + e^{-2h}v_4^2 + m^2v^2\} \tag{69}$$

And source density (65) yields

$$\sigma = e^{-2h}\{v_{44} + 2h_4v_4 + \frac{s_4v_4}{s}\} + m^2v. \tag{70}$$

4.2. Solution of the field equations

From equations (66) and (67), we get

$$\zeta = 0. \quad (71)$$

Hence there is no contribution to magnetic field and this result leads to the result found and studied by Mohanty and Sahoo [25].

5. Conclusion

The work reported in this paper can be considered as extension of to the work of Mohanty and Sahoo [25]. In view of recent interest of readers in electromagnetic field in free space we took an attempt to develop the idea of perfect fluid and cosmic string considering non-static plane symmetric cosmological model in bimetric theory of gravitation. The model obtained in case-1 is a geometric string model and in this model the universe starts from a constant volume and blows up at infinite future. Also the model has anisotropic in nature at the beginning and will continue throughout the evolution. The rate of expansion in the model gradually decreases as time increases and there will be no expansion at infinite time.

It is observed that there is no contribution to electromagnetic field in both second and third case but the vacuum model and the false vacuum model are established in second case. It is also seen that in case of false vacuum model the rate of expansion of the universe is increasing and the universe will collapse at infinite future.

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